Gas jet hitting a wall in a cross flow

British Gas

1 Introduction

This report concerns the problem brought to the 1995 Study Group by Dr. John Morgan of British Gas. The problem concerned the flow of a gas jet that was directed downwards towards a planar surface. A number of different people worked on the problem, and it emerged that there were a number of different ways of modelling the flow, each having varying degrees of complexity. This report therefore gives some basic details of the flow in question, and then examines three distinct modelling approaches in detail, namely a 'vortex model' where the main concern is the location of the horseshoe vortices in the wake behind the jet, a 'balance model' where some simple conservation arguments are used to determine the properties and upstream penetration of the jet and a two-dimensional model for the particle trajectories is proposed. Next, a 'dimensional analysis' model is considered where alternative approaches to the problem are examined. Finally some conclusions are discussed and some recommendations are made.

2 Details of the Physical Problem

A schematic drawing of the flow and coordinate system used is shown in figure (1). The jet is assumed to have originated from some 'adverse event' and, at its source is pure CH_4 (methane) having a density $\rho_m = 0.665$ kg/m^3. The jet issues into air (density $\rho_a = 1.205$ kg/m^3, all values calculated at an ambient temperature of 20 deg. C) vertically downwards from a tube of radius $a = 5.4$ mm at a speed of $U_j = 128$ m/sec. The mouth of the tube is at a distance of 138mm from an impermeable plane $z = 0$ and the cross-flow (of pure air) has a speed $u_e = 5$ m/sec in the positive x-direction. The kinematic viscosities of air and methane are taken to be equal, having the value of $\nu_m = \nu_a = 1.510^{-5}\text{m}^2/\text{sec}$.

The methane jet at the mouth of the tube has a Reynolds number (based on the tube radius) of 46000 and thus the jet is turbulent. As the jet spreads before hitting the plane, turbulent entrainment decreases the methane concentration. After striking the plane, the jet begins to spread radially, its direction influenced by the cross-flow. The main parameter of interest to British Gas is the position of the 8 per cent methane contour, since methane/air mixtures existing below this critical concentration do not burn. The 8 per cent contour may thus be thought of as the 'safety limit' in any hazardous adverse event.

A number of previous studies have considered related problems. Birch et al. (1988) made experimental measurements of concentration profiles in a turbulent methane jet using Raman spectrometry, but no cross-flow was present. Fairweather et al. (1990) developed a three-dimensional numerical model for a jet exhausting from a plane vertically upwards into a cross-flow. Using a k-ϵ model to simulate the turbulence in the flow, they achieved satisfactory agreement with experimental measurements. A simple integral model was also developed by Cleaver and Edwards (1990). The existence of the wall below the jet renders the problem very similar to the V/STOL (Vertical Short-Takeoff and Landing) problem, where the extent of the jet of a landing aircraft must be known (a) to avoid injuring troops that may be on the ground beneath the aircraft and (b) to avoid the problems that might arise if hot air is sucked into the engine air intakes. A large amount of literature exists, and the simple study of Skifstead (1969) is typical of the sort of basic model that has been used to predict jet trajectories. A numerical study of ground effects was undertaken by Bray and Knowles (1989). They used a large commercial code (PHOENICS) to make comparisons with experimental results, finding satisfactory agreement in some cases.

Figure (2) (originally provided by British Gas) shows some typical concentration profiles and gross flow features. One of the most noticeable features of the flow is the presence of 'horseshoe' vortices which curve around the front of the descending jet and travel downstream, entraining air. We therefore begin by considering a model for the position of these vortices.

