MODELLING AND OPTIMAL CONTROL OF PLATE EVAPORATORS

Evaporators are an important stage in the extraction of sugar from sugar cane. A model of a simple evaporator is developed and then extended to multiple stage evaporation. An approximate solution and an iterative solution to the equations are developed. From the properties of these models a control strategy is developed.

1. Introduction

In the extraction of sugar from sugar cane, the cane is first crushed to produce a sugar solution (known as juice) from which water is evaporated to produce a concentrated solution. Sugar is then crystallised from the concentrated solution. The evaporation stage is a major energy consumer in the production of sugar. CSR sugar mills use multiple stage (known as multiple effect) evaporators with up to 6 units arranged in series. These concentrate the juice from about 15% sugar to 70% sugar which is then sent to the crystallisers.

Most plants are currently using evaporators of the Roberts type (figure 1). These evaporators involve a vessel containing a section with vertical tubes that are held between tube plates with liquid inside the tubes and steam on the outside of the tubes. Below the tubes is a reservoir for juice that supplies the inside of the tubes. The Roberts type evaporators have relatively long residence times (several minutes) for the juice, however steam entering the unit produces output steam with a very minimal delay. The juice boils as long as steam is fed into the unit and they are relatively insensitive to variations in flow of the juice. Level controls aim to maintain a constant level and hence constant volume of juice in the unit.

Sugar mills are now installing a new type of more efficient evaporator, the plate evaporator (figure 1), in combination with the Roberts evaporators to increase capacity. The newer plate evaporators consist of multiple parallel plates with the steam on one side and the boiling juice on the other. There is no large volume of juice so the operation of plate evaporators is almost instantaneous making these units sensitive to short term changes in the feed flow rates.

An evaporator set consists of several evaporator units arranged in series. The first vessel in the evaporator set is supplied with juice and low pressure steam (typically around 100kPa) and operates under pressure. Vaporisation of the juice is achieved through heat exchange processes driven by the condensation of the steam. Vapour and liquid flow co-currently down the set with the vapour
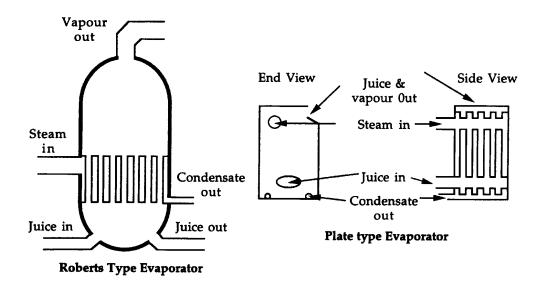
Figure 1: Schematic diagram of Roberts and plate type evaporators.

generated in one vessel providing the energy, through condensation, to drive the evaporation of juice in the next vessel. Operating pressures and temperatures decrease down the set and the final vessel operates under a vacuum (typically 90kPa less than atmospheric). Vapour from the final vessel is condensed in a direct contact (with water) condenser. This evaporator arrangement has two benefits; energy efficiency (as 1kg of steam evaporates approximately 4kg of water in a quadruple set) and concentrated juice is boiled at the lowest temperature to prevent caramellization of sugar.

The difference in responses of the two types of units of evaporators has made plants containing both types difficult to control. In particular with the existing control system the time required to stabilise the plant after disturbances (which do occur regularly) is significantly longer in plants containing both types of units.

The problem posed by CSR was to construct a mathematical model of a multiple effect evaporator set that predicts the steam flows, juice flows, and sugar concentrations within the unit from the initial flows, concentration, and specified end conditions, and to use the model in the development of a control strategy for a mixed set of plate and Roberts evaporators.

2. An evaporator model

A multiple stage (effect) evaporator consists of coupled components with the condensation of steam in one stage providing the energy to boil juice and thus produce steam for the next stage.

For simplicity we assume that all the components of a particular type (steam, or juice) in a particular evaporator stage have identical operating conditions so that one can model each stage as a two compartment vessel shown schematically in figure 2, with a single heat exchange interface separating the steam and juice compartments.

