Underreamer mechanics

Problem presented by

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Executive Summary

In the oil and gas industry, an underreamer is a tool used to extend and enlarge the diameter of a previously-drilled bore. The problem proposed to the Study Group is to obtain appropriate mathematical models of underreamer dynamics, in forms that will lead to feasible computation. The modes of dynamics of interest are torsional, lateral and axial. This report describes some initial models, two of which are developed in more detail: one for the propagation of torsional waves along the drill string and their reflection from contact points with the well bore; and one for the dynamic coupling between the underreamer and the drill bit during drilling.

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Contents

1 Introduction

1.1 Problem description

(1.1.1) In the oil and gas industry, an underreamer is a tool used to extend and enlarge the diameter of a previously-drilled bore. Appropriate mathematical models are needed of underreamer dynamics, in forms that will lead to feasible computation. The modes of dynamics of interest are torsional, lateral and axial. There are various key components to be modelled.

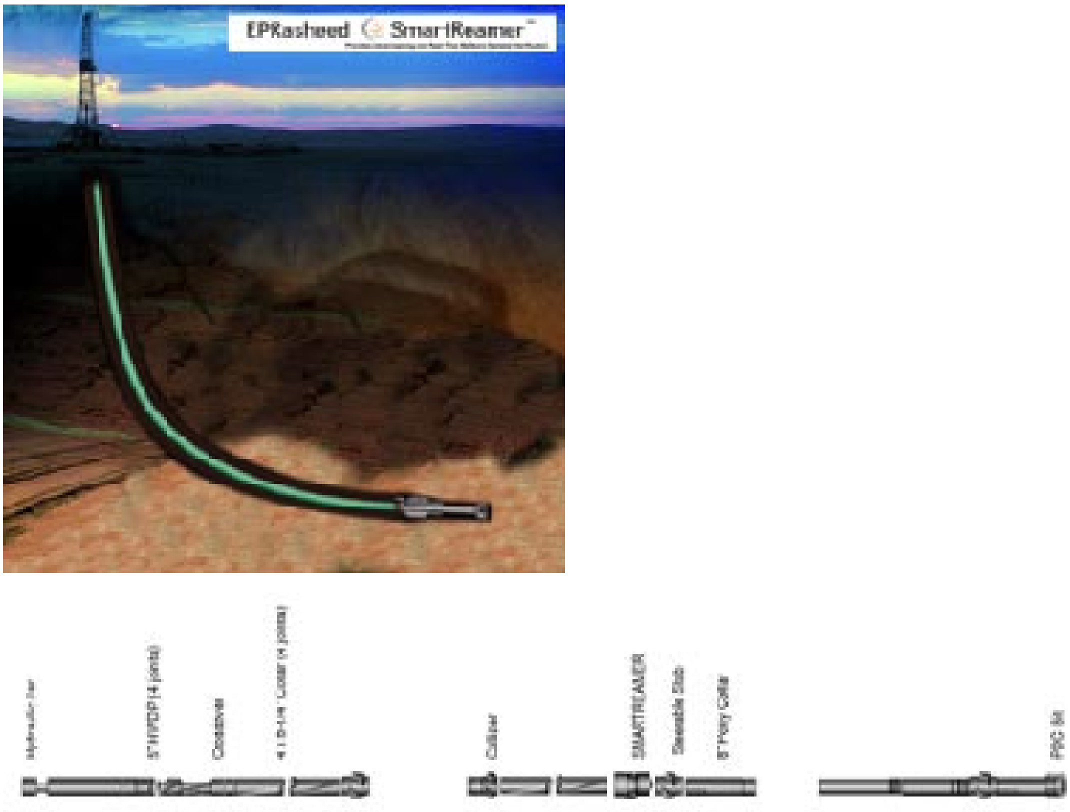

Figure 1: Schematic diagram

(1.1.2) Underreamer: The underreamer may be 12 feet long and is illustrated in Figure 1. It can pass through a pipe, of diameter say $8\frac{3}{4}$ inches, with its 3 cutter blocks retracted, and then the blocks can be expanded hydraulically to enable it to enlarge the bore to, say $9\frac{7}{8}$ inches. Further ahead of the underreamer is the leading drill bit, so rock cuttings are already in the mud flow past the underreamer. The cutters are positioned at 120° to each other round the axis, and the main back flow of mud and cuttings past the underreamer goes through the 3 "junk slots" in the circumferential positions between the cutter blocks. A jet of drilling mud is directed through a nozzle ahead of each cutter block in a further attempt to give it clear access to the rock face. It is expected that the main elastic deformation of the underreamer during drilling is in bending.

(1.1.3) Cutter blocks: The cutter blocks are mounted with cutting elements made of PDC (polycrystalline diamond compact). These will be subjected to highest loads, vibration and loads will be applied across different points of the cutter blocks.