3 A Vortex Model

Evidently the horseshoe vortices are an important mechanism for the dilution of methane in the flow. To predict the position of these vortices, we assume that the 'horseshoe vortex system' arises from a point source of vorticity positioned where the jet strikes the plane, and has tangent, normal and binormal $\boldsymbol{t}$, $\boldsymbol{n}$ and $\boldsymbol{b}$ respectively. Assuming that the horseshoe vortex lies close to the plane $z = 0$, we consider the total velocity $\boldsymbol{V}$ of the system to be given by

$$\boldsymbol{V} = u_e \boldsymbol{i} + \frac{\Gamma}{2h}\boldsymbol{n} + \Gamma\kappa \log\epsilon\, \boldsymbol{b} + v_w \boldsymbol{k}, \tag{1}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit coordinate vectors. The first term in this expression arises simply from the cross-flow, whilst the second represents the contribution from the horseshoe vortex image 'beneath' the plane at $z = -h$. The assumption that the contribution to the total velocity arising from the image may be expressed in this way is tantamount to a statement that h is small. The third term in (1) arises from an expansion of the Biot-Savart integral that sums the contributions to the total vortex motion from each part of the vortex (see, for example Batchelor (1985) p. 510). The local curvature κ is multiplied by a term $\log\epsilon$, which represents the core radius of a line vortex (in practice small, but non-zero). Finally, the fourth term in (1) arises from the fact that the spreading jet produces an 'effective vertical velocity' as it spreads out, invading the cross-flow.

For the horseshoe vortex system to remain at its current position, it must have a velocity in the tangential direction only, so that $\boldsymbol{V} \wedge \boldsymbol{t} = 0$. For small h, this implies that to leading order the image balances the cross-flow, so that $h = O(\Gamma/u_e)$ and, as well as providing a relationship between u and u_e, we find that the vortex is 'flat' to lowest order. To next order, we find that, upon taking the scalar product with $\boldsymbol{k}$ and assuming that $v_w = O(-h \log \epsilon)$, we have

$$-\Gamma \kappa \log \epsilon + v_w = 0.$$

This equation allows the curvature of the horseshoe vortex system to be calculated, provided that the velocity v_w is known, by solving the nonlinear differential equation

$$\Gamma \log \epsilon \frac{Y''}{(1 + Y'^2)^{3/2}} = v_w(x, Y(x))$$

for the function $Y(x)$ which represents the position of the horseshoe vortex system. [Thus if $v_w \equiv 0$, the vortex would be a straight line.] In practice, however, it seems that a much more readily available quantity would be the surface pressure on the plane $z = 0$. If this was known, then the fact that this pressure is related in a simple way to the x-derivative of the velocity potential ϕ would allow the z-derivative of ϕ (and thus v_w) to be calculated. In one dimension this gives a Hilbert transform, whose properties are well-known. In two dimensions the calculations required are more involved, but could, in principle, be carried out to yield the curvature and therefore eventually the height.

Once the position of the horseshoe vortex in the flow is known, the vorticial entrainment may be predicted using standard models. It would also be possible to estimate the entrainment into the layer by assuming (for example) that the layer entrainment was proportional to the width of the layer. In this way a complete model for the concentration of the gas mixture could be set up; it is somewhat doubtful, however, whether the experimental results that are required to propose this model would be easy to acquire.

Another important question regarding the flow upstream of the jet is the nature of the upstream spread-limiting mechanism. This caused much discussion during the week and two separate schools of thought emerged. Firstly, it was thought by some that the vortex sheet produced by the downward jet rolls up under its own influence, and this essentially determines the upstream distance to which the jet can penetrate. One problem with this hypothesis is that this would suggest that a similar instability might be present even *without* a cross-flow, and this seems somewhat unlikely.

Secondly, an alternative assumption was considered where the jet strikes the plane $z = 0$, spreads radially, and is simply retarded upstream by entrainment and competition with the cross-flow. Just before the cross-flow dominates and the jet turns back downstream, there will undoubtedly be a region where the jet becomes diffuse and may resemble a plume. After some discussions, however, it was decided to ignore this region and see if some simple predictions could be made using the assumption that the jet turns downstream when its speed has been reduced to that of the cross-flow.

