Maximizing the contribution of wind power in an electric power grid

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1 Introduction

The operator of an electric power grid must make continual adjustments in response to fluctuating conditions. On a short time scale (less than one hour), there have traditionally been two main kinds of variability for which the operator must compensate:

- Fluctuations in loads, offset by certain power plants with the ability to change their output quickly (frequency keepers).
- Unplanned outages (i.e. failures) of generators or transmission lines, offset by uncommitted power plants with the ability to begin generating quickly (spinning reserve).

Wind farms are also a source of variability, due to the fluctuating strength of the wind. In power grids where wind farms constitute only a small fraction of the total generating resource, the resulting variability can be included in the first category above, and dealt with as part of frequency-keeping. However, in a power grid with a large wind component (10-50% of time-averaged generation), wind-related variability may have to be treated as a third category of fluctuation, with specific operating procedures required to compensate for it. Both the amount of capacity needed, and the frequency with which it is required, are likely to fall somewhere between the traditional frequency-keeping and the spinning-reserve requirements.

This paper treats several different approaches to the problem of optimal grid operation in the presence of wind power. All the analyses are essentially economic in nature. That is, the cost of reserving spare capacity (either generation or transmission) is balanced against its expected benefits. Longer-term problems of optimal capital investment are not considered, although the methods and results here may be relevant to such problems.

The situation of the New Zealand power grid is particularly relevant. Although wind farms contributed only 1.5% of system energy in 2005 [1], it is anticipated that this share may increase to more than 10% over the next decade. New Zealand is also well-endowed with hydroelectric generation, which can play a key role in compensating for wind-related variability.

The essential problem at hand is the *dispatch problem* (also known as the *optimal power flow problem*). Suppose that each power station offers one or more tranches of power to the market, with each tranche being a fixed quantity of power at a fixed offer price. The problem is then to choose which tranches to accept (in whole or in part) to meet demand while minimizing the cost of power at the stated offer prices. (A similar problem would be solved even if there were no market; in that case, the offer prices would be replaced by some other estimates of the short-run marginal cost of generation.)

In practice, a system operator wishes to minimize the cost of power purchased over a fixed period of time (of duration T typically of the order of 5 minutes). If the behaviour of the power system is steady during this period, it suffices to solve the dispatch problem just once. The solution gives the constant power outputs that each station must maintain throughout the period.

Now consider the situation where some parameters of the dispatch problem vary with time. In particular, the quantity offered in a tranche (such as from a wind farm) may be a function of time (usually unpredictable in advance, i.e. a stochastic process). In this context, the dispatch problem becomes one of stochastic control. The actual cost to be incurred over the period is a random variable, and our objective is to minimize its expectation.

We consider several different ways to formulate such a time-varying dispatch problem. In Section 2, we study a simple model with three power generators. We explore the optimisation process to minimise the cost of meeting a power demand load. In Section 3, the available wind power may be a general stochastic process, but the response of the rest of the system is somewhat idealized (all ramp rates are treated as either zero or infinity). In Section 4, we briefly consider the problems of collection and analysis of wind speed data for use in wind-power generation planning. We conclude in Section 5.

2 The three-power-station model

Our initial model (Figure 1) incorporates just three power stations: a wind farm, a low-cost (thermal) generator that can adjust only slowly to new power levels, and a fast-ramping but high-cost (hydro) generator. To simplify the issues we assume that there are no significant transmission losses or constraints, so that all power is effectively generated and consumed at a single location. We also assume a constant load. As the rapid fluctuations in wind-power generation occur, the power output of the low-cost generator is adjusted to try to balance the load. As this low-cost generator is slow-ramping, it cannot immediately compensate for rapid changes in the wind, and the balance must be met by the fast-ramping high-cost generator. (A more general interpretation for this system could be as a model for the interaction of power provision sectors rather than of individual power stations.)

Power losses due to a drop in wind need high-cost power while the low-cost generator ramps up to its new required level. The costs due to the use of this high-cost power can be reduced by running the low-cost generator at a higher base level, and by not using some of the available wind power. Similarly an increase in wind power can be anticipated by using a base of high-cost power to allow more rapid adjustment to use the additional resource.

Within this model we may study how best to balance the sources of power for the cheapest long-term operation. This will depend upon the costs of the different generators, and the

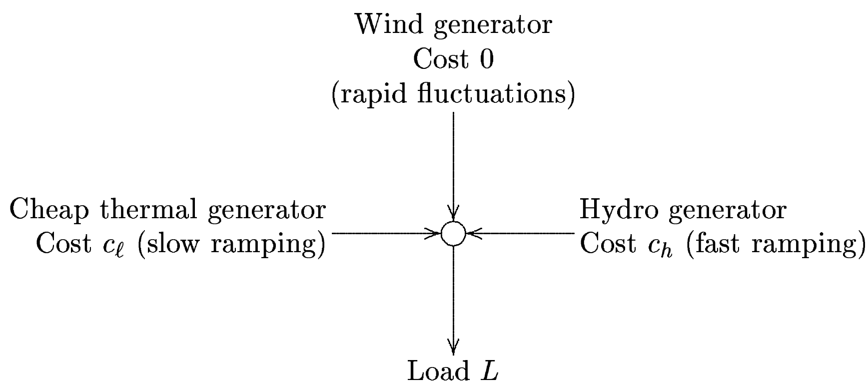


Figure 1: The preliminary power system model.

nature of the variation in wind power. The mathematical solution typically involves finding the minimum point of a U-shaped function.

Notation for this section:

L	Load, the amount of power required or that can be transmitted.
$W(t)$	Wind speed by time
$P_w(t)$	Wind power available,
$P_u(t)$	Wind power used,
h	Wind-power headspace, the maximum amount of wind power allowed
ΔP_w	Step change in wind power.
$P_\ell(t)$	Power from low-cost generator by time,
ΔP_ℓ	Excess power from low-cost generator,
c_ℓ	Cost of power from low-cost generator,
r	Maximum rate of change for low-cost generator.
$P_h(t)$	Power used from high-cost generator by time,
ΔP_h	Excess power from high-cost generator,
c_h	Cost of power from high-cost generator.
T	Total time considered.

We make the following general assumptions. Power output from the low-cost generator may be increased or decreased at a constant rate r. The changes in output of the high-cost generator, and the wind generator, are taken to be instantaneous. The model is parametrised by h, the portion of the load which wind power is allowed to serve. The system attempts to cover any load not served by wind power from the low-cost power source, with occasional use of high-cost power to match fluctuations in the wind.

In order to optimise h we require the costs for each of the power generators. For the purposes of these models, the cost of wind power was taken to be 0, while the costs of units of low- and high-cost power were taken to be c_ℓ and c_h, respectively. (We can set the cost of wind power generation equal to zero as it is the marginal costs that are involved in the optimisation. Wind power is likely to be the cheapest power source.)

The simplest kind of model is reactive, in which the system responds to current wind-power generation. Upon a sudden drop in wind power, it takes time to ramp up the low-cost generator to meet the power demand. The deficit must be met by the high-cost power station. Even with unlimited wind power, there is likely to be a point at which it is more cost effective to balance wind power with a contribution from the low-cost generator and to constrain the

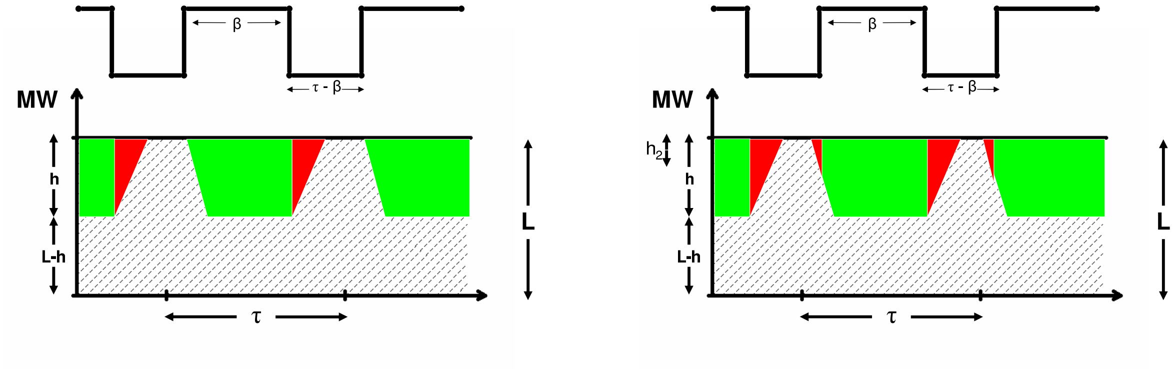


Figure 2: The square-wave model. The left diagram shows the reactive model while the right diagram shows the anticipating model. The upper part of each diagram shows the square wave output of the wind farm. The lower part shows how wind (green), low-cost (grey) and high-cost (red) generators meet the load.

amount of high-cost power needed should the wind drop.

A more sophisticated approach can be taken if changes in wind-power generation can be anticipated. Then, it may be worthwhile to deliberately increase power output from the high-cost generator and ramp down the low-cost generator prior to the onset of increased wind-power generation. This enables the wind to be more fully used as soon as it is available. Similarly, preparations could be made for the drops in wind-power generation.