In the following we use the notation (see figure 2):

c concentration of dissolved solids in the in-flowing juice,
c' concentration of solids in the out-flowing juice,
F mass flow of juice into the juice side of the vessel,
F' mass flow of juice from the juice side of the vessel,
P pressure in the steam side of the vessel,
P' pressure in the juice side of the vessel,
P_S pressure of steam source,
S mass flow of steam into the steam side of the vessel,
S' mass flow of vapour from the juice side of the vessel,
T temperature of the steam side of the vessel,
T' temperature of the juice (and steam) in the juice side of the vessel,
T_J temperature of the juice entering the stage,
T_S temperature of the steam entering the stage.
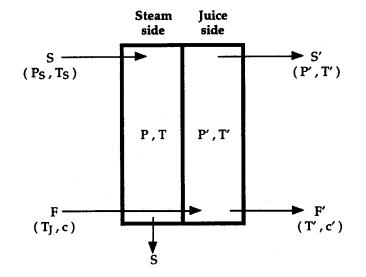

Figure 2: Two compartment model of an evaporator vessel.

We note that by conservation of mass in steady state operation, the mass flow of the condensate from the steam compartment must equal S.

The flow of steam is governed by the pressure difference between the pressure P_S of the steam source and the pressure P in the first steam compartment through the pressure drop relationship:

$$S = k\sqrt{(P_S - P)} \tag{1}$$

where k is determined by the control valve setting.

The thermodynamic properties of the system are governed by the pressure and temperature of the steam and juice compartments denoted respectively by P, T and P', T'.

Then if we assume that the system is in a state of thermodynamic equilibrium, we require T to be the boiling point of water at pressure P, and P' to be the vapour pressure of juice at temperature T' and concentration c'.

In addition if we assume that normal heat transfer conditions apply, the heat flux between the steam and juice compartments is given by:

$$h = UA(T - T') \tag{2}$$

where U is the heat transfer coefficient and A is the effective heat transfer area.

The above assumptions are approximations. For example, boiling and bubbling of juice results in the heat transfer coefficient changing as the fraction of bubbles changes, and the temperature of the vapour leaving the vessel may not be the same as the temperature of the juice leaving the vessel. Nevertheless the model is considered a good first approximation over the range of operation of interest.

It is not difficult to write down a set of evolution equations that describe the dynamics of the system in terms of the volume contents within the system (i.e. change of contents equals in-flow minus out-flow). In practice however, the rates of accumulation of juice (as the volume of juice is held constant), steam and heat are small compared with the corresponding fluxes, so that the natural time constants of the system are relatively small. It follows that the dynamic response of the system is demand driven by the supply tank fluctuations and the control system response to these changes. Process charts show typical fluctuations that are much longer than the process time constants and also indicate that the fluctuations originate from the feed.

In the remainder of this article we assume that steady state conditions apply. Equations connecting the system variables are thus derived from the heat and mass balance conditions, and from the thermodynamic relations described in the following section. It should be noted that these equations apply to both the Roberts and plate type evaporators.

3. Heat, mass and thermodynamic balances

In the following we use the additional notation:

$J(T, c)$ the heat content (or more correctly enthalpy) of juice at temperature T and concentration c,

$V(T)$ the heat content (enthalpy) of vapour (i.e. steam) at temperature T.

The heat balance in the steam compartment then requires that:

$$\{V(T_S) - J(T, 0)\}S = h \tag{3}$$

where S is defined in Section 2, and h, given by equation (2), is the heat flux between the two compartments. For simplicity the heat loss in the steam compartment is assumed to be included in the overall unit heat loss term included in the next equation.

Similarly in the juice compartment:

$$J(T_J, c)F - J(T', c')F' - V(T')S' = -h(1 - u) \tag{4}$$

where we have assumed a heat loss term (hu) proportional to the heat flux.

The mass balance requires that the mass flow of the juice into a vessel is equal to the sum of the juice and vapour out of the vessel:

$$F = F' + S'. \tag{5}$$

Similarly conservation of the mass of the dissolved solids requires:

$$Fc = F'c'. \tag{6}$$

As mentioned above there are thermodynamic (balance) relations between pressure, temperature, and concentration:

$$P' = P_v(T', c') \tag{7}$$

where $P_v(T', c')$ denotes the vapour pressure of the juice at temperature T' and concentration c'. A second relation derivable by inverting the above at $c' = 0$ is:

$$T = T_b(P) \tag{8}$$

where $T_b(P)$ denotes the boiling point of water at pressure P.