4 A Simple Balance Law Model for the Flow

First, we consider the region between the mouth of the jet nozzle and the plane $z = 0$. There are a number of ways in which the calculation of the concentration, jet radius and velocity in this region may be undertaken, but the simplest assumes the 'ten degree' law, which states that, according to experimental observation, a turbulent jet spreads by a semi-angle of $\theta_s = 10$ degrees. At the jet nozzle the concentration flux $F = \rho Acu$ is given by $F = \rho_m \pi a^2 u_j = 7.798 \times 10^{-3}$ kg/sec whilst the momentum flux $M = \rho_m \pi a^2 u_j^2 = 0.998$ kg m/sec^2. At a distance z from the jet nozzle the jet radius is $z \tan\theta_s$, and we have

$$\rho = \rho_a(1-c) + \rho_m c$$

$$(\rho_a(1-c) + \rho_m c)Au^2 = M, \qquad (\rho_a(1-c) + \rho_m c)AMc^2 = F^2.$$

Thus the concentration c is given by

$$c^2(\rho_a(1-c) + \rho_m c) = \frac{F^2}{M\pi z^2 \tan^2\theta_s}.$$

The distance from the jet nozzle to the plane is 138 mm, but we expect the radially spreading jet on the plane to have a height of around 20-30 mm. (This value could be altered if required.) At a distance $z = 110$mm below the jet nozzle we therefore have

$$c = 0.218, \quad u = 27.90\,\mathrm{m/sec}, \quad A = 1.182 \times 10^{-3}\,\mathrm{m}^2,$$

whilst with $z = 138$ mm we find that

$$c = 0.172, \quad u = 22.01\,\mathrm{m/sec}, \quad A = 1.860 \times 10^{-3}\,\mathrm{m}^2.$$

These results are significant as they show that, even by the time the jet has struck the plane $z = 0$, turbulent entrainment has forced the methane concentration to drop by a factor of around 5.

An independent check of the results given above may be carried out by assuming conservation of momentum and concentration flux as before, but modelling the turbulent entrainment using the law

$$\frac{d}{dz}(\rho Au) = 2\pi a \rho_a E \mid u_e - u \mid \tag{2}$$

rather than by assuming the (somewhat contrived) 'ten degree' law. In (2), u_e is the external flow velocity (zero for the downward jet), a is the jet radius and E is a dimensionless entrainment coefficient which has been found experimentally to be approximately 0.1 in a number of different flows (see Turner 1973, page 173). Using (2) together with the previous definitions of F and M, we find that the concentration at a distance z below the jet nozzle is given by

$$\frac{d}{dz}\left(\frac{1}{c}\right) = \frac{2\rho_a E}{F}\sqrt{\frac{\pi M}{\rho}}$$

and thus

$$-\frac{\sqrt{1-\alpha c}}{c}-\frac{\alpha}{2}\log\left(\frac{2-\alpha c-2\sqrt{1-\alpha c}}{\alpha c}\right)=-2\sqrt{\pi M\rho_a}\frac{Ez}{F}+K$$

where

$$\alpha=1-\frac{\rho_m}{\rho_a},\qquad K=-\sqrt{1-\alpha}-\frac{\alpha}{2}\log\left(\frac{2-\alpha-2\sqrt{1-\alpha}}{\alpha}\right).$$

This gives $c = 0.144$ at $z = 110$mm, a lower value than the 'ten degree' rule, and also yields $u = 18.43$m/sec, $A = 2.607 \times 10^{-3}\,\text{m}^2$.

A simple calculation explains the difference between these results, for if we use the entrainment model with $\rho_a = \rho_m$, we find that $\alpha = 0$ and c is given by

$$c=\frac{F}{F+2\sqrt{\pi M\rho_a}Ez},$$

whilst the 'ten degree' rule gives

$$c=\frac{F}{\sqrt{\rho_a M\pi}z\tan\theta_s}.$$

Comparing these two results for large z shows that $\theta_s = 10$ degrees gives $E \sim 0.09$, confirming that the 'ten degree' rule is appropriate only for single density flows.