2.1 The square-wave wind-power generation model

If we assume that the potential wind-power generation is a square-wave of period τ we can conduct an exact analysis of the optimal strategy. Of course the actual function will be much more complicated but this simpler form still enables us to see some of the effects of variation. For time duration β the wind power P_w is at its upper value P_{wmax} and for $\tau - \beta$ it is at its lower value P_{wmin}. If the maximum wind-power generation P_{wmax} is always below the load L, then the low-cost generator permanently meets the difference $L - P_{wmax}$. So to further simplify the problem, without losing the features of primary interest, we assume $P_{wmax} = L$ and likewise $P_{wmin} = 0$, so that

$$P_w(t) = \begin{cases} L, & \text{if } 0 < t < \beta \\ 0, & \text{if } \beta < t < \tau \end{cases} \qquad P_w(t + \tau) = P_w(t). \tag{1}$$

First, consider what happens when the low-cost generator changes in direct response to the wind-power fluctuations. The maximum wind power used is h and the remainder $L - h$ of the load is always met by the low-cost generator. This simple reactive model is illustrated in Figure 2(a) for a case where the slow-ramping is more rapid than the fluctuations. Specifically, $h/r < \beta$ and $h/r < \tau - \beta$. With the square-wave of equation (1), the wind power used P_u, low-cost power P_ℓ and high-cost power P_h are given by

$$
\begin{aligned}
\left[P_u, P_\ell, P_h\right](t) &= \begin{cases} [rt, L-rt, 0], & \text{if } 0 < t < h/r \\ [h, L-h, 0], & \text{if } h/r < t < \beta \\ [0, L-h+r(t-\beta), h-r(t-\beta)], & \text{if } \beta < t < \beta + h/r \\ [0, L, 0], & \text{if } \beta + h/r < t < \tau \end{cases} \\
\left[P_u, P_\ell, P_h\right](t) &= \left[P_u, P_\ell, P_h\right](t+\tau).
\end{aligned} \tag{2}
$$

Initially, there is no wind-power generation and the load is entirely met by the low-cost generator. As there is potential for wind-power generation the low-cost generator ramps down to allow the demand to be met by a mixture of wind and low-cost power. When the wind-power generation ceases again the high-cost power generator must initially meet the demand while the low-cost generator ramps up to meet the full load. This cycle is repeated. In order to determine the optimum value of h, we calculate the total cost per time period τ, which is a quadratic,

$$c_\ell(L\tau - h\beta) + c_h h^2/(2r). \tag{3}$$

The minimum for this is readily found to be at $h = r\beta c_\ell/c_h$, which corresponds to a total cost:

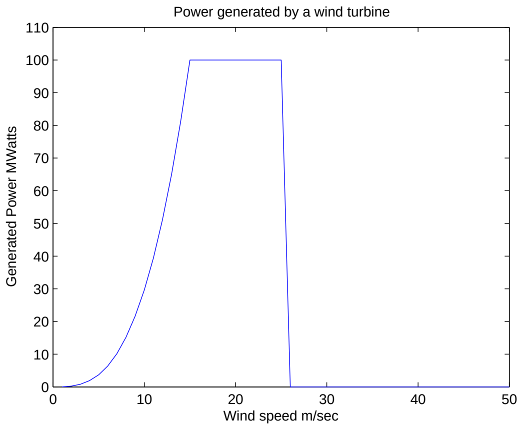
$$c_\ell L\tau - c_\ell^2\beta^2 r/(2c_h). \tag{4}$$

If the difference in cost between low-cost and high-cost generators is too small this may not be the cheapest strategy: the combination of power sources switching directly between wind and high-cost power may be collectively cheaper than low-cost power ($c_\ell\tau < c_h(\tau-\beta)$). A further minor complication for this reactive model occurs when there are ranges of h for which the slow ramping is too slow for the low-cost generator to fully decrease power by h during the bursts of wind (of duration β), or to fully increase power by h during the lulls in the wind (of duration $\tau-\beta$) or both of these cases. That is, if either $h/r > \beta$ or $h/r > \tau-\beta$, or both. For these ranges of h the total cost becomes a linear function and so reaches its extreme values at the limits of the domain.

In the limiting case there is no ramping of the low cost generator. (This is used in the analysis in Section 3.) Then the total cost function is linear over the entire range of h from 0 to L and so the optimum value of h will be either 0 or L. When $h = 0$, the entire power production is by the low-cost generator and will cost $c_\ell L\tau$ over the period τ. When $h = L$, this low-cost generator is never used, the production being entirely by the wind generator when this is operating or by the higher cost expensive generator when it is not. The total cost for this case is $c_h L(\tau-\beta)$ per period and so $h = 0$ is cheaper than $h = L$ when $c_\ell\tau < c_h(\tau-\beta)$.

The linearity, for no ramping, has resulted in the lowest cost option occurring at one of the extremes of either having no wind generation or no low-cost generation. However, for more general wind forms, this will not be the case. For example, if, instead of the square wave form for wind power, we use a triangular wave form, then the optimal value of h in the no-ramping case is again given by a quadratic expression.

This simple square-wave model also illustrates the possible savings if changes in wind generation can be predicted. For example suppose that we anticipate the rises in wind power (but not the falls, although their anticipation can also be used for savings (Section 2.3)). In this case we may increase power generation from the high-cost generator and ramp down the low-cost generator prior to the onset of the increased wind generation (Figure 2(b)). This enables the wind to be more fully used as soon as it is available. We will also suppose that



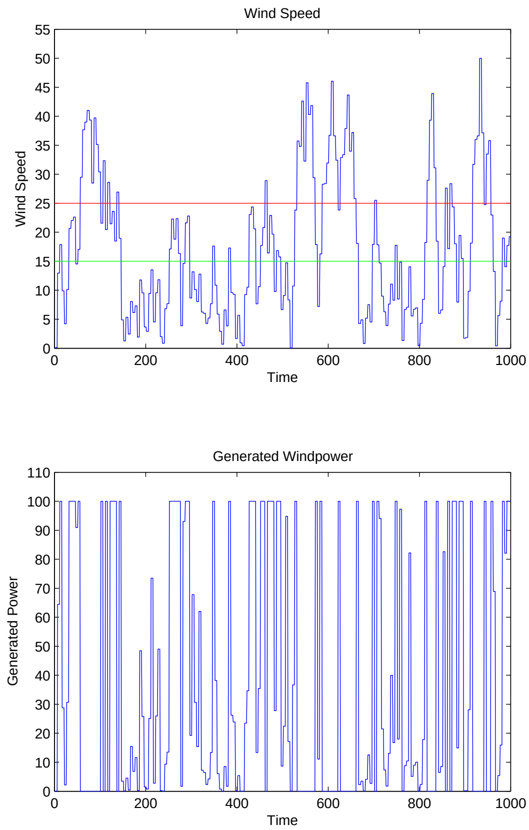


Figure 3: Numerical simulations. (a) (*above*) The wind-power response curve of equation (8). (b) A sample wind speed time series (*right above*) with wind-power response cut-offs W_{limit} (green) and W_{max} (red), and (*right below*) the corresponding wind-power.

the slow-ramping is fast enough to complete between changes in the wind-power generation and that the high-cost power is sufficiently costly to justify ramping. Let the slow-ramping down of the low-cost generator begin at r/h_2 before the onset of the wind-power generation so that at the beginning of wind-power generation the low-cost generator is producing $L - h_2$ power. Then the cost function equation (3) is modified to

$$c_\ell(L\tau - h\beta - hh_2/r) + c_h(h^2 + h_2^2)/(2r). \tag{5}$$

This is a quadratic form in the two parameters h and h_2. The minimum cost now occurs at the higher value of $h = r\beta c_\ell/(c_h - c_\ell^2/c_h)$ with $h_2 = c_\ell h/c_h$. Substituting this into equation (5) above we see that the second (negative) term in the total cost (equation (4)) has increased in magnitude by the same factor $(1/(1 - c_\ell^2/c_h^2))$ as h. So the total cost with anticipation is

$$c_\ell L\tau - c_\ell^2\beta^2 r/(2(c_h - c_\ell^2/c_h)). \tag{6}$$

This is a saving of $c_\ell^4\beta^2 r/(2c_h(c_h^2 - c_\ell^2))$ per period τ over not anticipating the wind.

2.2 Numerical simulations

The three-power-station model was simulated for the reactive case. For each simulation a constant minimum level of low-cost generation was maintained. As far as possible, wind generation is used to meet the remaining load h. If, during operation, low-cost and wind outputs together are insufficient to meet the load then high-cost power is used to meet the balance while the low-cost generation ramps up to compensate. If, instead, the wind and low-cost power is greater than the load, then wind is spilled and the low-cost generator ramps down (unless it is already at its minimum level).