(1.1.4) Drillstring: The drillstring between the surface and the underreamer is so long that it undergoes significant torsional motion. It is rotated at a steady speed at the surface, say 60 or 120 rpm, but the rotation at the underreamer and bit will be unsteady and can have peaks of up to 1000 rpm. There is also up to 50 m of drillstring from the underreamer to the bit. The drillstring is free to move within the bore and may make contact at various points along its length. Such points known as torque or drag hotspots could be identified.

(1.1.5) Rock and bore: Prior to the passage of the underreamer the rock faces of the bore will already be uneven. The bit is steerable, so the centreline of the bore is generally neither vertical nor straight: it curves and may have horizontal sections.

1.2 Requirements of model

(1.2.1) The underreamer dynamics are expected to show various kinds of wear and instabilities depending on both the surface parameters — rotation rate, weight on bit (WoB), flow rate *etc.*— and on the downhole parameters — formation hardness, wellbore inclination *etc.*

These parameters may adversely affect the underreaming operation and the resulting hole size, concentricity and rugosity. The model should specifically take into account:

- All types of loading and bending moments based on stabilisation points and wellbore contact points.
- Resonant and non-resonant vibration, *i.e.* torsional, axial, lateral as well as eccentric, sudden, nonresonant vibration modes releasing stored torque.

(1.2.2) The model could also be refined to model bit and underreamer wear in varying formations and investigate the effects that differing formation loading would have on both. The main challenge is to construct a model of underreamer dynamics, meeting the above requirements, that can be used both to highlight wear and avoid premature failures due to inherent tool design weak spots or usage related to surface or downhole parameters; and to optimise the rate of penetration and hole cleaning.

2 Basic mathematical problems and models

We here describe some of the basic geometrical and mechanical models that are involved in this problem.

2.1 Drillstring dynamics

(2.1.1) The fundamental situation we consider to start with is illustrated in Figure 2. Given the geometry of the bore (which is not known accurately),