An estimate of the upstream penetration of the jet may now be made. Assuming that when the jet hits the plane we have $c_0 = 0.144$, $u_0 = 18.43$ m/sec and $r_0 = 2.881 \times 10^{-2}$m (as calculated above), we assume that although momentum is destroyed by the plane, energy and mass are conserved. Thence the quantities ρA and u must both be conserved. The jet spreads out radially, and its area just before hitting the plane is πr_0^2. Just after contact, area is therefore preserved if the spreading layer has a height $h_0 = r_0/2$. We therefore conserve the quantities

$$M=\rho u^2h2\pi r,\qquad F=\rho uch2\pi r \tag{3}$$

and model the entrainment by

$$\frac{d}{dr}(2\pi r\rho uh)=E\rho_a 2\pi r\mid u-u_e\mid. \tag{4}$$

As a simple first approximation, we take $u_e = 0$. Some simple calculations now show that

$$\frac{1}{c^2}-\frac{1}{c_0^2}=\frac{2E\rho_a M\pi}{F^2}(r^2-r_0^2).$$

Assuming that the jet is effectively halted when u drops to 5 m/sec, we find that c at this point has the value 0.039, so that $r = 0.223$m and the jet penetrates 223mm upstream. Naturally this is an over estimate, since the cross-flow has been neglected for entrainment purposes. Conversely, an underestimate of the penetration distance may be calculated by assuming that the entrainment velocity is given by $\boldsymbol{u}_e = -u_e\boldsymbol{e}_r$. Using (3) and (4), we find that

$$\frac{1}{u_e}\left(\frac{1}{c_0}-\frac{1}{c}\right)+\frac{M}{Fu_e^2}\log\left(\frac{c_0(cM+Fu_e)}{c(c_0M+Fu_e)}\right)=\frac{\pi E\rho_a}{F}(r_0^2-r^2),$$

giving a penetration distance of 173mm. Experimental studies carried out by British Gas have indicated penetration distances of approximately 170mm, which is well in line with the two estimates produced above; evidently further models could be proposed that also included the effects of momentum entrainment from the cross-flow.

Finally, a two-dimensional model was proposed to predict the spread and concentration in the jet. Assuming that u and v are the velocities in the x and y directions respectively, and that both mass and momentum are entrained from the cross-flow, the obvious conservation laws give

$$\begin{aligned}(uh)_x + (vh)_y &= E \mid \boldsymbol{u} - \boldsymbol{u}_e \mid \\ (u^2h)_x + (uvh)_y &= Eu_e \mid \boldsymbol{u} - \boldsymbol{u}_e \mid \\ (uvh)_x + (v^2h)_y &= 0.\end{aligned} \tag{5}$$

These equations may be thought of as two-dimensional shallow water theory with no gravity, and must be solved subject to known conditions (see above) at the origin where the jet strikes the plane $z = 0$. At present the equations (5) do not include concentration conservation, and so also density changes; once their structure has been understood, however, concentration and density effects may easily be added. The equations have real (though not distinct) characteristics, and may be integrated numerically to determine particle paths: this was attempted at the meeting but there was not sufficient time available to produce a working code.

5 Dimensional Analysis Model

A separate analysis of the problem, based largely upon dimensional arguments, is also possible. Assuming simply that the maximum upstream penetration distance ℓ_m can depend only upon the the cross-flow speed u_e (of dimensions m/sec), the volume flux Q (of dimensions m^3/sec) and the specific momentum flux P (of dimensions m^4/sec^2), then the obvious characteristic length is given by

$$\ell_u = \frac{P^{1/2}}{u_e},$$

under the assumption that Q is not important. Assuming therefore that $\ell_m = a_0 \ell_u$ where a_0 is an order one constant, we find that, according to British Gas data $a_0 \sim 0.694$, giving a simple scaling law for the penetration distance. If more accurate results were required, then it would also be important to include density ratio effects.