4. Linearised model

The equations in the previous section can be simplified somewhat by expanding $V(T_S)$ in (3) around T:

$$V(T_S) \approx V(T) + (T_S - T)\frac{dV}{dT}(T). \tag{9}$$

Normally the last term in (9) is negligible compared with the latent heat defined by:

$$L(T, 0) = V(T) - J(T, 0) \tag{10}$$

where $L(T, c)$ is the latent heat of vapourisation of juice at temperature T and concentration c. It follows from (3), (9) and (10) that:

$$h \approx V(T, 0)S. \tag{11}$$

Similarly, if we assume that the difference between the heat content of the incoming and out-flowing juice is small compared with the latent heat of vaporisation then

$$L(T', c') \approx V(T') - J(T', c') \tag{12}$$

and the heat balance condition (4), using the mass balance condition (5), can be approximated by:

$$h(1 - u) = \{V(T') - J(T', c')\}S' + \{J(T', c') - J(T_J, c)\}F \approx L(T', c')S'. \tag{13}$$

An additional constant term can be calculated and added to this equation to correct for the approximations made here and in equation (11). The latent heat of the juice is included in the iteration given in the appendix in this manner.

Combining (11) and (13) we then obtain:

$$S' = \{L(T, 0)/L(T', c')\}(1 - u)S. \tag{14}$$

Again to a good first approximation the latent heat of condensation is equal to the latent heat of vaporisation, giving from (14):

$$S' \approx (1 - u)S. \tag{15}$$

It should be noted that in our simplification of the evaporator model the flow and concentration variables S, F, c and S', F', c' are simply related to one another through (5), (6) and (15) and thus do not depend on the thermodynamic variables P, T and P', T' in this approximation.

As we show in the following section a set of evaporator vessels arranged in series can be effectively modelled by repetitive application of the equations derived by simplifying our basic model.

5. Simplified model of the complete evaporator set

If we have a set of N evaporator vessels arranged in series (the normal configuration) as shown in figure 3, with output flows S_n, F_n and output concentration c_n from evaporator n providing input flows and concentration to evaporator $n+1$, equations (5), (6) and (15) can be used to derive the equation scheme for the set:

$$S_{n+1} = (1 - u)S_n, \quad F_{n+1} = F_n - S_{n+1}, \quad c_{n+1} = F_n c_n / F_{n+1} \qquad (16)$$

for $n = 0$ to N, where S_0, F_0 and c_0 denote the flows and concentration entering the first vessel. Equations (16) can be solved explicitly yielding:

$$S_n = (1 - u)^n S_0, \quad F_n = F_0 - g_n S_0, \quad c_n = F_0 c_0 / (F_0 - g_n S_0) \qquad (17)$$

for $n = 0$ to N, where:

$$g_n = (1/u - 1)\{1 - (1 - u)^n\} \qquad (18)$$

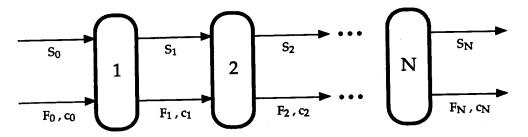is given explicitly in terms of the heat loss fraction u.

Figure 3: Notation for the complete evaporator set.

This allows us to set the input conditions S_0, F_0, c_0 and use (17) to compute the discharge flows and concentrations for each vessel making up the set.

In practice there are product requirements, for example the final concentration should be close to 0.7 . We discuss these conditions and other requirements in the next section.

6. Practical considerations

Since the final concentration is fixed by the product requirements and the initial concentration is known (typically about 0.15) the last of equations (17) can be inverted to give:

$$S_0/F_0 = (1 - c_0/c_N)(1/u - 1)(1 - (1 - u)^N) \qquad (19)$$

as the required initial ratio of steam and juice flows in terms of c_0/c_N and the loss fraction u.