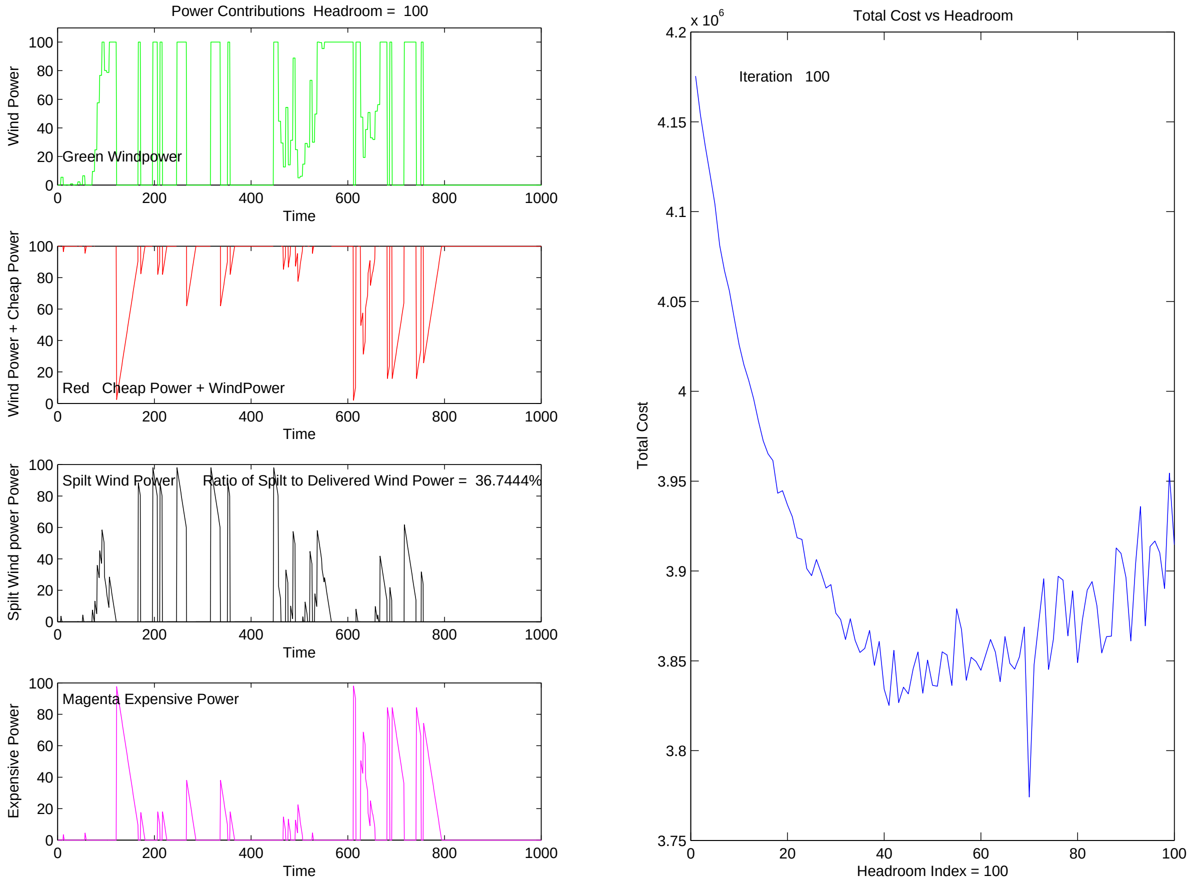


Figure 4: The four graphs on the left show the components of different power sources used to meet the load and the wind-power spilled for a sample wind-speed time series. The curve on the right shows the relationship between average total cost and h for an illustrative set of power prices.

The simulations were conducted with h values from 1% to 100% of the load. For each value of h there were 100 random realisations of the realistic wind speed time-series generated from the model

$$W(i+1) = 0.99W(i) + 2N(0,1) \tag{7}$$

where the $W(i)$ are the values of the wind at discrete time intervals and $N(0,1)$ is a Gaussian random sample [2]. The power generation response P_w to wind speed W is not linear (although this was used initially as a first approximation). During the initial rise in the wind the response is cubic but for high winds the power generation is bounded and for very high winds the wind must be spilled to protect the generator and so there is no power output. The power-generation response used takes the form

$$P_w = \begin{cases} \left| \frac{W}{W_{limit}} \right|^3 P_{max}, & \text{if } 0 < W < W_{limit} \\ P_{max}, & \text{if } W_{limit} < W < W_{max} \\ 0, & \text{if } W > W_{max} \end{cases} \tag{8}$$

which is typical of a single wind generator. The response curve is shown in Figure 3 together with a sample wind speed time series and its power output.

The cost of meeting the load's power demand is found for each realisation. These costs were averaged over the 100 simulations for each of the different h values. This average is shown in Figure 4. Again the relationship between cost and h is a U-shaped curve for which we have an optimal h. Numerical simulations of this kind can be readily extended and adjusted for more sophisticated models, as in Section 2.3.

2.3 Optimum settings for accommodating variation in wind power

Here we revisit the problem of choosing optimum settings for accommodating variation in wind-power for more realistic wind power forms which essentially change at random with the available wind. A step down in wind power needs to be replaced by other power, whereas a step up in wind power need not be used. For this reason it is necessary to consider separately the cases of a step down and a step up. This is done in the next two subsections.

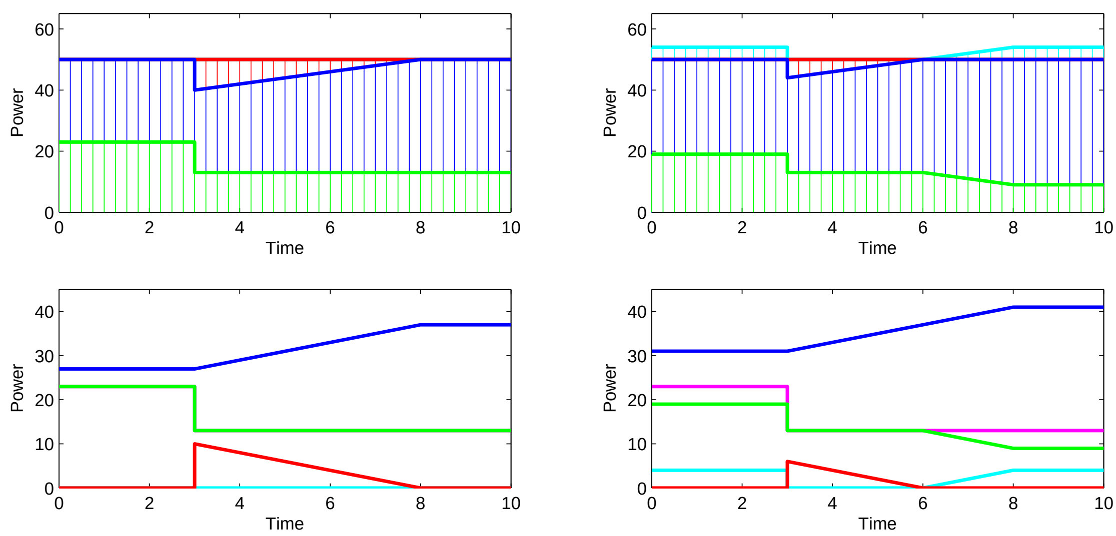


Figure 5: Power sources during a drop in wind power: (a) (*left*) no additional low-cost power; (b) (*right*) additional low-cost power replaces some wind power, reducing the need for high-cost power. The top diagram shows the total power output composition and the lower diagram shows the individual power outputs. (Green - wind power used; Cyan - unused wind power; Blue - low-cost power; Red - high-cost power; Magenta - available wind power).

2.3.1 Step change down in wind power ($\Delta P_w < 0$)

First, we consider the use of additional low-cost power to provide a buffer for the variation in wind power. This low-cost power replaces some of the wind power, reducing the amount of wind power used. However, it also reduces the need for high-cost power immediately following a drop in the wind.

Figure 5 shows the effect of a step down in wind power. A portion of high-cost power (red) is needed to replace the wind power until the low-cost generator can ramp up to take over from the reduced wind power. In Figure 5(a), the wind power is fully used, whereas in Figure 5(b), a higher level of low-cost power is used and this replaces a portion of the wind power. For the expense of replacing some of the wind power with low-cost power, the requirement for high-cost power is significantly reduced.

Thus additional low-cost power ΔP_ℓ is used to accommodate changes downwards in wind power ΔP_w at the expense of not using all the available wind power during normal operation. Initially we assume one step change down in the period T. It is noted that although there are continuing random changes in the wind the expected change is close to zero. So, we initially assume that the average of wind changes following the step change is zero.

There is no extra benefit from additional low-cost power above $-\Delta P_w$ and so this case can be ignored. The total cost is:

$$c_\ell \int_0^T P_\ell(t)dt + c_\ell \Delta P_\ell T + \frac{c_h}{2r}(-\Delta P_w - \Delta P_\ell)^2. \tag{9}$$

To find the minimum cost, we differentiate with respect to ΔP_ℓ and equate this to zero to obtain:

$$\Delta P_\ell = -\Delta P_w - rTc_\ell/c_h. \tag{10}$$

Thus the optimum ΔP_ℓ is always less than $-\Delta P_w$ (which is positive), and is limited by zero when the second term becomes too large. If high-cost power is sufficiently expensive then there will be an advantage in using additional low-cost power instead of wind power. The optimum amount of excess low-cost power also depends on the time interval T used, with longer time intervals indicating less excess low-cost power. The time interval used could also include a step in the opposite direction as considered in the next subsection.

Instead of assuming a fixed size of step change over the time interval T, now we assume that for step changes below $-\Delta P_\ell$ (negative and of a greater magnitude) the size is determined by the probability:

$$\mathrm{P}(\Delta P_w(u) : \Delta P_w(u) < -\Delta P_\ell; T) .$$

Rather than equation (9) the following total cost now applies:

$$\begin{gathered} c_\ell \textstyle\int_0^T P_\ell(t)dt + c_\ell \Delta P_\ell T + \\ \tfrac{c_h}{2r} \textstyle\int_0^1 \mathrm{P}(\Delta P_w(u) : \Delta P_w(u) < -\Delta P_\ell, T)(-\Delta P_w(u) - \Delta P_\ell)^2 du. \end{gathered} \tag{11}$$

To find the minimum cost, differentiate with respect to ΔP_ℓ (to get a simpler but approximate result, assume that the probability term is essentially constant, a method for correction of this and other approximations by adjusting the value of T is suggested in a later subsection):

$$c_\ell T - c_h/r \int_0^1 \mathrm{P}(\Delta P_w(u) : \Delta P_w(u) < -\Delta P_\ell, T)(-\Delta P_w(u) - \Delta P_\ell)du. \tag{12}$$

Set this to zero and solve for ΔP_ℓ:

$$\Delta P_\ell = \frac{-\int_0^1 \mathrm{P}(\Delta P_w(u) : \Delta P_w(u) < -\Delta P_\ell, T)\Delta P_w(u)du - rTc_\ell/c_h}{\int_0^1 \mathrm{P}(\Delta P_w(u) : \Delta P_w(u) < -\Delta P_\ell, T)du}. \tag{13}$$

After dividing by the denominator, the first term gives the average size of the larger step changes, while the second term is adjusted to account for the probability of a step change down occurring. This is unfortunately an implicit equation for ΔP_ℓ, but as it involves a single variable it is easily solved numerically for particular cases.

As noted above there is a problem in the choice of the value of T. This is considered in Section 2.3.4.