A simple model for the spread of the jet may also be proposed using dimensional arguments. Figure (3) shows the nomenclature used; r denotes the maximum extent of the jet in the θ direction whilst t represents the transit time for a particle in the jet to reach a given location. The main assumption in the model is that each direction is similar to the upstream direction, except that the opposing velocity is $u_e \cos\phi$. Assuming that both R and t are functions of $u_e \cos\phi$ and P, we find that

$$X = u_e t - R\cos\phi$$

$$Y = R\sin\phi$$

and thus

$$X = \ell_u(c_2\sec^2\phi - c_1)$$
$$Y = \ell_u c_1 \tan\phi$$

where c_1 and c_2 are constant. Finally, therefore, we have

$$\frac{X}{\ell_u} = b_0\left(\frac{Y}{\ell_u}\right)^2 - a_0.$$

Coefficients derived from British Gas raw data indicates that a good approximation to the constants is furnished by $a_0 = 0.694$ (as before) and $b_0 = 0.871$. The bounding shape of the jet is thus a parabola, with scaling law as given above.

Studies based on dimensional arguments were considered by Linden & Simpson (1994) and List (1979). Such models also propose an alternative explanation for the vortex structure that is seen at the edge of the spreading jet. Regarding each part of the jet as a 'component jet', each of which is subject to a cross-flow $u_e\sin\theta$ and an opposing flow $u_e\cos\theta$, the lateral extent of the jet is determined by the envelope of the particle paths. At points on the edge of the jet, the 'rays' converge and this requires that fluid is pushed vertically upwards before it can travel downstream. A vortex is thus produced at the envelope of the particle paths. It is hard to quantify this effect at present; undoubtedly some numerical solutions of the equations (5) would allow some firmer conclusions to be drawn.

6 Conclusions and Recommendations

Any study of this problem must acknowledge that this is a three-dimensional turbulent flow, and it is unlikely that any simple models will give all the required details of the flow. Recognizing that safety is the primary reason for studying the flow, it is probable that a combination of the methods given above will be required if realistic predictions are to be made. In many circumstances such as those pertaining here, political expediency demands that a study of the flow is undertaken using a commercial code such as PHOENICS or FLOW3D, in order that some wholly numerical data may be obtained. Safety contours may then be proposed by multiplying the relevant numbers by suitably chosen 'safety factors'. If calculations of this sort are undertaken, it should be stressed that, in spite of the 'good agreement with experiment' that has been reported in the literature, experience shows that the choice of turbulence model is crucial and different codes may give very different results. It is therefore strongly recommended that a number of computations are undertaken if commercial codes are to be used, and that these should be used in parallel with some of the methods described above.

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References

Batchelor, G.K. (1985) An Introduction to Fluid Dynamics, Cambridge University Press.

Birch, A.D., Brown, D.R., Dodson, M.G. & Thomas, J.R. (1978) The turbulent concentration field of a methane jet, J. Fluid Mech. **88** 431

Bray, D. & Knowles, K. (1989) Numerical modelling of jets in cross-flow and ground effect. Royal Military College of Science, Shrivenham, Aeromechanical Systems Group, rept. no. TN SMMCE/AeS/0305/1

Cleaver, R.P. & Edwards, P.D. (1990) Comparison of an integral model for predicting the dispersion of a turbulent jet in a crossflow with experimental data. J. Loss Prev. Proc. Ind. **3**(1) 91-95.

Fairweather, M., Jones, W.P. & Marquis, A.J. (1990) The concentration field of a jet in a cross-wind. Comb. Sci. & Tech. **73** 463-478.

Linden, P.F. & Simpson, J.E. (1994) Continuous releases of dense fluid from an elevated point source in a cross-flow. In: Mixing and Transport in the Environment, eds. K.J. Beven, P.C. Chatwin and J.H. Millbank. John Wiley & Sons. (See especially Chapter 21 pp. 401-418).

