

## PROBLEM 1

### THE FEASIBILITY OF CASTING STEEL IN A CONTINUOUS SHEET

#### 1. INTRODUCTION

The general concern of this problem was to investigate the feasibility of casting sheet steel by direct solidification of molten metal on a large, cooled, rotating drum or wheel. If such a procedure were possible, it would lead to considerable savings, notably on capital and energy costs associated with rolling steel slabs or billets down to sheet metal thickness. The typical design parameters that were envisaged by the clients (BHP Melbourne Research Laboratories) were to cast sheet steel between 1 and 10 mm thick and at peripheral drum speeds of about  $1 \text{ m sec}^{-1}$ . The Mathematics-in-Industry Study Group (MISG) was specifically asked to investigate the heat transfer and fluid mechanics in the immediate region where the molten metal was poured onto the wheel. We broadened this goal to consider heat transfer in the metal sheet up to the end of the solidification process, but did not consider subsequent cooling from solidification down to a temperature where coiling or finishing rolling could be applied.

Substantial preliminary work was undertaken before the MISG. In particular, a computerised literature search was made based on the key words "continuous", "casting" and "steel sheet or strip". The search unearthed more than 50 references (abstracts and some articles passed on to the BHP representative), although it was immediately clear that only a few of these were useful. It was apparent that several groups in the U.S.A. planned to spend of the order of U.S. 10 million on investigating the problem, but they did not appear to have pilot plants operating yet and their technical research findings were generally not published. One of the references (Swanson et al., 1984) was both technical and directed towards the same goals as the present

study. However, as explained later, we disagree with them on many aspects of the work.

The other major preliminary preparation was to have extensive discussions with Drs Day, Dunlop and Foley at the CSIRO Division of Applied Physics. This group have a lot of experience with the melt spinning of glassy metals: that is, the production of very thin ribbons of metal with amorphous structure by squirting molten metal directly onto a rapidly spinning small wheel. In particular, Dr Foley was of great assistance and gave a background talk on melt spinning at the MISG. The Applied Physics group had succeeded in slowing down their melt spinning process to such an extent that crystalline metal was formed, but only coarse samples of metal strips had been so obtained. Clearly, much more experimental work is required.

We would like to frame the rest of this report under the headings of general fluid mechanics considerations, (three) simple heat transfer calculations to give the puddle length, fluid mechanics stability considerations, and conclusions.

## 2. GENERAL FLUID MECHANICS CONSIDERATIONS

Careful geometric design should minimise the importance of fluid motion in the process. Basically, the molten metal should be poured onto the wheel in as smooth a fashion as possible, rather as shown in Figure 1(a) below. The thickness and mean speed of the feed stream of molten metal should be adjusted to approximate the design thickness of the steel sheet and the speed of the wheel's surface. Nozzles could be designed to minimise fluid effects: for example, a short nozzle would mean that fully developed plane Poiseuille flow would not exist at the exit of the nozzle. This would presumably ease the transition to free surface shear flow of the molten steel layer on the wheel. Attention to these precepts would mean that fluid motions above the solidifi-

cation front passing upwards through the metal layer could be neglected in a preliminary calculation to obtain the molten puddle length.

On the other hand, we believe it would be disastrous to squirt molten metal normal to the wheel from some distance as shown in Figure 1(b). This procedure would involve the risk of metal splashes, liquid metal going the wrong way (against the direction of the wheel), and transient effects such as waves and ripples on the surface of the molten metal sheet.

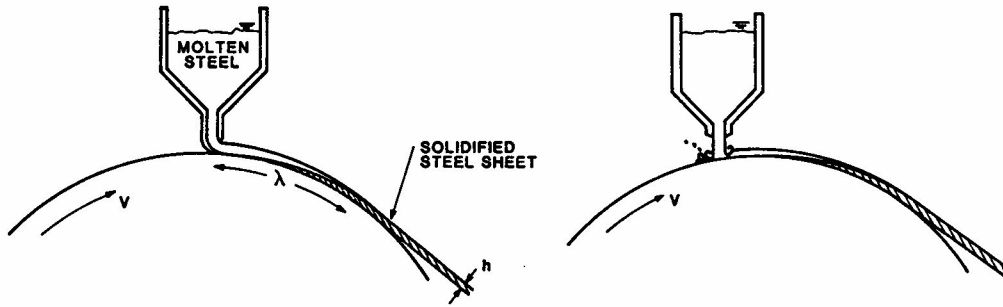


Figure 1(a). The correct way: molten steel poured smoothly onto the rotating wheel.