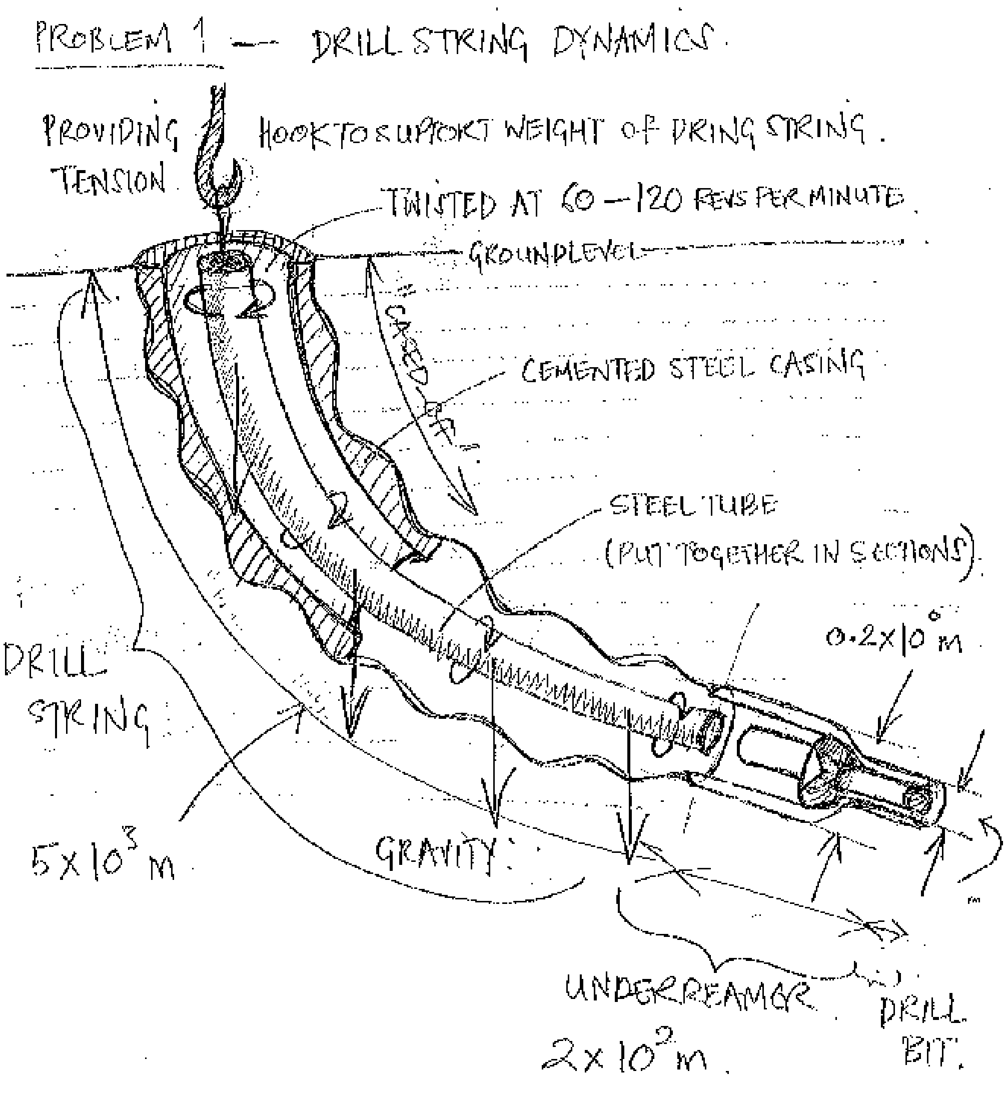

Figure 2: Schematic of drillstring dynamics problem

the material properties of the drillstring, the rotation rate or torque at ground level, and the hook tension, we would like to be able to predict the weight on bit (the normal force at the bottom of the borehole), the bending moment on the underreamer and the rotation rate or torque at the underreamer and bit.

2.2 Preliminary model

(2.2.1) A model that is obviously unrealistic but still illuminating is a planar model, in which there is no twisting or torque. It is, in effect, a planar model, and is illustrated in Figure 3. It is a clamped incompressible

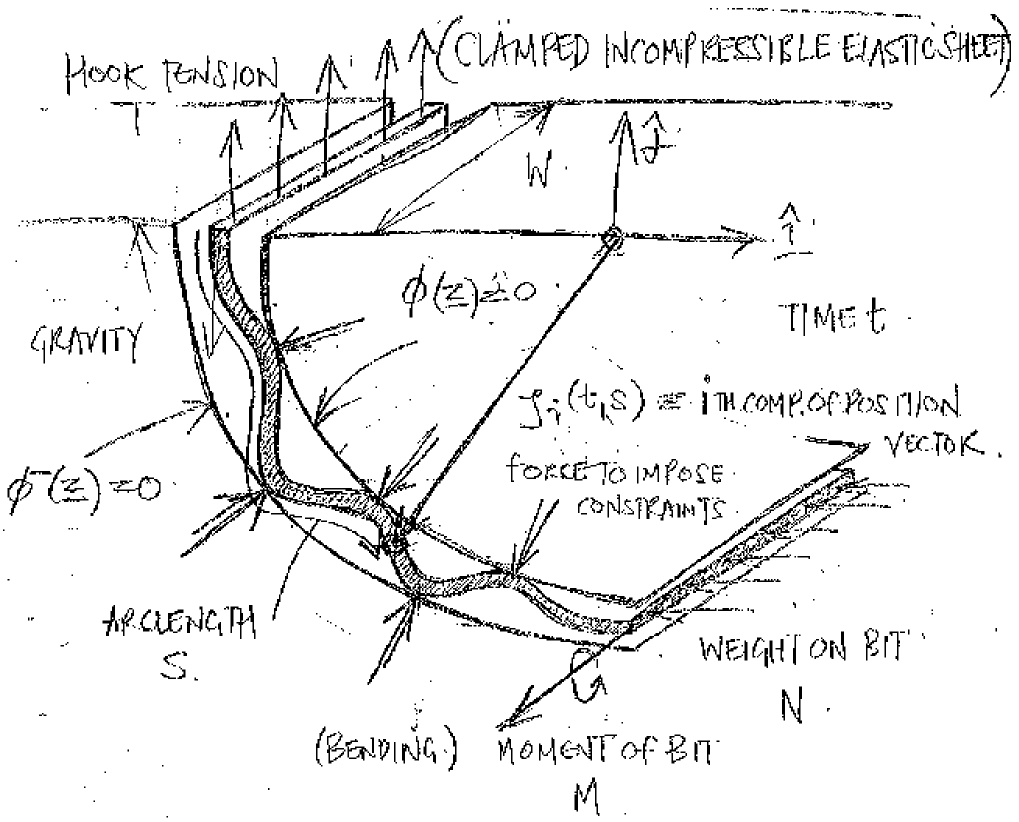

Figure 3: Schematic of planar model

elastic sheet confined between rigid walls $\phi^{\pm}(z) = 0$. Our plan is to describe the frictionless dynamics of this with a Lagrangian density, and then add in the frictional forces later. The Lagrangian density per unit width we write as

$$L = \tfrac{1}{2}\rho d \partial_t \zeta_i \partial_t \zeta_i - \rho d g \zeta_i \delta_{i2} - \tfrac{1}{2} e \partial_s^2 \zeta_i \partial_s^2 \zeta_i + \tfrac{1}{2}\Lambda(\partial_s \zeta_i \partial_s \zeta_i - 1) - \left[\Gamma^{\pm} H(\phi^{\pm})\right]_-^+, \tag{1}$$

where the first 3 terms represent the kinetic, gravitational and elastic energies, and the final 2 terms represent the inextensibility and confinement constraints. (In the last term H is the Heaviside step function.) The Lagrange multipliers Λ and $\Gamma^{\pm}$ will then be the tension in the string, and the reaction forces per unit length at the contacts on the upper and lower surfaces.