List, E.J. (1979) Turbulent Jets and Plumes. In: Mixing in Inland and Coastal Waters, eds. H.G. Fisher, E.J. List, R.C.Y. Koh, J. Imberger & N.H. Brooks, Academic Press. (See especially Chapter 9 pp. 315-389).

Skifstead, J.G. (1969) Numerical treatment of line singularities for modeling a jet in a low speed cross flow. In: Analysis of a jet in a subsonic crosswind, NASA SP-218

Turner, J.S. (1973) Buoyancy effects in fluids. CUP.

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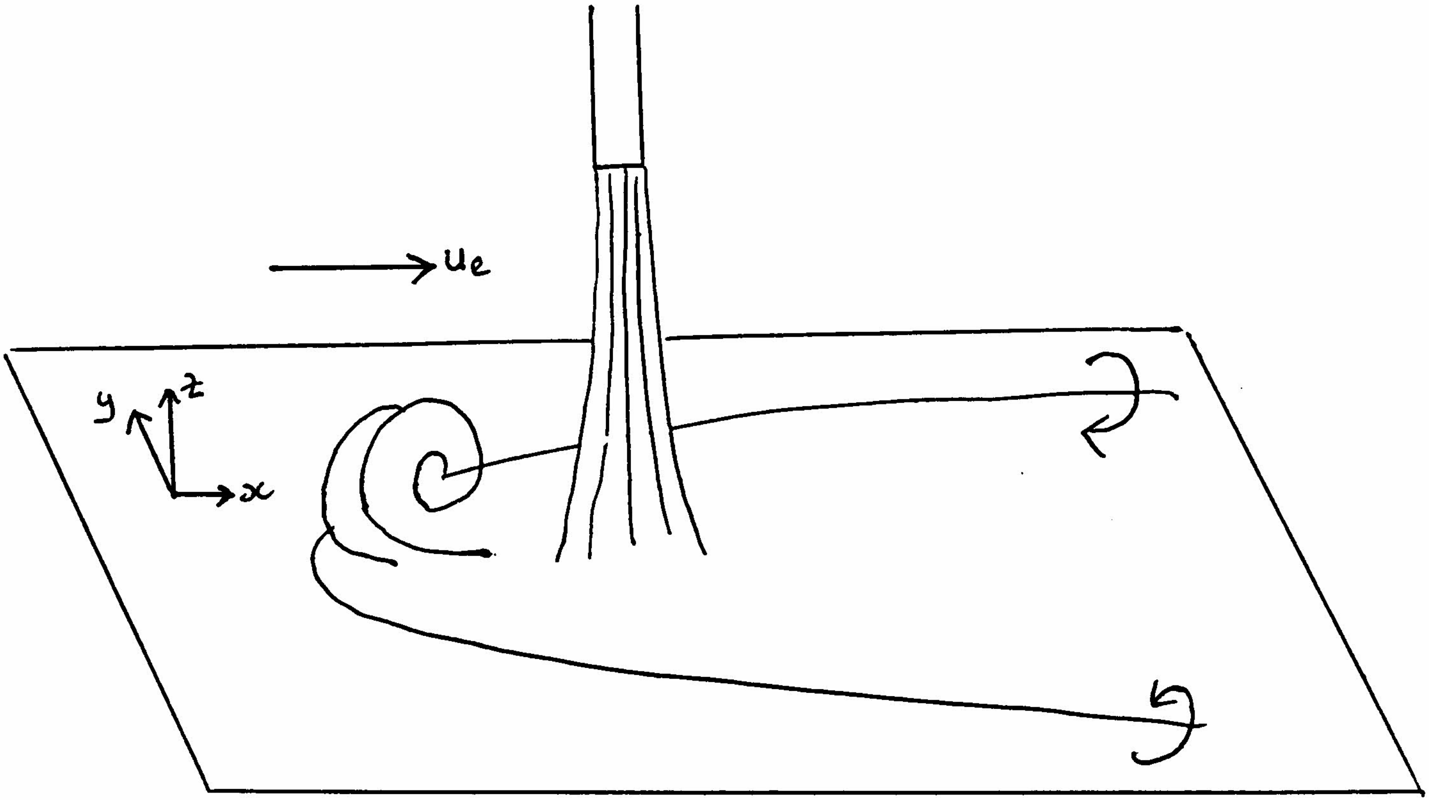
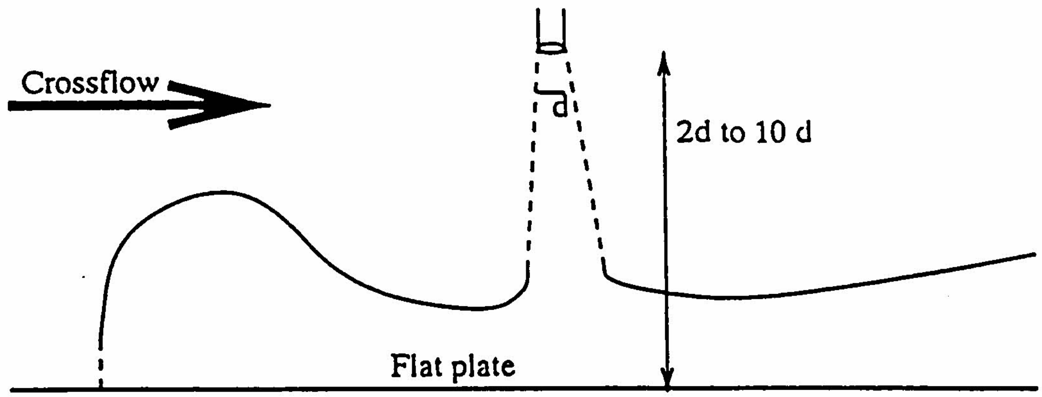
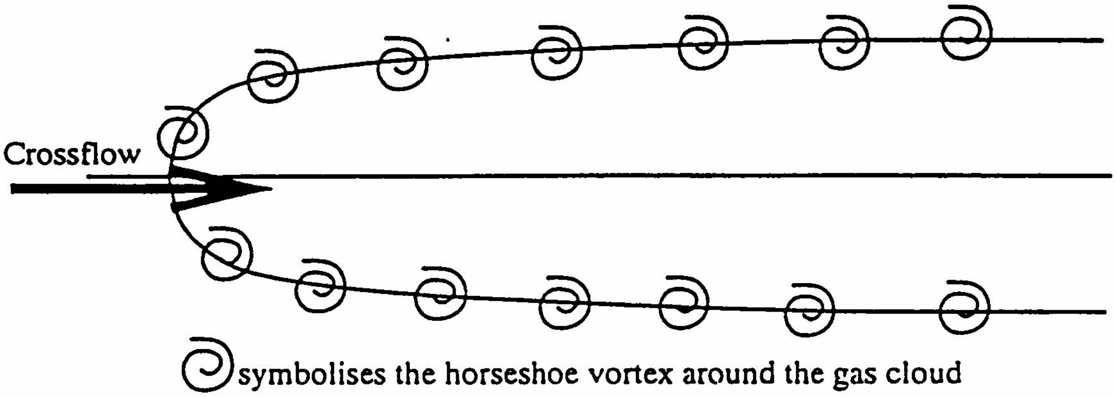