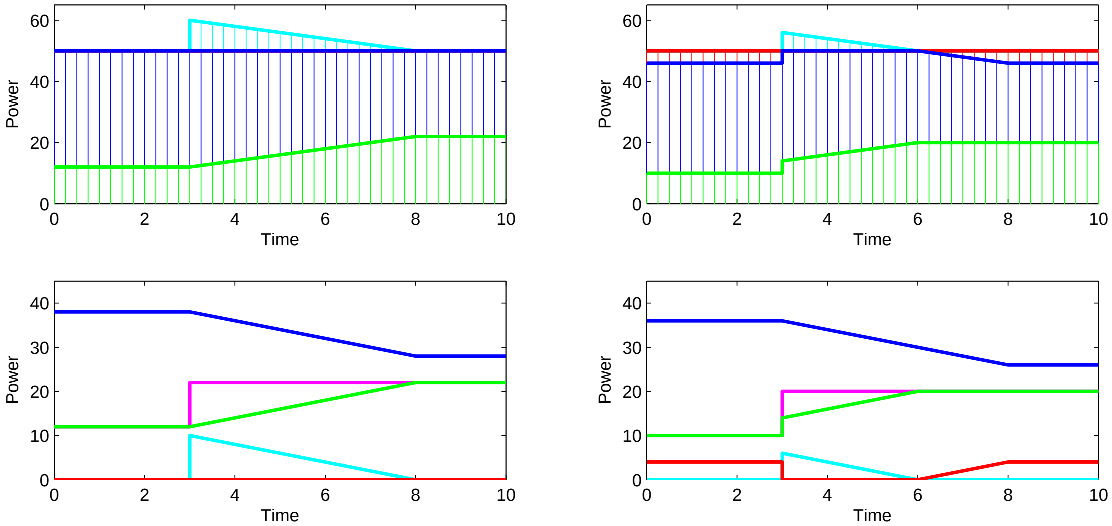


Figure 6: Power sources during a wind power increase; (a) (*left*) no high-cost power; (b) (*right*) some high-cost power replaces low-cost power. Diagrams are as in Figure 5 (Green - wind power used; Cyan - unused wind power; Blue - low-cost power; Red - high-cost power; Magenta - available wind power).

2.3.2 Step up change in wind power ($\Delta P_w > 0$)

High-cost power (which can be altered rapidly) could be used to replace some low-cost power to enable sore rapid utilisation of an increase in wind power. Figure 6 shows the effect of a step up in wind power. In Figure 6(a) some of the wind power cannot be used as the low-cost generator cannot adjust sufficiently fast. Figure 6(b) shows the same increase in wind power but with some high-cost power replacing part of the low-cost power. Now it can be seen that more of the wind power is used.

Using more high-cost power than the step in wind power is clearly a waste. The total cost when $\Delta P_h < \Delta P_w$ is:

$$c_\ell \int_0^T P_\ell(t)dt - c_\ell \Delta P_h T + c_h \Delta P_h T - \frac{c_h}{2r}\Delta P_w^2 + \frac{c_h}{2r}(\Delta P_w - \Delta P_h)^2. \tag{14}$$

As before, the value of ΔP_h at the minimum is found by differentiation:

$$\Delta P_h = \Delta P_w - r(c_h - c_\ell)T/c_h. \tag{15}$$

This always gives $\Delta P_h < \Delta P_w$ when $c_h > c_\ell$. When the difference between high and low-cost power is not too large, high-cost power can be used to take advantage of rapid increases in wind-power generation.

If we consider a cycle of a step up then a step down with both allowing time for the low-cost power to adjust so that $T > 2\Delta P_w/r$. Then by applying this inequality to equation (15) we have

$$\Delta P_h < \Delta P_w(2c_\ell/c_h - 1). \tag{16}$$

Thus it is seen that for ΔP_h to be positive we should have $c_h < 2c_\ell$.

With a probability distribution for $\Delta P_w(u)$ we have:

$$\Delta P_h = \frac{\int_0^1 \mathrm{P}(\Delta P_w(u) : \Delta P_w(u) < \Delta P_h, T)\Delta P_w(u)du - r(c_h - c_\ell)T/c_h}{\int_0^1 \mathrm{P}(\Delta P_w(u) : \Delta P_w(u) < \Delta P_h, T)du}. \tag{17}$$

Thus ΔP_h is always less than a weighted sum of the ΔP_w values.

The quantity ΔP_h is the reduction for the target power of the low-cost generator. The low-cost generator moves towards its target at its limited slow ramp rate. The actual power allocation will be the amount currently generated by the low-cost generator, plus either the wind power needed to reach the required total power, or, if this is not sufficient, the maximum wind power with the remainder coming from the high-cost generator. Again, selecting the value of T is a problem.

2.3.3 Simulation of wind power generation

Sections 2.3.1 and 2.3.2 have provided formulae that show how extra amounts of low and high-cost power can help reduce the expense of wind power variation management. Equations (13) and (17) can be used for a given value of T to determine the amounts of additional power to be generated. (It can be easily verified that these two formulae act independently.) These formulae will now be tested for a numerical simulation similar to that of Section 2.2.

A table can be constructed giving the values of ΔP_ℓ and ΔP_h for for each wind speed. For every case the distribution of wind speeds after the time interval T is also calculated. For the demonstration here a normal distribution with standard deviation 3.0 is used as T is assumed small enough for this to be sufficiently accurate. (Another distribution of changes in wind speed could be used if thought appropriate.) From the distribution of wind speeds, and a power versus wind speed relation, the distribution of power after time interval T can be determined. The power relation, typical of a single wind generator, can be seen as the magenta curve in Figure 7:

$$P_w = \begin{cases} 30(W/15)^3, & \text{if } W < 15 \\ 30, & \text{if } 15 \le W \le 25 \\ 0, & \text{if } W > 25\ . \end{cases} \tag{18}$$

Once the distribution of power changes is obtained, equations (13) and (17) can be solved to obtain target values of ΔP_ℓ and ΔP_h as illustrated in Figure 7.

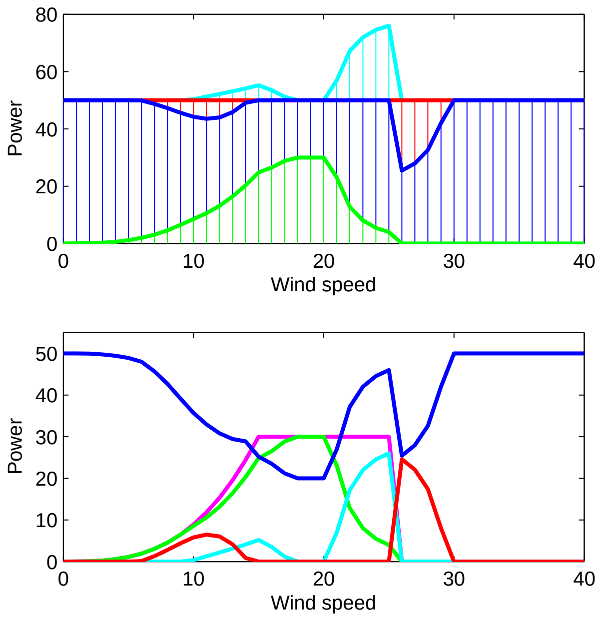


Figure 7: Targets for power generation given wind speed: Magenta - potential wind power; Green - wind power target; Cyan - unused wind power; Blue - low-cost power target; Red - high-cost power target. This illustration is for the values: $L = 50$; $r = 2$; $c_\ell = 40$; $c_h = 80$; $T = 2$.

Due to the slow dynamic performance of the low-cost generator, it may not be able to provide the specified target value for a given wind speed. In this case any deficit is made up first by wind power, if available, and then by high-cost power, while the low-cost generator is ramped towards the target.

Given the table of target values, a simulation can be performed of the wind generation over time. The formula for the wind speed:

$$W_i = \alpha W_{i-1} + 3N(0,1), \tag{19}$$

is similar to that given in [2]. We chose α (0.9798) to give 15 as the standard deviation of the wind speed. (A Fourier Analysis was done of wind speed data in [2]. The Power Spectrum had two clear peaks corresponding to 24 and 12 hour periods and a possible indication of a component with an 8 hour period. The wind speed models, equations (7) and (19), in part arise from this analysis. That paper also found several different forms for the daily variation, rather than a typical average behaviour.)

Figure 8 shows a portion of the simulation results. The variation in wind speed is seen to be amplified in the wind power P_w. These variations then drive the variations in the other power values.

2.3.4 Determination of the time interval

There remains the value of T to be determined. This can be done by a direct search using simulated wind speeds. Ten thousand time steps were used with the same random wind profile

$c_\ell = 40,\ c_h = 60$

$T(\Delta P_h)$	$T(\Delta P_\ell)$ 2	8	14
2	1773.6	1765.7	1765.2
8	1779.0	1770.1	1769.5
14	1780.3	1771.6	1771.0

$c_\ell = 40,\ c_h = 80$

$T(\Delta P_h)$	$T(\Delta P_\ell)$ 2	8	14
2	1843.4	1843.1	1843.2
8	1836.5	1835.7	1835.9
14	1836.7	1836.2	1836.5

$c_\ell = 40,\ c_h = 100$

$T(\Delta P_h)$	$T(\Delta P_\ell)$ 2	8	14
2	1902.2	1912.3	1914.4
8	1888.8	1898.2	1900.9
14	1888.7	1898.2	1901.0

Table 1: Cost of generation for different power costs and T values

for each run.

Table 1 gives the effect of a range of values values for T and for the cost of high-cost power. The value of T for a step up in wind power has been assumed independent to that for a step down. Except where noted the same values as in the previous subsection have been used. In this table it should be noted that a higher value of T corresponds to reduced additional power.

In the first case the cost of high-cost power is not much more than that of the low-cost power. It is seen here that low values of $T(\Delta P_h)$ give the lowest cost, corresponding to an addition of high-cost power. Additional low-cost power (low $T(\Delta P_\ell)$) increases the costs in this case.

The second case has the high-cost power twice the cost of the low-cost power. Here we see the effect of the additional low-cost power (low $T(\Delta P_\ell)$) is close to zero, but too much high-cost power (low $T(\Delta P_h)$) increases cost.

The final case has a higher value for the high-cost power. Here we see the advantage of using additional low-cost power to avoid the use of the high-cost power. Additional high-cost power (low $T(\Delta P_h)$) increases the costs.