Another important end condition derives from the fact that the vapour outlet at the end of the set is condensed while in contact with a reservoir maintained at a fixed temperature. In general from (2) and (11) we see that if T_n is defined to be the temperature of the vapour leaving the vessel n, then to a first approximation:

$$L(T_n, 0)S_n = UA(T_n - T_{n+1}).\tag{20}$$

Assuming a linear relationship:

$$L(T, 0) \approx a + bT \tag{21}$$

for the latent heat (10), (20) can be solved for for T_n in terms of T_{n+1} and S_n, yielding:

$$T_n = (aS_n + UAT_{n+1})/(UA - bS_n) \tag{22}$$

where:
$$S_n = (1 - u)^n S_0.$$

If T_N and UA are known, equation (22) can be evaluated backwards (i.e. starting with the last stage $n = N$) to determine the initial temperature T_0 and hence the initial steam pressure needed to drive the set. Note that T_N is the final stage output temperature, and UA may vary from vessel to vessel in the set, particularly for a mixed set of Roberts and plate evaporators. Estimates of the temperature allow more accurate calculation of the heat transfers and thus the calculation of more accurate flow, concentration and temperature values. This can be iterated as in the spreadsheet in the appendix to obtain an exact solution of the equations.

Alternatively one can use the stepwise calculation backwards to derive a relation between T_0, S_0 and T_N which could be used in conjunction with (19) to set the required initial conditions.

In addition to the above considerations there are also fluid dynamic relations derived from (1) and thermodynamic requirements from (7) and (8) that need to be checked for consistency with physical constraints as the flow proceeds from one vessel to the next.

7. Control policies

In a mixed set of Roberts and plate evaporators it is necessary to supply a consistent ratio of juice and steam to the plate evaporators, or their operation

suffers considerably, possibly to the extent that no steam for the next stage is produced. The Roberts evaporators are able to tolerate flow variations, and the normal control is to maintain a constant level in the unit, which is done at the expense of varying flows between the units. This however creates unsatisfactory flow conditions for any plate evaporator stages.

As the basic dynamics of each evaporator is very fast (being determined by steam flows) and the modelling shows that in steady state the process stream flows maintain nearly constant ratios, the control of a mixed Roberts and plate evaporator set needs to maintain the correct relation between the inter unit flows and avoid excursions from steady state behaviour. This can be done by ratioing the juice flows to the preceding juice flow, with the juice feed being ratioed to the steam input (see figure 4). The levels in the evaporators can then be maintained by adjusting the ratio of the discharge juice stream flow. The ratio of juice feed to steam is adjusted, as in current practice, to ensure the required final juice concentration.
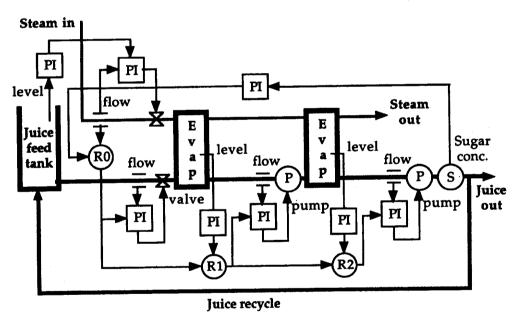

Figure 4: Proposed control loops for two stage evaporator.

S — sugar concentration measurement, PI — proportional + integral controller, P — pump, R0 R1 R2 — ratios for flows set by controllers.
Notes:
Level control applies to Roberts internal level and to plate discharge tank level.
Level controllers should have at most a small integral component.
Juice feed level control could have a dead band in the middle tank levels.
Controllers setting ratios should have output limited to a sensible range.
This scheme extends to more stages by duplicating the middle section.

A recycle of juice from the final stage to the feed tank or the feed of an earlier stage is needed so that sugar concentrations below that needed are not produced during start up, and possibly during major disturbances. However the aim should be to tune the control system so that it tolerates disturbances without the production of out of specification product.

This proposed control scheme (given schematically in figure 4) places less emphasis on maintaining precise level control in the Roberts evaporators. However it is believed that more than adequate level control is possible as the feed forward adjustment of flows will result in less disturbance to the levels. As the level in the Roberts evaporators is set to an optimum value for efficient operation, minor excursions from the optimum will result in very little loss in efficiency. This very minor loss (if any) is the price necessary to obtain major gains in the operation of the associated plate evaporators.

Two further improvements to the proposed control system are possible. Firstly instead of simply ratioing the flows a more complex scheduling of the flows based on model predictions could be used. It would first be necessary to adequately verify the model using plant data.