Figure 1(b). The wrong way: molten metal squirted onto the rotating wheel.

It is clear that solidification would need to be completed before the metal sheet was removed from the wheel and before the sheet achieved such an angle to the horizontal that surface instabilities had a chance to grow and become significant. Accordingly, we believe that the most important calculation is to obtain the length of the molten puddle on the wheel (neglecting fluid motions above the solidification front), and this problem is attacked in 3 different ways in the next section. It is believed that curvature of the wheel's surface and surface tension do not affect the approximate heat transfer calculations, although these effects should be considered in a full

feasibility study. Some possible sources of fluid instabilities are described in Section 4.

### 3. HEAT TRANSFER CALCULATIONS FOR THE PUDDLE LENGTH

A useful heat transfer calculation has been given by Kuiken (1977). His model considers a moving belt or wall maintained at a constant temperature below the freezing temperature emerging into a semi-infinite fluid at uniform temperature above freezing. Solidification occurs adjacent to the moving surface, and Kuiken has analysed the temperature and velocity fields using the boundary layer equations for fluid motion and by looking for similarity solutions. This enables a calculation to be made of the heat transfer through the moving wall, and thereby provides an input to the calculation of Swanson et al. (1984). Swanson et al. considered heat transfer from a solidifying steel layer to a conducting metal wheel or drum, and arrived at numerical results for molten puddle length and wall temperature for various values of the peripheral wheel speed and wheel diameter and at a fixed small sheet metal thickness. Unfortunately, Swanson et al. did not give information for a range of values of the sheet metal thickness. What we desire is an expression for the molten puddle length for general values of sheet thickness, wheel speed, and temperature drop from the molten metal to the deep core temperature in the wheel. We attempt to provide this expression by three different arguments in this section.

#### Model 1

The first model comes from the theory of heat transfer in solid rollers. Suppose that the heat to be extracted comes only from latent heat liberated as the liquid steel sheet solidifies. The latent heat extraction required per unit time per unit width of the sheet is

$$Q_L = \rho_S L V h$$

where  $\rho_s$  is the steel density,  $L$  the specific latent heat per unit mass,  $V$  the peripheral speed and  $h$  the sheet thickness. From strip rolling theory (Yuen, 1985), the heat transferred per unit time per unit width to the roller is

$$Q_c = \frac{2}{\sqrt{\pi}} \beta k_c (V\lambda/\alpha_c)^{\frac{1}{2}} (T_s - T_c)$$

where  $\beta = k_s \sqrt{P_s} / (k_c \sqrt{P_c} + k_s \sqrt{P_s})$ ,  $k$  is the thermal conductivity,  $P = V\lambda/\alpha$  the Peclet number,  $\alpha$  the thermal diffusivity,  $\lambda$  the molten puddle length,  $T_s$  the solidification temperature,  $T_c$  the deep core temperature in the drum, and subscripts  $s$  and  $c$  refer to the sheet steel and roller (copper) properties respectively. It is assumed that the wheel or drum is sufficiently thick that Yuen's results for solid rolls are applicable and we have neglected radiative heat transfer from the top surface of the steel sheet.

After algebraic manipulation, we obtain the expression

$$\lambda = \frac{\pi}{4} \left[ \frac{\sqrt{\alpha_c}}{k_c} + \frac{\sqrt{\alpha_s}}{k_s} \right]^2 \rho_s^2 L^2 V h^2 (T_s - T_c)^{-2}.$$

By substituting the approximate numerical values  $\alpha_c = 10^{-4} \text{ m}^2 \text{ s}^{-1}$ ,  $k_c = 400 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ , (appropriate for copper),  $\alpha_s = 4 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ ,  $k_s = 20 \text{ W m}^{-1} \text{ }^\circ\text{C}^{-1}$ ,  $\rho_s = 7.6 \times 10^3 \text{ kg m}^{-3}$ ,  $L = 2.7 \times 10^5 \text{ J kg}^{-1}$ ,  $V = 1 \text{ ms}^{-1}$ ,  $h = 0.01 \text{ m}$ ,  $T_s = 1400 \text{ }^\circ\text{C}$  and  $T_c = 150 \text{ }^\circ\text{C}$ , we find the estimate for  $\lambda$  of 3.8 metres.