(2.2.2) The Euler-Lagrange equation then gives the dynamics in the form

$$(\rho d\partial_t^2 + e\partial_s^4)\zeta_j + \partial_s\left(\Lambda\partial_s\zeta_j\right) = -\left[\Gamma^{\pm}\delta(\phi^{\pm})\frac{\partial\phi^{\pm}}{\partial\zeta_j}\right]_-^+ - \rho dg\delta_{j2}. \qquad (2)$$

This has to be solved along with the inextensibility constraint $\partial_s\zeta_i\partial_s\zeta_i = 1$ and the containment constraints that $\phi^{\pm}(z) = 0$ for z on the upper and lower boundaries.

(2.2.3) A possible solution method would be to use the tangential component of the field equation to solve for the tension, and the normal component to solve for the curvature. The constraints have to be built in, and there can be two kinds of contact of the sheet with the wall: one where it is in contact at just one point, where there is a point reaction; and a second kind of contact where there is contact over an interval, with point reaction forces at each end of the interval and a distributed reaction force along the interval.

(2.2.4) If this problem is rescaled, it is natural to scale s and ζ with L, the length of the borehole, Λ and Γ with eL^-2, and then scale time t with $\sqrt{\rho dL^4/e}$. This results in the dimensionless equation becoming

$$(\partial_t^2 + \partial_s^4)\zeta_j + \partial_s(\Lambda\partial_s\zeta_j) = -\left[\Gamma^{\pm}\delta(\phi)\hat{n}_j\right]_-^+ - \alpha\delta_{2j}. \qquad (3)$$

The dimensionless constant that appears is $\alpha = \rho dgL^3/e$ (which we expect to be large since it very similar to the combination of parameters that occurs in the problem of pipelaying from a reelship).

2.3 Model with twisting and torque

(2.3.1) We now need to modify this kind of model to include the fully 3-dimensional geometry as illustrated in Figure 4. The centre-line of the drillstring can be described by the Frenet-Serret vectors, the curvature and geometrical torsion *etc.* However, for the purposes of a mechanical model like this, the *geometrical* torsion is not what matters, and instead one has to use the *mechanical* torsion. We do not write down the full equations, but they will take a similar form to those given for the planar model.

(2.3.2) It is then necessary to add in the frictional forces, using some appropriate model for their dependence on the normal forces at the contact points. (The purpose of using the Lagrangian approach for the frictionless case is that it gives a reliable way of getting the interaction between the geometry and the mechanics correct.)

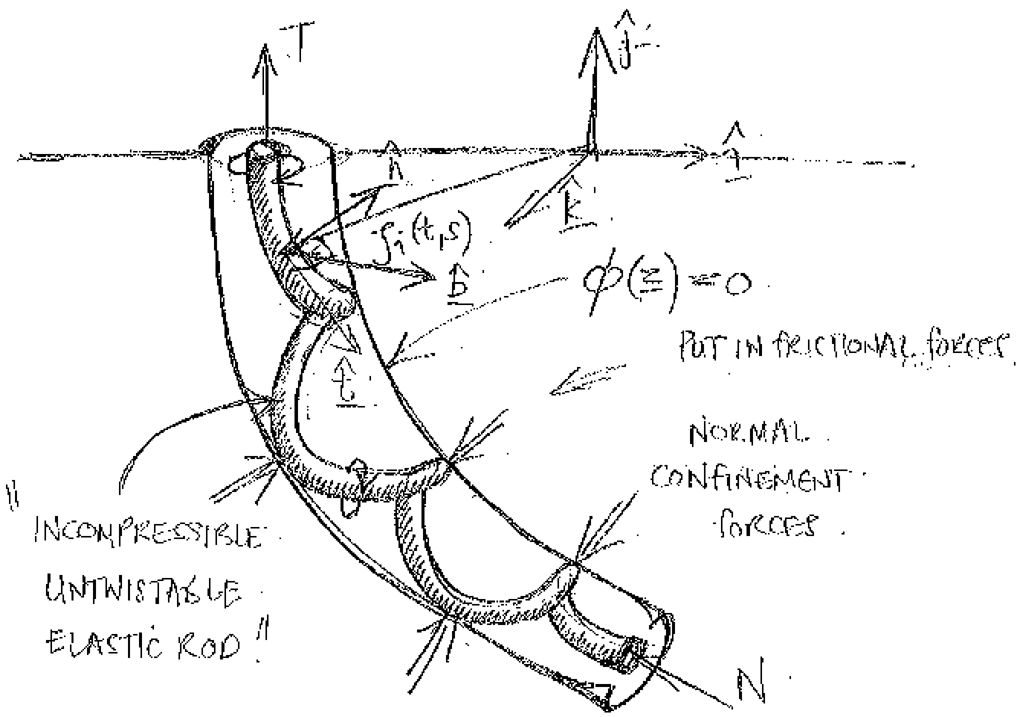

Figure 4: Schematic of three-dimensional model

(2.3.3) If this model is pursued, there would be two distinct regimes to consider. In one case there is more-or-less steady drilling, with the bit rotating and removing material. But in the other case the bit is stuck, and the applied torque builds up until eventually the combination of the torque and weight on the bit overcomes the formation hardness and drilling resumes.