The second possible improvement relates to maintaining optimum conditions in the plate evaporators (it is known that 30% level in the tubes provides near optimum performance in the Roberts evaporators and this can be set via the level set point). Conditions in the plate evaporators are set by the feed rates of steam and juice and the plant configuration (namely the number of stages in the plant and the number of plates used). The ratio of steam to juice is set by the product requirements leaving only the feed rate as a possible adjustment. An experimental program is needed to determine the optimum conditions for the plate evaporators (the equations developed indicate that the performance should be close to optimum over a wide range of conditions), then the plant configuration needs to be set so that the optimum performance is attained at the required flow rate. Finally, the juice feed tank level control can be altered so that levels over a region in the middle of the tank always give the flow rate corresponding to optimum conditions. This has the effect of using part of the feed tank volume to smooth the feed flow, while still ensuring that, when necessary, the tank outflow is adjusted to match the incoming flow.

Physical or financial constraints may limit the full implementation of this control scheme. However in such cases, a reduced version may still give useful benefits. For instance, depending or the type of pump used, the inter stage pump power may be useable as an alternative to the flow meters.

8. Conclusions

A model of the evaporator set using mass, heat and thermodynamic balances has been developed. The major time constants are determined by the steam flow and are very short. Hence it is only necessary to consider steady state for the initial control design.

A control scheme based on maintaining the juice flows in the appropriate ratios is proposed. The ratios are adjusted to maintain levels and the required final sugar concentration. Possible extensions of the control scheme which will need a more detailed knowledge of plant operation are proposed.

Acknowledgements

Appendix: Sample spreadsheet calculation

This spreadsheet calculates the flows, temperatures, and pressures in a five stage evaporator. The steam input, fraction solids (sugar) in the feed juice and product juice, the final vacuum level, and the initial juice temperature are given by the user. This then specifies all the remaining values. The specific heat of the juice has been included as this makes a small adjustment to the values, however the specific heat of steam has not been included.

In some cases the initial steam temperature and pressure may become too high in which case the input steam flow needs to be reduced. Eight stages of iteration are used to calculate the final values, however the initial iteration shown below can be seen to give a reasonable approximation to the final values in rows 1 to 13.

Details of the spreadsheet numeric values and the formulas used (adapted from *Heat transfer performance in evaporators, Part I Theory and mechanisms*, Sugar Research Institute, Mackay, Internal report No 191) follow.

	A	B	C	D	E	F	G	H
1		Units	Input	Out 1	Out 2	Out 3	Out 4	Out 5
2	Steam Bleed	tph		35.00	0.00	0.00	0.00	0.00
3	Steam	tph	90.00	46.88	45.85	44.53	43.00	42.22
4	Fr. Solids	Fr.	0.15	0.20	0.25	0.32	0.44	0.70
5	Juice	tph	327.71	245.83	199.98	155.44	112.45	70.22
6								
7	U	W/m2/C		3000	2000	2000	2000	1500
8	Area	m2		3000	2000	2000	2000	1500
9	Efficiency	-		0.95	0.95	0.95	0.95	0.95
10								
11	Juice temperature	C	90	98.49	94.52	90.69	86.81	73.22
12	Steam temperature	C	100.18	98.31	94.17	89.90	84.29	49.33
13	Pressure	kPa	101.98	95.39	82.05	69.92	56.31	12
14								
15								
16	Ratio	-		0.95	0.95	0.95	0.95	0.95
17	Const	-		0.00	0.00	0.00	0.00	0.00
18	Iteration 1							
19	Steam	tph	90.00	50.50	47.97	45.58	43.30	41.13
20	Fr. Solids	Fr.	0.15	0.20	0.25	0.32	0.45	0.70
21	Juice	tph	335.34	249.84	201.86	156.29	112.99	71.86
22	Juice temperature	C	90	100.82	95.83	91.36	87.12	73.22
23	DTb	(C)		0.43	0.61	0.94	1.72	5.69
24	Dtbt	C		0.18	0.37	0.84	2.71	23.89
25	U*A*ef/Si			342000	270891	285148	300156	177724
26	Steam temperature	C	102.52	100.64	95.46	90.52	84.41	49.33
27	Pressure	kPa	110.76	103.67	86.03	71.59	56.57	12
28	dHSteamOut/kg	kJ/kg	2459	2460	2463	2466	2470	2485
29	dHJuice/kg	kJ/kg		-46	21	19	18	59
30	New Ratio	-		0.95	0.95	0.95	0.95	0.94
31	New Const	-		-4.64	1.73	1.19	0.82	1.70

D2 to H2 Input values of steam bled from output of each stage.