Sensible heat effects can also be incorporated in a crude way. If the temperature profile in the solidified sheet is considered to be linear (as is also assumed in model 2 below), the heat extraction due to sensible heat is

$$Q_s = \frac{1}{2} \rho_s V h C_s (T_s - T_w)$$

where  $C_s$  is the specific heat of the sheet steel and  $T_w$  the wall temperature given by (Yuen, 1985)

$$T_w = T_c + (T_s - T_c) \beta.$$

If  $Q_c$  is now equated to the sum of  $Q_L$  and  $Q_s$ , the following expression for  $\lambda$  is obtained

$$\lambda = \frac{\pi}{4} \left[ \frac{\sqrt{\alpha_s}}{k_c} + \frac{\sqrt{\alpha_s}}{k_s} \right]^2 \rho_s^2 \left[ L + \frac{1}{2} C_s (T_s - T_c)(1-\beta) \right]^2 V h^2 (T_s - T_c)^{-2}.$$

With  $C_s = 600 \text{ Jkg}^{-1} \text{ } ^\circ\text{C}^{-1}$  and the same data used above,  $\lambda$  is approximately 17 metres. Note that these estimates for  $\lambda$  would be decreased by allowing for radiative heat transfer at the top of the steel sheet.

### Model 2

The previous model gave an estimate of the molten puddle length based on the amount of heat that could be transferred away through the underlying material. The second model looks at heat transfer processes in the solidified steel sheet, and balances the latent heat released at the solidification front against the heat which can be conducted through the solidified steel to the underlying material. Again, we neglect fluid motion above the solidification front; indeed, we assume that the overlying molten steel is all at a uniform temperature just above the solidification temperature of the metal. If it is further assumed that the temperature profile in the solidified sheet is linear and if we balance the rate of latent heat production against heat flux through the solid sheet, we have the equation

$$\rho_s L \frac{d\delta}{dt} = k_s (T_s - T_w) / \delta$$

in which  $\rho_s$ ,  $L$ ,  $k_s$ ,  $T_s$  and  $T_w$  are as before, and  $\delta$  is the distance of the solidification front above the wall. The above equation is a first order ordinary differential equation for  $\delta$  and it can be integrated to give  $\delta$  as a function of  $t$ . If  $t$  is identified with  $\lambda/V$ , the length of the puddle when the thickness is  $h$  is found to be

$$\lambda_0 = \frac{\rho_s L V h^2}{2 k_s (T_s - T_w)}$$

If we now substitute the previous numerical values and  $T_s - T_w = 800^\circ\text{C}$ , we get the estimate for  $\lambda_0$  of approximately 6 metres.

This model ignores thermal and flow conditions in the molten metal zone above the solidification front, and it forces a linear temperature profile in the solidified sheet. The validity of the linear temperature profile can be tested by using results of Carslaw and Jaeger (1980, pp.286-289) for the advancement of a solidification front into a stationary liquid. Carslaw and Jaeger's results show that the puddle length depends on the ratio of sensible to latent heat requirements according to the expression

$$\lambda = \lambda_0 \left(1 + \frac{2}{3}\gamma + \gamma^2\right)$$

where

$$\gamma = \frac{1}{2}(T_s - T_w)C_s/L$$

If we use the above data and  $C_s = 600 \text{ Jkg}^{-1}\text{C}^{-1}$ , we find that  $\gamma = 0.9$  which indicates that it is unwise to assume a linear temperature profile in the solidified sheet. It also implies that  $\lambda_0$  is a crude underestimate of the molten puddle length.

### Model 3

It is clear that the above two models need an improved calculation for the temperature profile in the solidified steel sheet. A model which looks at the coupled heat transfer in the sheet and the underlying material is now presented.

Again, we assume that the molten steel above the solidification front is at a constant temperature and that all the latent heat liberated at the front is conducted through the metal sheet. Moreover, assume there is perfect thermal contact between the solidified sheet and the underlying wheel. In coordinates moving with the speed of the wheel's surface, the temperature equations to be solved are

$$\begin{aligned} \text{sheet, } 0 < y < \delta(t), & \quad \frac{\partial T}{\partial t} = \alpha_s \frac{\partial^2 T}{\partial y^2}, \\ \text{wheel, } -\infty < y < 0, & \quad \frac{\partial T}{\partial t} = \alpha_c \frac{\partial^2 T}{\partial y^2}, \end{aligned}$$

subject to the conditions

$$\begin{aligned} \text{solidification front: } y=\delta(t), T=T_S, \quad L\rho \frac{d\delta}{dt} &= k_s \frac{\partial T}{\partial y} \Big|_{-}; \\ \text{metal interface: } y=0, T=T_W, \quad k_s \frac{\partial T}{\partial y} \Big|_{+} &= k_c \frac{\partial T}{\partial y} \Big|_{-}; \\ \text{core condition: } y \rightarrow -\infty, T &\rightarrow T_c. \end{aligned}$$