2.3.5 Additional comments

This illustration uses the power curve for a single wind generator. The largest adjustment to power and hence costs, are seen close to the steepest parts of the wind power curve (Figure 7). In practice the power output of a wind farm will be the total from multiple wind generators and so will be smoother than that used here. This will reduce the benefits to be gained.

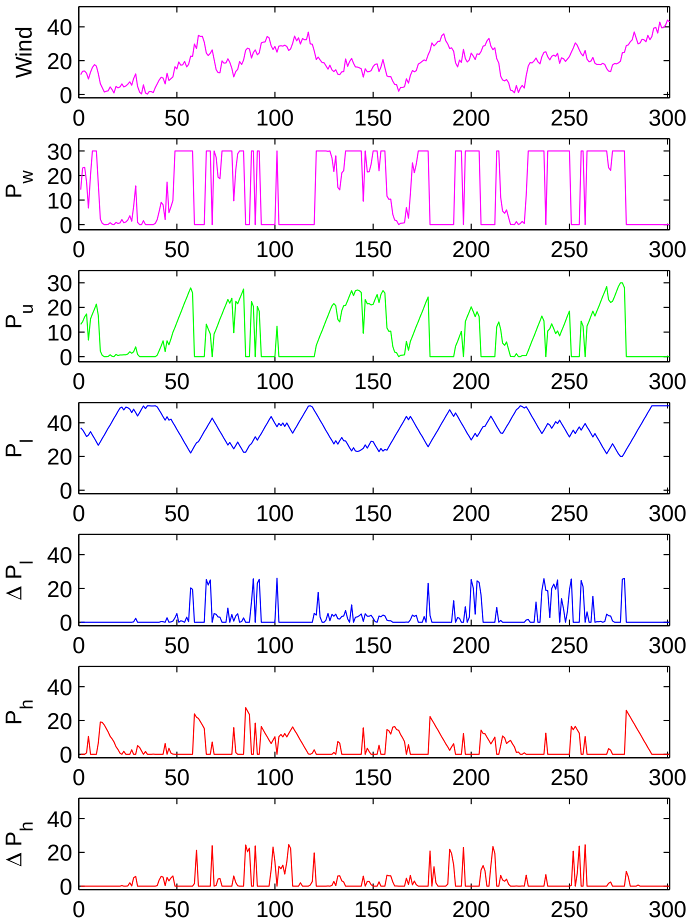


Figure 8: Simulation results showing the effect of variation in wind speed on power generation

In some cases the high-cost power cost is large, and then there are significant benefits in scheduling a margin of low-cost power. Avoiding high-cost power here decreases the marginal power costs that determine what customers pay.

By using additional low-cost power to buffer possible drops in wind generation, the wind farm is being penalised for the variability of its supply by not having all the available power scheduled for use. Replacing some low-cost power with high-cost, to better utilise possible increases in wind power, is only of benefit when the costs of high and low-cost powers are close.

3 Optimal dispatch with simple ramp rates

In this section again, as earlier, we make the important simplifying assumption that wind farms share the network with power stations of two kinds: fast and slow. The fast stations

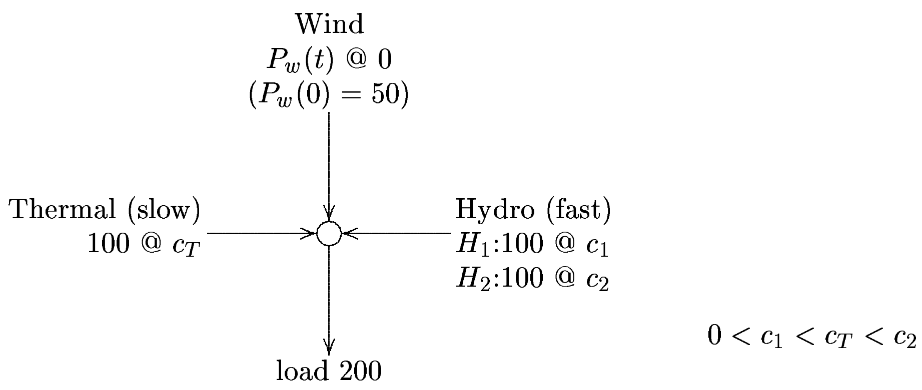


Figure 9: The one-node model.

effectively have an infinite ramp rate as before; they can quickly be re-dispatched in response to any fluctuations in the output of wind farms. However, here, the slow stations are taken to have essentially zero ramp rate; their dispatch, once chosen at time $t = 0$, may not be altered during the time period $0 \le t \le T$ considered. Broadly speaking, the "fast" behaviour is an idealized model of a hydroelectric station, while the "slow" behaviour models a thermal power station.

In such a model, the conventional least-cost dispatch serves only to minimize the instantaneous cost at time $t = 0$. However, the true objective is to minimize the expected overall cost over the period $[0, T]$, allowing for the adjustments that will become necessary as the wind fluctuates. (From the perspective of $t = 0$, the actual cost to be incurred is a random variable, due to stochasticity in the wind; its expectation value is our objective.) To minimize cost in this sense, a different initial dispatch may be required.

3.1 A one-node model

Again we begin by considering an example in which all power is effectively generated and consumed at a single location (Figure 9).

A total load of 200 must be met from the following generator offers. A wind farm offers a time-varying quantity $P_w(t)$, at zero price (marginal cost). We suppose that $0 \le P_w(t) \le 100$ and $P_w(0) = 50$. A thermal station offers 100 at price c_T, and a hydro generator offers tranches of 100 at price c_1 and a further 100 at price c_2. The offer prices satisfy $0 < c_1 < c_T < c_2$.

The decision to be made at time $t = 0$ is essentially the quantity x to dispatch from the thermal station. At subsequent times $t \in [0, T]$, the power required will be made up as follows: x from the thermal, $P_w(t)$ from the wind farm, and the balance from the hydro.

The least-initial-cost solution would be $x = 50$, i.e. dispatch the wind farm, the cheaper of the two hydro tranches, and (partially) the thermal. If the available wind power should subsequently rise, it will displace H_1 hydro generation, saving water with value c_1. In the event of a decrease in available wind power, hydro generation from H_2 will be used to make up the shortfall.

However, if c_2 is very much greater than the other prices, this seems like a poor solution. One wonders if it might be better, in that case, to choose a higher initial thermal dispatch, and reserve some of the H_1 tranche for possible future use. On the other hand, if c_1 is very low (perhaps almost zero) and c_2 is not prohibitively large, a good initial dispatch might include some of H_2 (as well as all of H_1), so as to increase the likelihood of fully using the cheap water. Note that both of these situations involve "out-of-order" dispatching, in which an apparently expensive tranche is dispatched while a cheaper one is not. In fact, both can occur, as the following analysis shows.

Formally, the instantaneous cost of the generation at time t is

$$c(x, P_w(t)) = \begin{cases} xc_T + 100c_1 + (100 - x - P_w(t))c_2, & \text{if } x + P_w(t) \leq 100 \\ xc_T + (200 - x - P_w(t))c_1, & \text{if } x + P_w(t) \geq 100 \; . \end{cases} \tag{20}$$

The instantaneous marginal cost is found by differentiating with respect to x:

$$c_x(x, P_w(t)) = \begin{cases} c_T - c_2, & \text{if } x + P_w(t) < 100 \\ c_T - c_1, & \text{if } x + P_w(t) > 100 \; . \end{cases} \tag{21}$$

We want to minimize the expected average cost over the period $0 \leq t \leq T$, which is:

$$C(x) = \mathrm{E}\left[\frac{1}{T}\int_0^T c(x, P_w(t))\, dt\right]. \tag{22}$$

This expression involves averaging over both time (the integral) and a probability space (the expectation). We can make the notation less cumbersome by defining τ to be a random variable distributed uniformly on $[0, T]$, independently of $P_w(t)$, and $V = P_w(\tau)$. Then

$$C(x) = \mathrm{E}\left[c(x, V)\right]. \tag{23}$$

When V has a well-behaved distribution, we may differentiate inside the expectation to obtain the marginal cost with respect to x:

$$\begin{aligned} C'(x) &= \mathrm{E}\left[c_x(x, V)\right] \\ &= (c_T - c_2)\mathrm{P}(V \leq 100 - x) + (c_T - c_1)\mathrm{P}(V \geq 100 - x) \\ &= (c_T - c_1) - (c_2 - c_1)\mathrm{P}(V \leq 100 - x). \end{aligned}$$

This is clearly an increasing function of x, and so $C(x)$ is convex in x. The optimal x can be found by setting this expression equal to 0.

For a more specific example, assume that $P_w(t)$ follows a Brownian motion model, so that $P_w(t) \sim N(P_w(0), \sigma^2 t)$. (We neglect the bounds $0 \leq P_w(t) \leq 100$.) Then $V \sim N(P_w(0), \frac{1}{2}\sigma^2 T)$. Solving gives an optimal initial thermal dispatch

$$x^* = P_w(0) - \left(\frac{\sigma^2 T}{2}\right)^{1/2} \Phi^{-1}\left(\frac{c_T - c_1}{c_2 - c_1}\right), \tag{24}$$

where Φ is the standard normal cumulative probability distribution function. This expression may be either more or less than the least-initial-cost solution ($x = 50$, here), depending on the relative costs of the various tranches. into play.

