FLOWS IN GAS PIPELINES

1. Introduction

This problem was brought to the 1989 Mathematics in Industry Study Group by Mr R. Calvert from The Pipeline Authority (TPA). TPA owns and operates a natural gas pipeline from Moomba to Sydney (with some smaller lateral pipelines supplying areas of population such as the ACT). Because of increasing demand, TPA has installed a compressor station at Bulla Park and it is expected that a further station will be commissioned at Young by the middle of this year. Safe and efficient use of compressors in a pipeline network is quite complex since high pressures and pressure fluctuations may damage the pipeline. As a consequence, the problem posed to the study group was to examine to what extent mathematical modelling can be used to provide guidance for the operation of compressor stations.

Gas flows in pipelines have been studied extensively and sophisticated software is available for calculating flows. For example, TPA use SIROGAS, a package developed by CSIRO, which has been extremely useful to TPA in their simulations of flows. Nevertheless, it was felt by the study group that a worthwhile first step was to examine some flow models with a view to gaining insight about time and length scales involved.

2. Simple Flow Models

Let us examine a pipeline of length L with one inlet and one outlet. This is a reasonable assumption as the bulk of the flow is between Moomba and Sydney. We shall denote variables pertaining to the inlet and outlet by subscripts 0 and 1 respectively. The notation is summarized at the end of the report.

Under the assumption that the flow is spatially one dimensional (in the direction of the pipe) we have by continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial U}{\partial x} = 0 \qquad (1)$$

Conservation of momentum can be written as (see, for example, Wilkinson, et. al., 1964)

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\left(\frac{U^2}{\rho}\right) + \frac{\partial p}{\partial x} + g\rho\frac{\partial h}{\partial x} + \frac{f\,U|U|}{2D\rho} = 0 \qquad (2)$$

while the equation of state is

$$p = \rho \frac{RT}{M} Z \tag{3}$$

where Z is a super compressibility factor which is used to account for deviations from the ideal gas law. In practice, the regression

$$Z = \frac{1}{1 + w\rho}, w = aT + b \tag{4}$$

is used where a and b are determined experimentally. A further equation is required to fully specify the system, namely an equation for conservation of energy. Unfortunately the coefficients for heat transfer between the pipeline and its environment might be expected to be poorly determined and a more appealing approach is to assume that the gas is in thermal equilibrium with its surroundings – an assumption that is not unreasonable except in a neighbourhood of the compressor and possibly the inlet. For simplicity, we made the further assumption that the gas temperature is constant.

We now make further simplifications to the momentum equation (3). Specifically we neglect the term $g\rho h_x$, which is the effect on elevation. Simulations using SIROGAS prior to the study group showed that variability in elevation has only a very minor effect on the flow. Thus, (2) becomes

$$\frac{\partial U}{\partial t} + \left(\frac{\partial p}{\partial \rho} - V^2\right)\frac{\partial \rho}{\partial x} + 2V\frac{\partial U}{\partial x} + \frac{fV|U|}{2D} = 0$$

where $V = U/\rho$ is the gas velocity. Now

$$c^2(\rho) = \frac{\partial p}{\partial \rho}$$

is the square of the 'isotropic speed of sound' in the gas and hence we also neglect the term $V^2\rho_x$ in comparison to p_x.

To make further progress, we need to address the question of what information we wish to obtain from our model. In this instance we are interested in the transport of gas and the appropriate length scale is L, the length of the pipeline. However, we expect the term VdU/dx to be important only over a length scale of

$$l_1 = \frac{D}{f} \ll L.$$

Similarly, the term U_t is important only over a length scale of

$$l_2 = \frac{CD}{Vf}$$

where $\bar{V}$ is a typical gas velocity. It turns out (see the calculations for steady state) that $l_1 \ll l_2 \ll L$ and we therefore neglect these terms. The momentum equation therefore is simplified to

$$\frac{\partial p}{\partial x} + \frac{fV|U|}{2D} = 0 \tag{5}$$

which is simply a balance between pressure gradient and frictional losses.