This problem has the similarity solution

$$\begin{aligned} \text{sheet metal: } T &= T_W + (T_S - T_W)\phi(\xi), \quad \xi = y(\alpha_s t)^{-\frac{1}{2}}; \\ \text{wheel: } T &= T_c + (T_W - T_c)\psi(\eta), \quad \eta = y(\alpha_c t)^{-\frac{1}{2}}. \end{aligned}$$

where  $\phi$  and  $\psi$  satisfy

$$\begin{aligned} \frac{d^2\phi}{d\xi^2} + \frac{1}{2}\xi \frac{d\phi}{d\xi} &= 0, \\ \frac{d^2\psi}{d\eta^2} + \frac{1}{2}\eta \frac{d\psi}{d\eta} &= 0. \end{aligned}$$

The solidification front is given by  $\xi=A$ , that is  $\delta(t)=A(\alpha_s t)^{\frac{1}{2}}$ ; and the conditions there are

$$\xi=A, \phi(A)=1, \frac{1}{2}AR=\phi'(A)$$

where the property  $\alpha_s=k_s/(\rho_s C_s)$  has been used to obtain

$$R = \frac{L}{(T_S - T_W)C_s}$$

At the interface between the sheet metal and the wheel, the conditions are found to be

$$\xi=0, \eta=0, \phi(0)=0, \psi(0)=1, \gamma\tau\phi'(0) = \psi'(0)$$

where

$$\gamma = \frac{(k_s \rho_s C_s)^{\frac{1}{2}}}{(k_c \rho_c C_c)^{\frac{1}{2}}} \quad \text{and} \quad \tau = \frac{(T_S - T_W)}{(T_W - T_c)}.$$

As  $\eta \rightarrow -\infty$ , we also have  $\psi(\eta) \rightarrow 0$ .

The solutions to the above equations are

$$\begin{aligned} \phi(\xi) &= \frac{\text{erf}(\xi/2)}{\text{erf}(A/2)}, \\ \psi(\eta) &= 1 + \text{erf}(\eta/2). \end{aligned}$$



Then, the interfacial conditions may be used to eliminate  $T_w$  in favour of A and other parameters; this gives (after simplification)

$$\frac{1}{2}\sqrt{\pi} A e^{A^2/4} [\gamma + \text{erf}(A/2)] = C_s (T_s - T_c)/L.$$

This transcendental equation has to be solved for A given the parameters  $\gamma$ ,  $C_s$ ,  $T_s$ ,  $T_c$  and L. Now, in this model, the molten metal will have disappeared when  $h = A(\alpha_s t)^{\frac{1}{2}}$  or, equivalently, when

$$\lambda = \frac{Vh^2}{A^2 \alpha_s}.$$

Again, this estimate for the puddle length is proportional to  $Vh^2$ , but it is now required to estimate A.

To this end, we note the previous numerical values and the additional approximate values  $C_s = 600 \text{ Jkg}^{-1}\text{C}^{-1}$ ,  $C_c = 400 \text{ Jkg}^{-1}\text{C}^{-1}$ ,  $T_s = 1400 \text{ }^\circ\text{C}$ ,  $T_c = 150 \text{ }^\circ\text{C}$ ,  $\kappa_s = 5 \times 10^{-6} \text{ m}^2\text{s}^{-1}$ ,  $V = 1 \text{ ms}^{-1}$ ,  $h = 0.01 \text{ m}$ . The constants  $\gamma$  and  $C_s(T_s - T_c)/L$  are found to be approximately 0.25 and 2.7 respectively, and the root of the transcendental equation is about  $A = 1.6$ . We find that  $\lambda$  is about 7.7 metres and  $T_w$  is about  $460 \text{ }^\circ\text{C}$ .