3.2 Line reservations

The *line reservation* technique considers only initial dispatches that arise in a particular way: by reserving capacity on transmission lines. That is, we solve a dispatch problem which differs from the conventional one only in that the assumed line capacities may be less than the actual

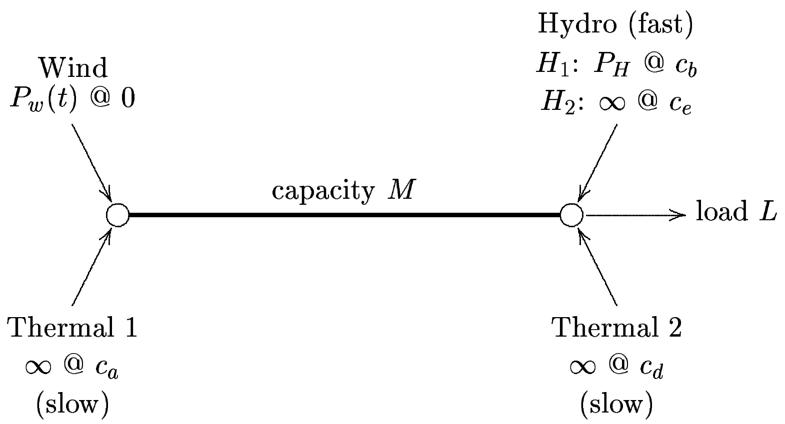


Figure 10: The two-node network.

capacities. The "reserved" line capacity is then available to accommodate fluctuations in the wind during the period $0 < t \leq T$.

The line-reservation technique is only a heuristic method, and the initial dispatch found in this way may not be optimal for our real objective (that is, another initial dispatch may have lower expected overall cost over the period $[0, T]$). Notice, for example, that line reservations would be of no help with the problem in the previous subsection, in which there are no lines. However, it does at least have the advantage of being easy to implement.

3.3 Wind matching over a single line

We shall now consider networks in which the power stations and the load are each located at one of a number of nodes which are interconnected by transmission lines of finite capacity. We begin with the wind-matching problem on a two-node network (Figure 10). In this network, a lossless transmission line of capacity M connects two nodes; a load L is taken off at the right-hand node. Generators offer power as follows: at the left-hand node, a wind farm offers a time-varying quantity $P_w(t)$, at zero price (marginal cost). A thermal station offers unlimited quantities at price c_a. At the right-hand node, a hydro station offers a relatively cheap tranche of quantity P_H at price c_b, and a more expensive tranche at price c_e. Another thermal station offers unlimited quantities at price c_d. We assume $M < L$, $0 < c_a < c_d$, and $0 < c_b < c_d < c_e$.

The least-initial-cost solution would dispatch the generators in the order of their offer prices, starting with the cheapest. This might mean, for example, dispatching the wind and H_1 hydro tranches, then Thermal 1 to the extent allowed by the transmission capacity, and finally Thermal 2 for the remainder. This leaves the transmission line constrained, with no capacity to spare. If the wind output should subsequently rise above its initial level, the excess wind power must be spilled, as there is no way to make use of it.

An alternative approach would be to leave some "headroom", via a reduction x in the quantity dispatched from Thermal 1 and a corresponding increase in the quantity dispatched from Thermal 2. This leaves unused capacity x in the line. Any subsequent rise in available wind power (up to x above the initial level) can then be used to displace hydro generation from H_1, saving water with value c_b. Only if the wind rises above its initial level by more than x must the excess be spilled.

Note that under either solution, a decrease in the wind farm's output must be compensated for by increased hydro generation from the H_2 tranche. If c_e is large, the use of H_2 water might represent a sizable contribution to the overall expected cost. However, having reserved line capacity does not help with this situation. (One would instead need to reserve spare capacity directly within the H_1 hydro tranche, as in the example in Section 3.1.)

Determining the amount of headroom to leave requires solving an optimization problem. Suppose a decision is made at time $t = 0$ to leave an initial headroom $x \geq 0$, by shifting generation from Thermal 1 to Thermal 2. This commits the system to an ongoing additional cost (per unit time) of $(c_d - c_a)x$, relative to the least-initial-cost solution. At a time $t > 0$, the instantaneous further cost (or benefit if negative) of additional water used (or saved) due to wind shifts is

$$f(t) = \begin{cases} (P_w(0) - P_w(t))c_e, & \text{if } P_w(t) < P_w(0) \\ -(P_w(t) - P_w(0))c_b, & \text{if } P_w(0) \leq P_w(t) \leq P_w(0) + x \\ -xc_b, & \text{if } P_w(t) > P_w(0) + x \ , \end{cases} \tag{25}$$

or, more compactly,

$$f(t) = c_e(P_w(0) - P_w(t))_+ - c_b \min(x, (P_w(t) - P_w(0))_+), \tag{26}$$

where the notation z_+ denotes $\max(z, 0)$. Thus at time 0, the expected average cost per unit time incurred over the time interval $0 \leq t \leq T$ is:

$$\begin{aligned} C(x) &= \mathrm{E}\left[\frac{1}{T}\int_0^T ((c_d - c_a)x + f(t)) \ dt\right] \\ &= \mathrm{E}\left[(c_d - c_a)x + c_e(-\delta)_+ - c_b \min(x, \delta_+)\right], \end{aligned}$$

where $\delta = P_w(\tau) - P_w(0)$, with τ, as before, a random variable independent of $(P_w(t))$ distributed uniformly on $[0, T]$.

Suppose we wish to choose x so as to minimize $C(x)$. Assume for simplicity that $P_w(t)$ has a continuous probability distribution. We see that for $x > 0$,

$$\begin{aligned} \frac{d}{dx}C(x) &= (c_d - c_a) - c_b \,\mathrm{E}\left[\frac{d}{dx}\min(x, \delta_+)\right] \\ &= (c_d - c_a) - c_b \,\mathrm{E}\left[1_{\delta > x}\right] \\ &= (c_d - c_a) - c_b \,\mathrm{P}(\delta > x). \end{aligned}$$

It is clear that this is a continuous increasing function of x, and hence that $C(x)$ is a smooth convex function of x. If $\mathrm{P}(\delta > 0) \leq (c_d - c_a)/c_b$ then $C(x)$ increases with x for all $x > 0$, and so the optimal choice is $x = 0$. Otherwise, the optimal x can be found by solving the equation

$$\mathrm{P}(\delta > x) = (c_d - c_a)/c_b. \tag{27}$$

In some situations (e.g. when $P_w(0)$ is neither 0 nor the maximum output of the wind farm, but somewhere in between), it may be reasonable to suppose that $\mathrm{P}(\delta > 0) = \frac{1}{2}$. (In other words, the wind is as likely to rise as to fall.) In this case, it is worth reserving headroom only if

$$c_d - c_a < c_b/2. \tag{28}$$

One can see this intuitively: the first unit of headroom costs $c_d - c_a$ to create, whereas the average cost of not having it is c_b for half of the time.

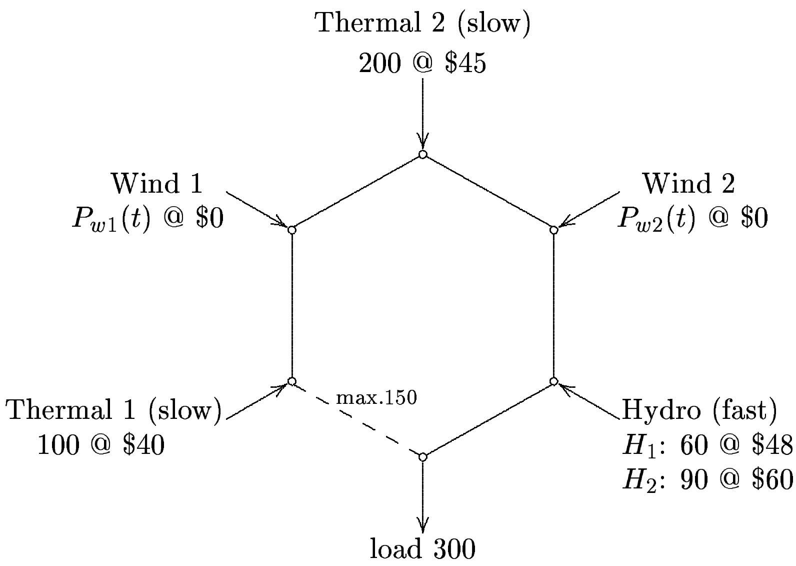


Figure 11: The six-node loop network.

3.4 Wind matching and sloshing in a network loop.

We now apply a similar analysis to that in Section 3.3 to a more complicated problem (Figure 11). Here we have two wind farms sharing a six-node loop network with two slow stations and a fast hydro. The only transmission capacity constraint of significance is the maximum flow of 150 on the dashed line in the diagram.

The six lines are assumed to be lossless, but with equal reactances. This means that power flowing from any station to the load will divide itself between the two possible paths in inverse proportion to the number of lines traversed. That is, $\frac{5}{6}$ of the power from Thermal 1 flows to the load via the limited-capacity line, while $\frac{1}{6}$ flows via the other five lines. For Wind 1, the flows divide in proportion $(\frac{2}{3}, \frac{1}{3})$; for Thermal 2, $(\frac{1}{2}, \frac{1}{2})$; for Wind 2, $(\frac{1}{3}, \frac{2}{3})$. The Hydro has the most advantageous position: only $\frac{1}{6}$ of its power flows over the limited-capacity line, with the other $\frac{5}{6}$ taking the more direct route. The network flows due to different power stations may be linearly superposed.