FIGURE (1) **Schematic diagram of jet flow**



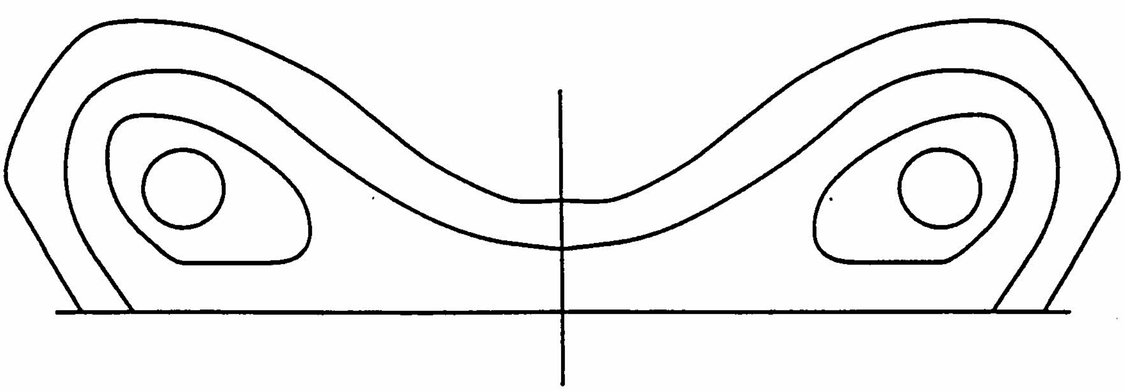
SIDE VIEW ALONG THE AXIS OF SYMMETRY SHOWING THE EXTENT OF THE GAS CLOUD



symbolises the horseshoe vortex around the gas cloud

PLAN VIEW OF THE GAS CLOUD

Sketches of the geometry of the gas cloud.



A sketch of gas concentration curves across the cloud, below the point of jet impaction.

FIGURE (2) : Observations of gross flow features from British Gas experimental data

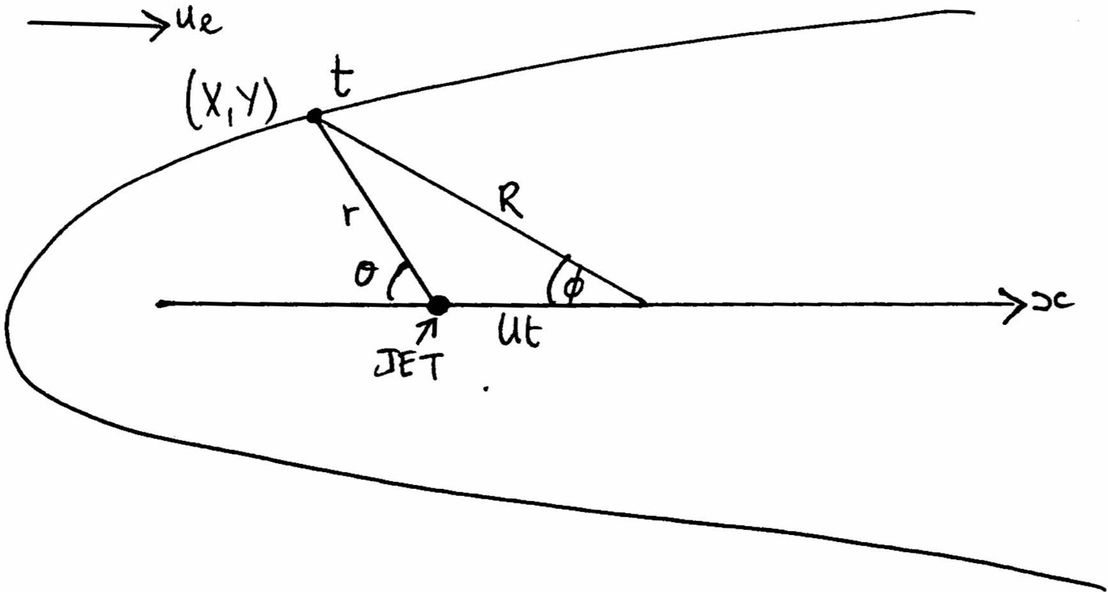


FIGURE (3) : Nomenclature for dimensional analysis jet extent calculation.