3 Torsional waves in a simplified drillstring model

Based on the work of Tucker and Wang [1, 2, 3], we model the drill-string as a long slender rod under the standard assumptions of Cosserat theory (arbitrary deformations but small strains).[1]

3.1 Torsional wave model

(3.1.1) By considering only the twisting of the drill-string, we arrive at the torsional wave equation

$$J_s u_{tt}(x,t) = G u_x x(x,t) \tag{4}$$

[1]Tucker and Wang come up with a very general model, including axial, torsional and lateral motions of the drill-string. However, it is too complicated to do anything with in a week!

with boundary conditions given by a constant driving torque at the top and Coulomb friction ($F(u_t)$) at the base

$$\begin{aligned} u_t(0,t) &= \Omega, && (5) \\ J_b u_t t(L,t) &= \Gamma u_x(L,t) - F(u_t(L,t)). && (6) \end{aligned}$$

The Coulomb friction is approximated by a $\tanh(u_t/\epsilon)$ term with small ϵ. Using a travelling wave ansatz, these equations together can be reduced to a neutral delay differential equation of the form $\dot{v}(t) = f(v(t), v(t-\tau), \dot{v}(t-\tau))$ where τ is the time taken for a torsional wave to propagate from the tip of the drill-string to the top and back again (see [4]). However, the results are unphysical due to the lack of dissipation in the system.

(3.1.2) To overcome this lack of reality, viscous damping is added to the torsional wave equation to give

$$J_s u_{tt}(x,t) = G u_x x(x,t) - c u_t(x,t). \tag{7}$$

The addition of the damping term prevents the transformation to a delay equation (the dispersion relation becomes non-trivial and a delay equation would be needed for each individual mode, each with different delay times). Instead, we coded up a spectral PDE solver that uses Chebyshev polynomials for the spatial discretisation and Matlab's ode23s stiff solver for the time direction.

(3.1.3) Figure 5 shows a simulation of (7) with representative parameters taken from the literature [1]; the value for the damping constant is unknown and so we just pick one but experimentation suggests that there are no qualitative changes in dynamics when you change the damping. Also the parameters for the friction terms are a little dubious. The torsional wave propagation can clearly be seen. However, the torsional waves eventually die out (there is a question here as to the ratio between static and dynamic friction constants; in the undamped model torsional waves only persist if the static friction constant is sufficiently larger than the dynamic friction constant).

(3.1.4) To simulate the presence of contact points along the length of the drill-string, boundary conditions are introduced with additional friction terms. Figure 6 shows the results of the simulations with a single contact point at (a) the top, (b) the middle and (c) the tip. Contact points near the top of the drill-string appear to have very little influence. However, contact points near the tip and middle of the drill-string are much more influential on the dynamics. In particular, contact points near the middle of the drill-string appear to create additional torsional waves that interfere constructively/destructively with the torsional waves created at the tip of the drill-string leading to much larger changes in the angular velocity.

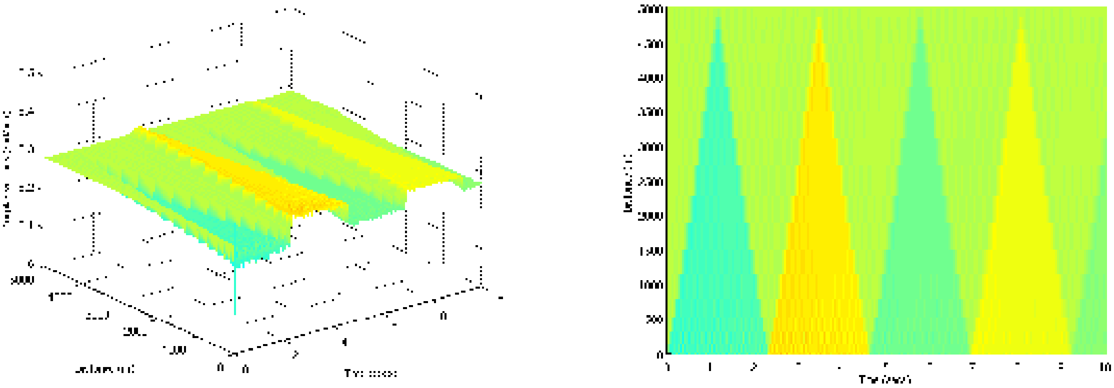

Figure 5: A simulation of (7) with a small weight-on-bit. Torsional waves can clearly be seen bouncing between the top and the tip. The torsional waves were instigated by the initial conditions and eventually die out. The panel on the right is a top-down view of the panel on the left.