Let us suppose that the initial wind outputs are $P_{w1}(0) = P_{w2}(0) = 60$. The corresponding least-initial-cost dispatch is perhaps not apparent by inspection, but it can be found by solving a linear program. If we let P_1,P_2,P_3,P_4,P_5, and P_6 denote the quantities dispatched from, respectively, Thermal 1, Wind 1, Thermal 2, Wind 2, Hydro H_1, and Hydro H_2, then the linear program is

$$\begin{array}{lll} \min & 40P_1 + 45P_3 + 48P_5 + 60P_6 & \\ \text{s.t.} & P_1 + P_2 + P_3 + P_4 + P_5 + P_6 & = 300 \\ & \frac{5}{6}P_1 + \frac{2}{3}P_2 + \frac{1}{2}P_3 + \frac{1}{3}P_4 + \frac{1}{6}(P_5 + P_6) & \leq 150 \\ & 0 \leq P_1 \leq 100, & 0 \leq P_2 \leq 60, \\ & 0 \leq P_3 \leq 200, & 0 \leq P_4 \leq 60, \\ & 0 \leq P_5 \leq 60, & 0 \leq P_6 \leq 90. \end{array}$$

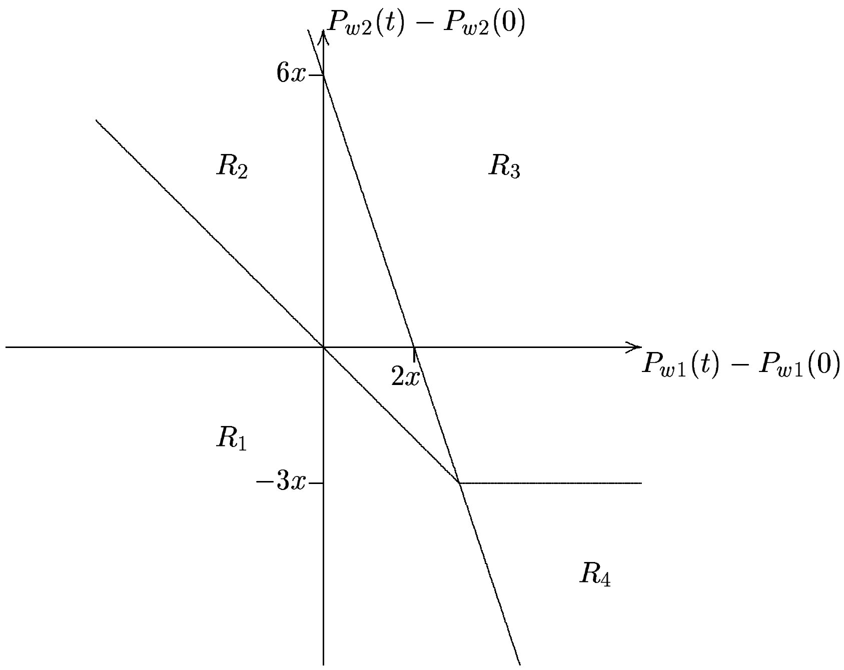


Figure 12: Response of the six-node network to wind changes.

The solution is $P_1 = P_2 = P_3 = P_4 = P_5 = 60$ and $P_6 = 0$. (Note that even though the thermal offers are cheaper than the hydro, the transmission constraint has prevented their full use.)

However, this leaves the limited-capacity line constrained, which will make it difficult to accommodate subsequent fluctuations in wind power. If the output of either wind farm (or both together) should increase, the extra wind power cannot be used to displace hydro power from H_1, as this would increase the flow on our constrained line. Even a slosh from Wind 2 to Wind 1 (i.e. a decrease in output at Wind 2, and an equal increase at Wind 1) could not be dealt with, although a slosh from Wind 1 to Wind 2 could be.

It might, perhaps, be a better idea to choose a different initial dispatch which does not put the network into such a constrained configuration. Suppose we require the initial dispatch to reserve spare capacity (headroom) x in the limited-capacity line, for later use in responding to changes in the wind. The best way to achieve this can be found by changing the right-hand-side of the inequality constraint to $150 - x$ in the dispatch problem, and re-solving. Furthermore, the additional cost (at the margin) of creating the headroom can also be read off: it is the value at optimality of the dual variable corresponding to the inequality constraint of the dispatch problem.

We will assume that $0 \leq x \leq 20$; to create each unit of headroom within this range requires shifting 3 units of generation from Thermal 1 to Thermal 2; the cost is thus $3 \times (45 - 40)$, or \$15 per unit headroom per unit time. (To create more than 20 units of headroom, some generation must be moved from Thermal 2 to Hydro H_2; this requires further analysis, which we omit.)

Now consider the situation at time $t > 0$. The power available from the wind farms is now $P_{w1}(t)$ and $P_{w2}(t)$; we may adjust the Hydro dispatch, but not the Thermals, in response.

The optimal response is the solution to a new version of our linear program in which only the Wind and Hydro dispatches are variables, with the Thermal dispatches being constants. Less formally, we can observe that each unit of headroom on our limited-capacity line allows the network to carry 2 units of extra power from Wind 1, all else being equal. (The extra wind power displaces hydro power, effectively requiring a new flow from Wind 1 to the Hydro to be superposed onto the existing flows; half of this new flow travels via the limited-capacity line.) Similarly, each unit of headroom allows 6 units of hydro power to be displaced by wind power from Wind 2. The network will thus remain unconstrained provided

$$\frac{1}{2}\Delta P_{w1}(t) + \frac{1}{6}\Delta P_{w2}(t) < x, \tag{29}$$

where $\Delta P_{wi}(t) = P_{wi}(t) - P_{wi}(0)$ for $i = 1, 2$.

The responses are shown in Figure 12. The network is unconstrained in regions R_1 and R_2. In R_1, the total available wind power has declined ($\Delta P_{w1}(t) + \Delta P_{w2}(t) < 0$), requiring Hydro H_2 to be dispatched to make up the shortfall; in R_2, additional wind power has displaced generation from Hydro H_1. In regions R_3 and R_4, the network is unable to carry all of the available wind power, so some must be spilled. It is always better to spill wind at Wind 1 than at Wind 2, since the amount that must be spilled is smaller (by a factor of 3); the amount of usable wind power is thus constant along horizontal lines in these two regions. In R_3, there has been a decrease in hydro dispatch, while in R_4 there has been an increase. Note that in part of R_4 the total *available* wind power has increased, but the total *usable* wind power has decreased, due to sloshing from Wind 2 to Wind 1.

Let $\Delta P_w(t) = (\Delta P_{w1}(t), \Delta P_{w2}(t))$. The change since $t = 0$ in the wind power used by the system is

$$u(x, \Delta P_w(t)) = \begin{cases} \Delta P_{w1}(t) + \Delta P_{w2}(t), & \text{if } \Delta P_w(t) \in R_1 \cup R_2 \\ 2x + \frac{2}{3}\Delta P_{w2}(t), & \text{if } \Delta P_w(t) \in R_3 \cup R_4 \ . \end{cases} \tag{30}$$

The instantaneous cost of generation at time t (relative to the least-initial-cost solution) is thus

$$c(x, \Delta P_w(t)) = 15x - \rho(u(x, \Delta P_w(t))), \tag{31}$$

where

$$\rho(z) = \begin{cases} 48z, & \text{if } z \geq 0 \\ 60z, & \text{if } z \leq 0 \ . \end{cases} \tag{32}$$

The overall expected cost of reserving headroom x for the time period $0 \leq t \leq T$ is then

$$C(x) = \mathrm{E}\left[\frac{1}{T}\int_0^T c(x, \Delta P_w(t))\, dt\right] = \mathrm{E}\left[c(x, \delta)\right], \tag{33}$$

where $\delta = \Delta P_w(\tau)$, with τ, as usual, a random variable independent of $\Delta P_w(t)$ and uniformly distributed on $[0, T]$.

If $(P_{w1}(t), P_{w2}(t))$ has a continuous probability distribution, then

$$\begin{aligned} C'(x) &= \mathrm{E}\left[c_x(x, \delta)\right] \\ &= \mathrm{E}\left[15 - \rho'(u(x, \delta))u_x(x, \delta)\right] \\ &= 15 - 2(48\mathrm{P}(\delta \in R_3) + 60\mathrm{P}(\delta \in R_4)). \end{aligned}$$

Note that $C'(x)$ is increasing in x, and hence $C(x)$ is convex in x. To find the optimal solution for x, we must attempt to solve the equation $C'(x) = 0$, that is, to find $x \in [0, 20]$ with

$$2(48\mathrm{P}(\delta \in R_3) + 60\mathrm{P}(\delta \in R_4)) = 15. \tag{34}$$

The left-hand side of this equation represents the marginal value of the water that might be saved by allowing an additional unit of headroom; the right-hand side the marginal cost of the headroom. A solution will fail to exist only if $C'(0) > 0$ (in which case the optimal solution is $x = 0$) or $C'(20) < 0$. In the latter case, it may be that $x = 20$ is optimal, or it may be that even more than 20 units of headroom are called for.

4 Analysis of wind farm data

Data on wind velocity and power output, from existing and potential sites, is useful for two purposes. First, it is needed as input into models for parts of the grid that are affected by wind-farms. Secondly, it can provide understanding of how wind and power are related across sites, in time; it will inform us about sloshing, evening-out and possibly forecasting. During the MISG week, group members examined the wind speed data provided from the Tararua wind farm, and also the Garrad Hassan report [3]. The first item raises questions about the properties that wind speed data needs so that it is fit for purpose. The second raises questions about further analysis of the dataset that it uses.

4.1 Desirable qualities in wind-speed data

The main purpose for wind speed data is to estimate wind-power output, and to investigate how this rises and falls. Hence the need is for time-series that mimic the behaviour of a turbine (or farm). The wind meter or meters need to be in the right location and at a suitable height. The series also need to be collected at time intervals or by equipment that makes the power estimates from them behave like turbine output.

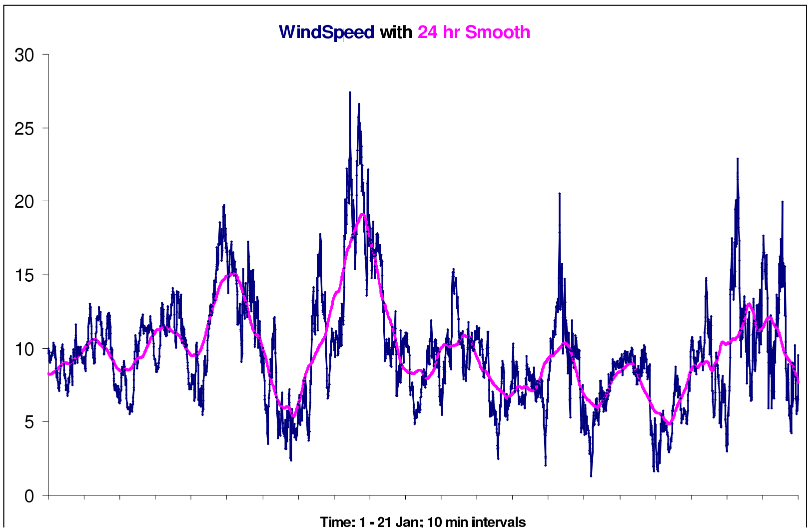
The Tararua data is measured at 10 minute intervals, and is very volatile over these intervals. The data was studied for a 21 day period, which gives 3024 ten-minute intervals, 144 per day. This data was converted to a smoothed time series, a day effect, and residuals. The smoothed series was calculated as the moving average of 144 values, and centred by averaging adjacent pairs of the moving average values. The day effect was calculated after subtracting the centred moving average.