#### Conclusions based on the simple heat transfer calculations

At our level of approximation, the most that can be said of all these models is that they give estimates for the puddle length of about 10 metres at the design parameters of 0.01 m thickness and 1 m/sec speed. The results of model 3 should be more reliable than those of models 1 and 2. The results are considerably more pessimistic than those of Swanson et al. who predicted a puddle length of about 0.1 m for a sheet of thickness 0.005 m cast at 1 m/sec, whereas model 3 would give a puddle length about 20 times longer. Moreover, their results did not show the linear dependence with V that we have calculated and they did not mention the h dependence of their results.

#### 4. REMARKS ON THE STABILITY OF THE MOLTEN STEEL LAYER

Clearly, it is necessary that the molten steel layer should solidify before surface instabilities have time to grow and become significant. The most obvious instability that needs to be considered is the surface instability in fluid flowing down an inclined plane. This problem was analysed in the low Reynolds number, low wavenumber limit by Benjamin and Yih in the 1950s; they found that instabilities would occur (in the linear theory) when the Reynolds number  $R=2Uh/3\nu$  ( $U$  is the surface velocity) was greater than  $(5/6) \cot \theta$  where  $\theta$  is the angle of the layer to the horizontal. However, it was pointed out that the growth rates of these instabilities would be small, and the large Reynolds number behaviour of the neutral stability curve was unknown.

A second sort of stability was also discussed. This was related to unpublished work and examined the linear stability of profiles sketched in Figure 2 below. In the inviscid theory, this profile was stable to small disturbances for low Reynolds numbers, unstable at moderate Reynolds numbers, and stable again at large Reynolds numbers.

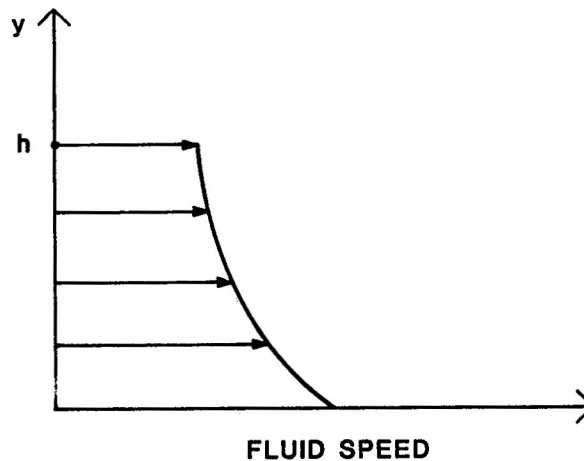


Figure 2. Typical velocity profile for the molten steel on the spinning wheel.

We therefore thought that many possible sorts of instability could manifest themselves, even without the complicating effects of solidification and surface tension. It was pointless trying to estimate growth rates for the possible instabilities without being guided by experiments as to which form of instability was the most important to look for. Therefore, we make the firm recommendation that an experimental program should be set up to look at possible instabilities on either a sloping surface or a rotating drum. In such a program, it would not be necessary to have the fluid solidifying, rather it would be important to get the nozzle geometry and the Reynolds number of the flow correct.

## 5. CONCLUSIONS

Our conclusions are as follows:

1. We feel that careful nozzle design should minimise fluid mechanical effects in the solidification process, and that it should then be possible to model the casting process by neglecting fluid motions and heat conduction in the molten layer overlying the solidification front.
2. Based on our estimates for the molten puddle length, it would seem to be impossible to continuously cast steel sheet at the design parameters of 0.01 m thickness and at a speed of about 1 m/sec on any reasonable size drum (say of diameter about 2 metres). Moreover, our estimates for the molten puddle length are much greater than those of Swanson et al.
3. However, all of our estimates for the puddle length are proportional to  $Vh^2$ ; hence, it may be possible to continuously cast thin strip of typical thickness 0.001 m.
4. An experimental program should be initiated to investigate possible instabilities in sheets of fluid flowing down inclined planes. Solidifi-

cation should not be considered here, rather the Reynolds numbers should be made to correspond to those anticipated in the continuous casting process.

5. Further analytical and numerical work is justified if instabilities do not appear to be troublesome and if thinner sheets and slower casting speeds  $V$  are acceptable. This work could include (a) the importance of fluid effects in the transition from the nozzle flow to the free surface shear flow on the drum, (b) temperature-dependent properties, (c) heat and mass transfer in the molten steel layer overlying the solidification front, and (d) surface tension (progressively more important for thinner sheets).

#### REFERENCES

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