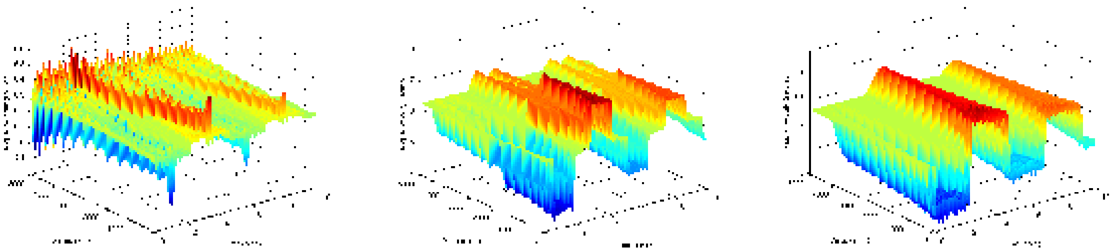

Figure 6: Simulations of (7) with contact points at the top, middle and tip (respectively). Note the scales on the axes. It appears that contact points near the top have very little effect on the overall dynamics. Contact points near the tip have much more of an effect; however, it is contact points near the middle of the drill-string that are of the most concern due to the constructive/destructive wave interference.

(3.1.5) Other literature that may be of interest is the work by Gert van der Heiden (UCL) on the constrained buckling of drill-strings (again a Cosserat approach but from the static perspective).

4 Coupled propagation of underreamer and drill bit

We now describe a different aspect of the problem under consideration, namely the coupling between the underreamer and the drill bit as drilling proceeds.

4.1 Progression of the underreamer and drill bit

(4.1.1) One aspect of underreaming explored at the study group was the progression of the underreamer and drill bit through a geological formation of varying resistance. The underreamer and the drill bit are both driven forward by a combination of the thrust from the weight of the drill string and the torque from the rotation supplied at the surface. These two aspects are both necessary when drilling through hard formation; rotation without weight on the bit will not lead to successful drilling, neither will merely pushing through the rock.

(4.1.2) Some mathematical modelling of drilling has been done in the past; see for example Wojtanowicz and Kuru [5] on bit wearing. However, this earlier work only considered an isolated drill bit, rather than a drill bit connected to an underreamer. One opportunity for practical and interesting mathematical modelling is to consider the progression through geological formation of a drill bit connected with an underreamer. Both the underreamer and the drill bit require torque and normal force in order to cut the formation, but the balance of forces between the underreamer and the drill bit will change over time in a manner that should be amenable to analysis.

4.2 Structure of the problem

(4.2.1) The underreamer is connected to the drill bit by a long (about 40 m) structure that contains various devices for steering the drill bit and performing other important functions. Effectively, this structure can be treated as a solid steel tube with an outer radius of about 10.8 cm. Steel has a Young's modulus of $2 \times 10^{11}\,\mathrm{Nm}^{-2}$.

(4.2.2) The drill bit and the underreamer both contain smaller cutters that act to chip away the formation. These cutters gouge into the formation and tear off chips of rock that are then flushed/floated back to the surface. The rate at which the drill bit and underreamer are able to progress through the formation is naturally dependent on the sharpness and arrangement of their cutters. For the rest of the work that follows, however, we will ignore this finer structure and simply assume that the rate of progression depends on the normal force acting on the surface of the rock and on the speed of rotation.

(4.2.3) Moreover, we will assume that the weight that drives the underreamer and drill bit through the formation can be treated as a simple pushing force acting at the underreamer. That is, we assume that the force supplied by the weight of the drill string is known at the underreamer and that the weight of the structure connecting the underreamer with the drill bit is comparatively small and able to be neglected. Similarly, we will assume that the rotation of the drill string can be modelled by treating the drill string as a torsional spring of known length and rigidity attached to the underreamer at one end and being rotated at constant velocity at the other end. In a fuller model, some description of the dynamics of the drill string would be required to supply the needed information about the weight-on-underreamer and the rotation of the string.

(4.2.4) During the study group, we discussed two variations of the underreamer progression problem. In the first simple model, we ignore torsion and assume that the drill bit and underreamer both turn at a constant rate that is sufficient for drilling. In the extended model, we also consider variations in the rate of drill rotation. As the rotation-variation model was not developed very far, we only discuss the weight-driven model in this report.

4.3 Weight-driven progress

(4.3.1) We are interested in the progression of the drill-bit and the underreamer through the formation with reference to some arbitrary starting point. Let $b(t)$ represent the distance drilled by the drill bit since $t = 0$. Thus, $b(0) = 0$ and b increases with time. Similarly, let $g(t)$ represent the length of the structure connecting the underreamer with the drill-bit, so that the location of the underreamer is at $b(t) - g(t)$. If g is greater than some natural length, g_0, there will be elastic forces pulling the drill-bit backwards and the underreamer forwards. Similarly, if $g < g_0$, the drill-bit will be pushed forwards and the underreamer pushed backwards.