The 21-day data is shown in Figure 13. The actual series contains: cycles of about three days in length, a small day effect and residuals that are much larger than the day effect and of similar size to the cycles. Figure 14 shows behaviour over one day. The five years of data could also be examined for an annual effect, but the MISG group did not have the resources for this.

The day effect appears to have two components: a smooth part that rises and falls in an autocorrelated way, so probably reflects a small but persistent effect over these 21 days, and a fluctuating part that one would expect in the means of 21 independent haphazard events.

The residuals show haphazard or volatile behaviour: short-term fluctuations from one reading to the next. They also show autocorrelated movements of length about half a day. The smoother has excluded these from the smoothed series of course. They need further investigation.

The main conclusion about this data is that the actual series has short-term fluctuations that are large compared with the features of real interest. For future data collections, these would need to be removed either by meter design or by logging data at short intervals and then processing it to separate the components. In wind generation, we are interested in change; if



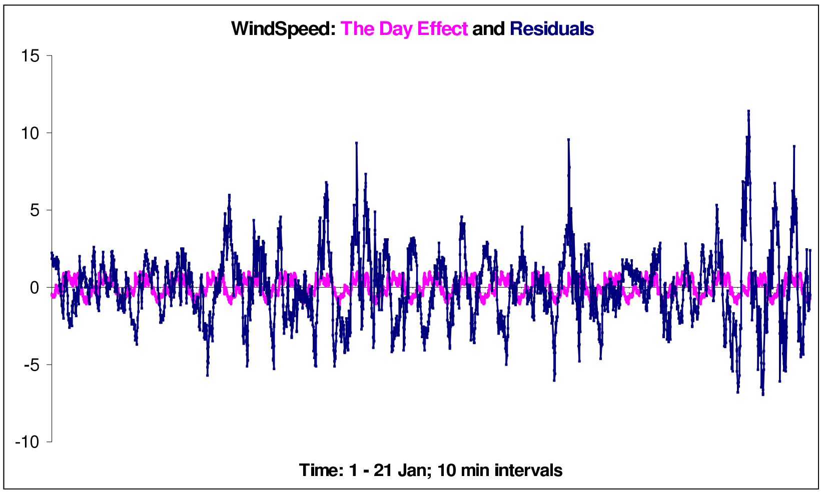
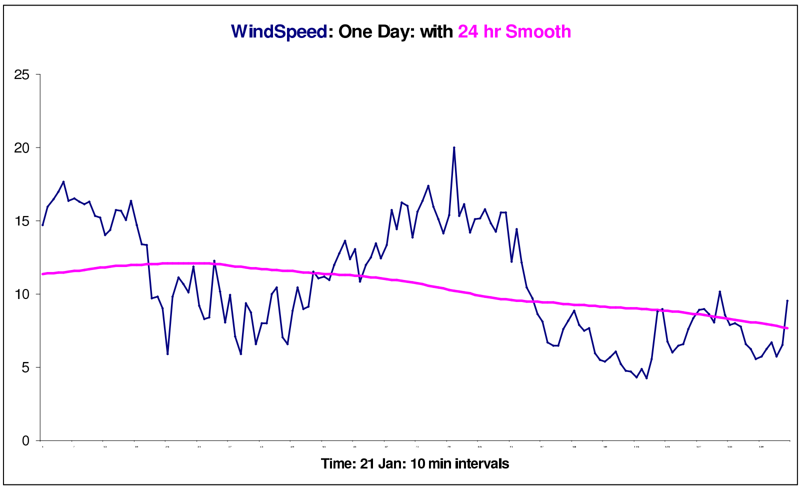


Figure 13: A 21-day part of the Tararua wind speed time series data; (a) (*above*) Blue - the actual data series; Magenta - a 24-hour smoothing of this; (b) (*below*) Magenta - the overall day (diurnal) effect repeated for each day; Blue - the residuals after subtracting the smoothed values and the day effect.



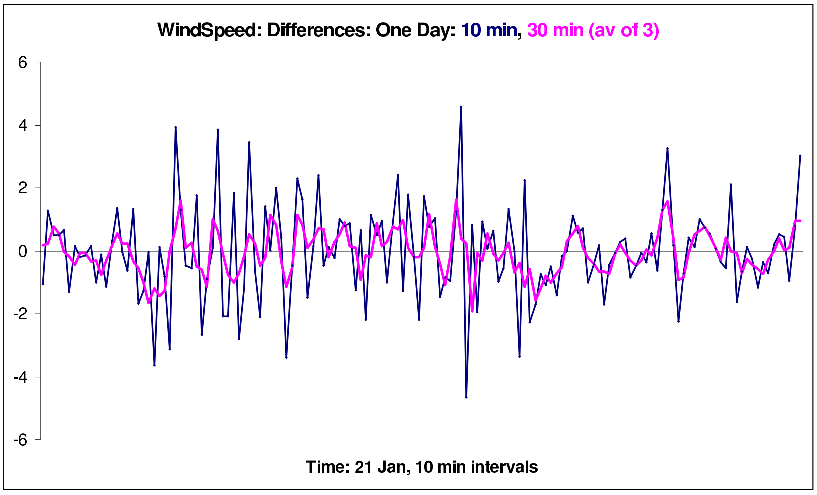


Figure 14: Data from a single day; (a) (*above*) Blue - wind speed; Magenta - a 24-hour smoothing of this; (b) (*below*) Differences: Blue - 10-minute intervals; Magenta - 30-minute intervals.

we look at change by differencing this series, the short-term fluctuations are even more of a nuisance (Figure 14).

The study helped clarify what we should look for in wind speed data and the issues include these: how does wind speed behave over time, and in particular how does it rise and fall? Hence the series of differences is important. Some of this behaviour may be predictable. We can expect (from meteorology) cycles of irregular lengths of a few days, daily cycles and annual cycles. These three features can be planned for. We would expect plenty of autocorrelation. Unfortunately, we also found plenty of short-term variability. This dominated the series of differences.

The study also clarified how data should be collected. A turbine has inertia, and will therefore smooth out short-term effects. A collection of turbines will have a further smoothing effect. There are two solutions. The first is to use a meter that mimics the behaviour of a turbine or collection; the second is to use a meter that measures velocity at short intervals (one minute or less), with its inherent volatility, but then smooth the data with a set of weights that mimics the smoothing effect of the turbine's inertia and size. Further smoothing could imitate the smoothing effect of the collection of turbines. The time-interval needs to be much shorter than the typical ramping-up or ramping-down time for a turbine.

In mimicking the behaviour of a turbine, we need, as well as smoothing, to transform speed into power. If speed and power are collected for an existing turbine, this data can be used immediately to plot the power/speed function for this type of turbine (at this location). In converting speed into power, we need to look for a lag effect: initially when wind-speed rises, some wind-energy may be accelerating the turbine rather than producing power output.

It could be useful to log wind-speed at a shorter interval (like one second), and then examine it for the short-term variability. This may occur in a particular frequency-band. The results would assist in design of data collection (instruments and intervals) for the future.

Variation and change is very important in wind generation, and leads to ramping of power output. If one site is more variable than usual, then it will be more complex and more expensive to manage substitutes for its down-times. Hence data collection design needs to enable analysis of variation.

4.2 Data for relationships among sites

The Garrad Hassan report uses a rich dataset that contains wind-speeds (and theoretical power outputs) at 10 minute and longer intervals, for 16 North Island and 5 South Island sites, for over a year. The report contains a thorough look at two aspects of this data: "correlations" between sites, and examination of events with large rates of change.

A common question at the workshop was "Are any of the correlations negative?". In fact the report uses R^2 values, which measure the strength of the linear part of the relationships. These relationships can be investigated further, by looking for non-linearities and relationships with time-lags. A second aim could be to compare variability of sites, since that affects the usefulness of the site as an energy source. North and South Islands have separate parts of the grid, so relationships within islands are the main interest.

A data visualisation approach would involve steps like the following.

Firstly, produce the matrix of scatterplots, and join the dots. The scatterplots reveal much about the relationships (strength, direction [positive or negative], linearity or non-linearity, presence of clusters and outliers). The dot-joining will reveal something about the behaviour of the relationship over time, such as whether cycles in two series are in phase or out of phase.

Secondly, plot two, more, or all series against time together, and look for whether they move in phase together, out of phase, or independently. Then lag one series by a range of time intervals and check whether this strengthens or weakens the relationship.

In the above steps, it may be necessary to choose the time interval between readings, and the length of series used, so as to best reveal the behaviours.

The approach could provide useful information on how a set of wind-farms behave, so that this behaviour can be fed into models of parts of the grid.

5 Conclusions

The team working on this MISG problem have considered issues relating to electric power grid management for the case when wind power is a significant proportion of the total power generation. The topic is of importance as the proportion of wind power generation in New Zealand may significantly increase in the future. A number of approaches and simple models have been used to study aspects of production and transmission. There is scope for further extension of the work.

The three-power generator models illustrates the problems in maintaining power supply at a reasonable cost as the wind power rapidly varies. We have approached the problem of load balance considering the case when the low-cost alternative to wind can only be ramped slowly.

In our later approach the model is simplified further by taking the low-cost power output to be constant. More complicated networks of power stations have been considered which have further included the problem of managing limited capacity in power transmission lines.

The MISG group also considered the use of measured data for predictability and time profile of the wind as this is important for wind power production.

Although wind-power generation provides great opportunities for meeting energy requirements, its planning and management presents new challenges. When these have been met we will be able to more fully utilise this resource.

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