Warping of Moulded Plastics

1. INTRODUCTION

During the injection moulding of plastics, the hot melt is forced into the mould under high pressure, which compresses the material considerably. The melt then solidifies, and during this process there may be large temperature gradients. The result of these effects is that on cooling the product may be under warping stress, to which it yields on release from the mould.

The course of events preceding solidification is complicated, and partly treated in another report in this volume. The state of the material may be characterised when molten by:

(a) the temperature distribution;

(b) the pressure distribution;

(c) the velocity distribution.

On freezing the velocity distribution is no longer required, but the pressure distribution is replaced by a more complicated stress distribution.

When partly frozen, in principle all these distributions must be calculated. We found it necessary to simplify by assuming that the stress distribution does not become important (except for a uniform pressure) until just after total freezing, and it is at this point in time that the analysis begins.

2. ELASTICITY THEORY

A linear stress-strain relation (Hookes' Law) is assumed, and also a linear dependence of thermal strain on temperature. In general the stress σ and the strain ϵ are second order tensors defined locally in the material, and the coefficient P relating them:

$$\sigma = P\epsilon \tag{1}$$

is a fourth order tensor.

The state immediately following freezing will be denoted with suffix 0, and after final release from the mould by suffix 1. Then σ_0 is just a uniform pressure tensor.

If there is material displacement v in going from state 0 to 1, then this affects the strain, but does not determine it, because the temperature change has shifted the equilibrium point. If f(T) is the tensor giving the rate of change of strain with temperature variation, and d is the apparent strain deduced from v, then the real strain ϵ_1 is given by:

$$\epsilon_1 = \epsilon_0 + d - \int_{T_0}^{T_1} f(T)dT \tag{2}$$

where d is derived from v by:

$$d_{ij} = \left(\frac{\partial v_i}{\partial x(j)} + \frac{\partial v_j}{\partial x(i)} - \frac{\partial v_k}{\partial x(i)}\frac{\partial v_k}{\partial x(j)}\right)/2 \qquad i,j = 1...3 . \tag{3}$$

The summation convention is used in this report, so terms involving the repeated suffix k are summed over k = 1 to 3.

The strain energy contained in a volume V is given by:

$$\text{Strain energy} = \int tr(\sigma^*\epsilon)dV = \int tr(\sigma^*P\epsilon)dV \tag{4}$$

where tr denotes trace (sum of diagonal elements) and * denotes transpose. The final shape adopted is that which minimises this energy.

This rather general tensor formulation can be used for anisotropic materials. The tensor P has only a few independent elements; the number depends on the extent of anisotropy. As indicated in the example below, for isotropic materials there are just two parameters which determine the tensor, and a simpler formulation is possible. The tensor f(T) also simplifies to just the temperature difference multiplied by the thermal coefficient of expansion, for many materials.

3. ESTIMATING THE INITIAL TEMPERATURE AND STRESS

It is assumed that in state 0 the melt is still fluid enough to move in response tc a pressure gradient, but does not move; therefore the stress everywhere is equal to the applied injection pressure.

The temperatures at the mould surfaces can be taken as known, and in state 0 the maximum, attained somewhere in the interior, is the freezing, or no-flow temperature. This is enough information to fit a quadratic distribution, at least where the mould surfaces are nearly parallel.

4. TESTING FOR MINIMUM STRAIN ENERGY

It is likely that one is not interested in the exact shape following warping, but rather in whether warping will occur at all; that is, in whether the desired shape minimizes the strain energy. The simplest way to test this is to calculate the strain energy for that shape, and then to calculate the energy for shapes warped in some plausible and easily computed way.

Example

Isotropic Material

Consider a sheet of isotropic material, laid on the z = 0 plane, thickness z_0. It has Young's modulus E and Poisson ratio σ. The Lamé constants λ and μ are defined:

$$\lambda = \frac{E}{2(1 + \sigma)}$$

$$\mu = \frac{E\sigma}{(1 + \sigma)(1 - 2\sigma)} .$$

Then the coefficients P_{ijkm} of the stress-strain tensor P are:

$$P_{ijkm} = \lambda\delta_{ij}\delta_{mk} + 2\mu\delta_{ik}\delta_{jm} .$$

[Love], where $\delta_{ij} = 1$ if i = j, 0 otherwise. This gives the strain energy per unit volume:

$$2W = \lambda(\text{tr } e)^2 + 2\mu \text{ tr } e^2$$

where tr denotes trace = sum of diagonal elements.

Suppose there was initially a uniform temperature gradient in the z-direction which produces, after freezing, a contribution to the strain given by

$$e' = h_1 z I$$

where I is the unit tensor (matrix), and h_1 is the coefficient of thermal expansion.

Then

$$2W = (9\lambda + 6\mu)(h_1 z)^2 .$$

Suppose the plate then deforms so that an additional strain tensor d is added (calculated from (3)). Then the new strain energy density is calculated using $e = e' + d$.

In this simple case, there is an explicit minimum. The displacement $u = -h_1 yz$, $v = -h_1 xz$, $w = -\frac{h_1}{2}(z^2 - x^2 - y^2)$ gives a tensor $d = -e'$, making the strain energy zero. The curvature then, which is a measure of warping, is just $2h_1$.

REFERENCE

A.E.H. Love, *The Mathematical Theory of Elasticity*, Cambridge University Press, Fourth Edition (1934).