(4.3.2) In the case where progress depends only on weight, we will assume that the velocity of the drill-bit, $b'(t)$ depends linearly on the difference between the force at the drill-bit and some critical force that is needed for motion. This critical force will depend on $R(b(t))$, a dimensionless measure of the 'drillability' (or inverse hardness) of the formation at the position $b(t)$. This leads to the following equation for the velocity of the drill-bit:

$$\frac{db}{dt} = k_{\text{bit}} \left(\left(\sigma_{\text{wob}} - E\,\frac{g(t) - g_0}{g_0} \right) A_{\text{bit}} - W_{\text{bit}}^{\text{crit}}\, R(b(t)) \right)^{+}, \tag{8}$$

where k_{bit} is a constant of proportionality that gives the velocity of the drill bit when the force acting at the bit is known, σ_{wob} is the stress due to the weight of the drill string, E is Young's modulus for steel, A_{bit} is the area of the bit, $W_{\text{bit}}^{\text{crit}}$ is a typical critical weight required for drilling and all other terms are as described above. The superscript $+$ represents the positive part function defined by

$$(x)^{+} = \begin{cases} x, & x > 0, \\ 0, & x \le 0. \end{cases} \tag{9}$$

(4.3.3) Similarly, the velocity of the underreamer is assumed to depend on the difference between the force experienced at the underreamer and some critical force of order $W^{\text{crit}}_{\text{reamer}}$. This leads to an equation of the form

$$\frac{db}{dt}-\frac{dg}{dt} = k_{\text{reamer}} \left(\left(\sigma_{\text{wob}} + E\,\frac{g(t)-g_0}{g_0} \right) A_{\text{reamer}} - W^{\text{crit}}_{\text{reamer}}\, R(b(t)-g(t)) \right)^{+}, \tag{10}$$

where k_{reamer} is a constant analogous to k_{bit} and A_{reamer} is the surface area of the underreamer.

(4.3.4) We can nondimensionalise b and g by scaling with respect to g_0 and we can nondimensionalise t with respect to $\frac{g_0}{k_{\text{bit}}\,W^{\text{crit}}_{\text{bit}}}$. This leads to the following dimensionless system of first order ordinary differential equations:

$$\begin{aligned} \frac{db}{dt} &= (\alpha - \beta\,(g-1) - R(b(t)))^{+} && (11)\\ \frac{dg}{dt} &= (\alpha - \beta\,(g-1) - R(b(t)))^{+} \\ &\quad - \gamma\,(\alpha + \beta\,(g-1) - \delta\,R(b(t)-g(t)))^{+}, && (12) \end{aligned}$$

where the nondimensional parameters α, β, γ and δ are defined as follows:

$$\alpha = \frac{\sigma_{\text{wob}}\,A_{\text{bit}}}{W^{\text{crit}}_{\text{bit}}}, \quad \beta = \frac{E\,A_{\text{bit}}}{W^{\text{crit}}_{\text{bit}}}, \quad \gamma = \frac{k_{\text{reamer}}\,A_{\text{reamer}}}{k_{\text{bit}}\,A_{\text{bit}}}, \quad \delta = \frac{W^{\text{crit}}_{\text{reamer}}\,A_{\text{bit}}}{W^{\text{crit}}_{\text{bit}}\,A_{\text{reamer}}}. \tag{13}$$

(4.3.5) It should be noted that all of these parameters require some information obtained from empirical tests; they cannot be determined purely from physical properties of the underreamer and drill-bit that can be measured at the surface. We also note that a value of α that is too small could lead to no progress being made.

(4.3.6) To illustrate the properties of this system of equations, consider the case where all of the parameters are taken to be one and $R(x)$ is given by a periodic function, $R(x) = 0.9 + 0.12\sin x$. In this case, the resistance of the formation occasionally creeps above the critical value of $R(x) = 1$, preventing easy progress from being made until enough stress builds up in the column connecting the drill-bit with the underreamer. The lurching progress of the drill-bit and underreamer is depicted below in Figure 7.