C3 Input tph of steam entering the circuit.
D3=D117 Calculated steam flows out of units after iteration.
E3 to H3 Copy formula D3 across.

C4 Input feed juice fraction solids, i.e. fraction dissolved sugar.
D4=$C5*$C4/D5 Fraction solids in output, solids input to stage over juice output.
E4 to G4 Copy formula D4 across.
H4 Input required final solids (i.e. sugar) fraction.

C5=(D$2+E$2+F$2+G$2+H$2+D3+E3+F3+G3+H3)*H4/(H4-C4)
    Juice feed from total steam produced and initial and final fractions solids.
D5=C5-D$2-D3 Juice tph from input juice tph minus steam produced in stage.

E5 to H5 Copy formula D5 across.

D7 to H7 Input heat transfer coefficient for each stage.
D8 to H8 Input heat transfer surface for each stage.
D9 to H9 Input efficiency fraction for each stage.

C11 Initial temperature of the juice.
D11=D120 Calculated juice temperature out of each stage after iteration.
E11 to H11 Copy formula D11 across.

C12=C124 Calculated wet steam temperature out of each stage (& into following
        stage).
D12 to G12 Copy formula C12 across.
H12=EXP(-7.32362+2.363801*LN(H13*1000)-
        0.164849*LN(H13*1000)^2+0.0042999*LN(H13*1000)^3)
        Wet steam temperature calculated from final pressure.

C13=C125 Calculated pressure (output of stage and input to next stage) after
        iteration.
D13 to G13 Copy formula C13 across.
H13 Input final steam pressure.

D16=D9 Ratio of output steam to input steam (initial value).
E16 to H16 Copy formula D16 across.

D17 to H17 Input initial estimates of constant term in linear formula for output
        steam from input steam tph (zero should be an adequate guess).

The following are the first stage of the iterative solution:
C19=C$3 Tph of steam entering the circuit.
D19=C19*D16+D17-D$2 Estimated steam (from linear approx.) going to next
        stage.
E19 to H19 Copy formula D19 across.

C20=C$4 Faction solids entering the circuit.
D20 to G20 Copy formulas from D4 to G4.
H20=H$4 Value of final fraction solids.

C21 to H21 Copy formulas from C5 to H5.

C22=C$11 Initial juice temperature.
D22=D26+D24 Steam temperature plus increase in boiling temperature for
        juice.
E22 to H22 Copy formula D22 across.

D23=-0.1379+2.22*D20/(1-D20)+0.1188*(D20/(1-D20))^2 Value for use in
D24.
E23 to H23 Copy formula D23 across.

D24=3D(D26+273)^2/((373/D23)^2+(100-D26)) Increase in juice boiling
temperature from steam temperature.
E24 to H24 Copy formula D24 across.

D25=D$7*D$8*D$9/C19*3.6 Coefficient for use in C26.
E25 to H25 Copy formula D25 across.

C26=0.3175+0.1559*D25-SQRT(0.02431*D25^2+0.09899*D25-777212-
0.3118*D25*D22) Temperature of input steam from heat transfer solved
for input temperature.
D26 to G26 Copy formula C26 across.
H26 Copy formula H12.

C27=EXP(18.23207-8.341154*LN(C26)+1.901357*LN(C26)^2-
0.0882262*LN(C26)^3)/1000 Pressure of wet steam from temperature.
D27 to G27 Copy formula C27 across.
H27=H$13 Final output pressure.

C28=(2492600-2.0363*C26-3.2071*C26*C26)/1000 Steam heat of vaporisation
from temperature.
D28 to H28 Copy formula C28 across.

D29=3D(4125.3-24.804*C20+C22*(0.067*C20+1.8691-0.009271*C22)+4125.3-
24.804*D20+D22*(0.067*D20+1.8691-0.009271*D22))*(C22-D22)/2000
Heat loss by juice in the stage.
E29 to H29 Copy formula D29 across.

D30=D$9*C28/D28 Updated ratio of input steam to output steam.
E30 to H30 Copy formula D30 across.

D31=D21*D29/C28 Updated constant term for prediction of output steam.
E31 to H31 Copy formula D31 across.

Copy formulas for line 18 to 31 seven times starting at line 32 (last line used is
129).