(4.3.7) The reasons for this lurching can be seen more clearly by looking at $g(t)$, the length of the material connecting the drill-bit with the underreamer. As shown in Figure 8, this 'spring' slowly extends over a large period of time as the drill-bit moves through the formation faster than the underreamer. Eventually, the extension of the 'spring' builds up to the extent that the tensile forces are able to overcome the resistance of the formation

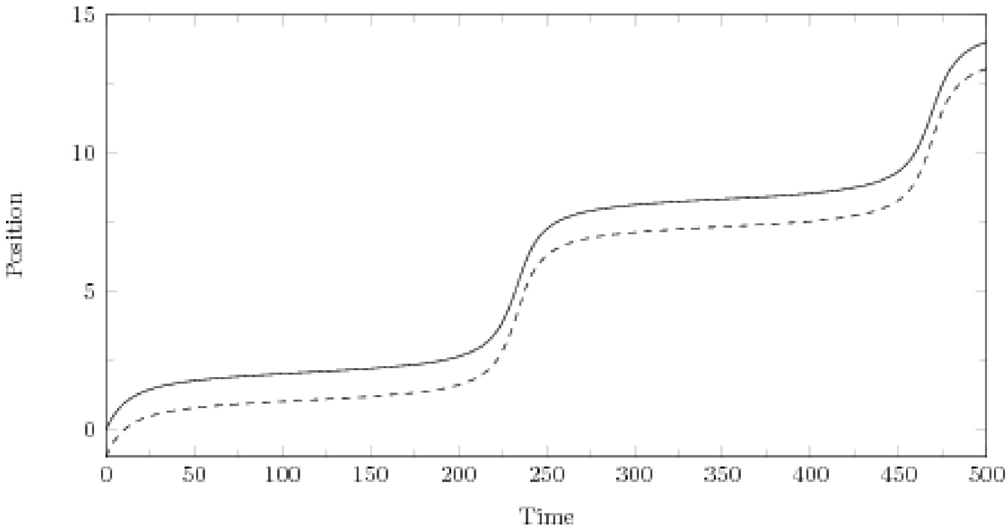

Figure 7: Progress of drill-bit (shown as continuous line) and underreamer (shown as dashed line), when $R(x) = 0.9 + 0.12 \sin x$ and all other constants are taken to be one. Note that the drill-bit and underreamer become effectively stuck for large periods of time, before breaking through the formation and continuing.

around the underreamer and the whole machine lurches forward, accompanied by a rapid decrease in the length of the material connecting the drill-bit with the underreamer.

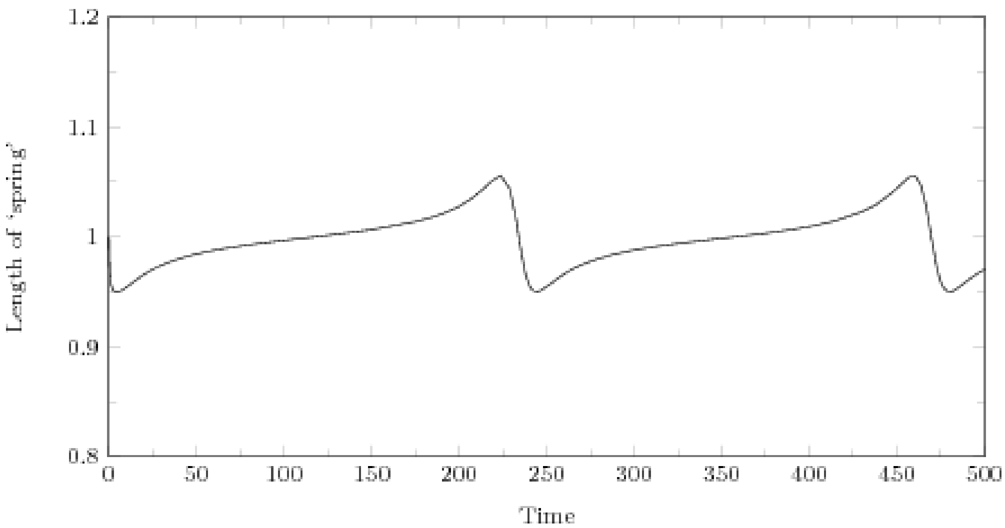

Figure 8: A plot of $g(t)$ over time, showing the changing length of the material connecting the drill-bit and the underreamer. This effectively acts as a spring, moderating the forces between the drill-bit and the underreamer.

(4.3.8) Hence, this model is able to capture some aspects of the underreaming process — specifically, the manner in which the progress through the formation is jerky and uneven. One interesting extension to this model would be to consider the case where the driving stress (as represented by σ_{wob}) is able to be varied. This could lead to an interesting control problem if there is an aim for the ideal speed of underreamer progress while the resistance of the formation is unknown.

5 Conclusions

5.1 General

(5.1.1) Clearly this is a modelling problem with many interconnected elements, and in this report we have begun to investigate some of the elements that go to make up the whole, but have not attempted to integrate them into a unified model. Nevertheless, some promising avenues for further investigation have been opened up by this work.

5.2 Control

(5.2.1) In particular, we have seen that the progression of the bit and underreamer through the rock can exhibit lurching, *i.e.* a behaviour in which the underreamer moves forwards in spurts, and this naturally could result in parts of the bore not being enlarged cleanly. To avoid this, some sensor and feedback control mechanism will be needed, either controlling the weight-on-bit or the rotation rate or both. Naturally, since the spatial variation in the resistance of the geological formation is not known beforehand, the control will have to be planned to cover the whole range of expected hardness.

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