On combining (1) and (5), we obtain

$$\frac{\partial \rho}{\partial t} = \sqrt{\frac{2D}{f}} \frac{\partial}{\partial x} \left(\frac{c\rho^{1/2}}{|\rho_x|^{1/2}} \rho_x \right).$$

We now introduce the scaling

$$\rho = \rho^* \bar{\rho} \quad , \quad c = c^* \bar{c}$$
$$t = \tau s \quad , \quad x = Ly$$

where $\bar{\rho}$ and $\bar{c}$ are typical values of ρ and c. Then,

$$\frac{\partial \rho^*}{\partial s} = \tau \bar{c} \sqrt{\frac{2D}{fL^3}} \frac{\partial}{\partial y} \left(\frac{c^*(\rho^*)^{1/2}}{|\rho^*_y|^{1/2}} \rho^*_y \right)$$

and it makes sense to choose the time scale τ so that

$$\tau = \frac{L}{\bar{c}} \sqrt{\frac{fL}{2D}}.$$

In this case,

$$\frac{\partial \rho^*}{\partial s} = \frac{\partial}{\partial y} \left(\frac{c^*(\rho^*)^{1/2}}{|\rho^*_y|^{1/2}} \rho^*_y \right) \tag{6}$$

Equation (6) is a nonlinear diffusion equation and some discussion during the study group focussed on the possibility of linearizing (6) about the steady state solution. However, no progress was made on this during the time available. Nevertheless, the form of (6) does allow us to make some qualitative predictions about the solution. For example, we expect variations in flux at Sydney to decay by the time they are felt at Moomba and furthermore expect 'fast' variations to decay more rapidly than slow variations. This is observed to be the case in practice.

3. Steady State Flow

When the flow is steady and a simple ideal gas law (i.e. equation (4) with $a = b = 0$) is used, equation (1), (3) and (5) yield

$$\begin{aligned} U &= \text{constant} \\ p^2 &= p_0^2 - \frac{c^2 f U^2 x}{D} \end{aligned}$$

Thus, the flux is constant and the square of the pressure varies linearly with the distance along the pipeline. To obtain an appreciation of some of the parameters involved, it is useful to consider the following data supplied by TPA

$$\begin{aligned} L = 1300\text{km} \quad &, \quad Q = 2.5 \times 10^5 m^3/\text{hour (at STP)} \\ D = .864\text{m} \quad &, \quad p_0 = 6.2 \times 10^6\text{Pa} \\ & \quad\; p_1 = 4.7 \times 10^6\text{Pa} \end{aligned}$$

Taking the density of gas at STP to be .75 Kg/m^3 we obtain

$$\begin{aligned} f &\simeq .01 \\ \bar{V} = 2.4\text{m/sec} \quad &, \quad c = 365\text{m/sec} \\ l_1 = 86\text{m} \quad &, \quad l_2 = 1.3 \times 10^4\text{m}. \end{aligned}$$

The time taken for the gas to travel from the inlet to outlet is

$$\begin{aligned} \tau_1 &= \frac{2L(\rho_0^2 + \rho_1\rho_0 + \rho_1^2)}{3U(\rho_0 + \rho_1)} \\ &\simeq 167 \text{ hours} \end{aligned}$$

which is the same order of magnitude as the time scale associated with the diffusion equation

$$\begin{aligned} \tau &= \frac{L}{c}\sqrt{\frac{fL}{2D}} \\ &= 86 \text{ hours} \end{aligned}$$

This last time scale is in quite good agreement with the time lag of 77 hours obtained by a time series analysis.

4. Similarity Solutions

Some analytical progress can be made with equation (6) by means of similarity solutions. While of limited utility for general pipeline operation, these solutions do provide some insight to pressure fluctuations in the vicinity of the compressors. Following Wooding et. al. (1977), let

$$\eta = x/s^{2/3}$$

Then (6) has a solution $\rho^* = \rho(\eta)$ where

$$-\frac{2}{3}\eta\rho' = \left((-\rho\rho')^{1/2}\right)' \tag{7}$$

and $\rho(\infty) = \rho_\infty$. The solution to (7) is shown for various values of $\rho(0)$ in Figure 1. They correspond to instantaneous changes in density at $y = 0$ and are valid only for relatively short time scales when end effects and pressure gradients in the initial flow can be ignored. Note however that the flux at $y = 0$ is $O(s^{-1/3})$ and becomes unbounded is $s \to 0$. This poses a problem when modelling the start up of a compressor.

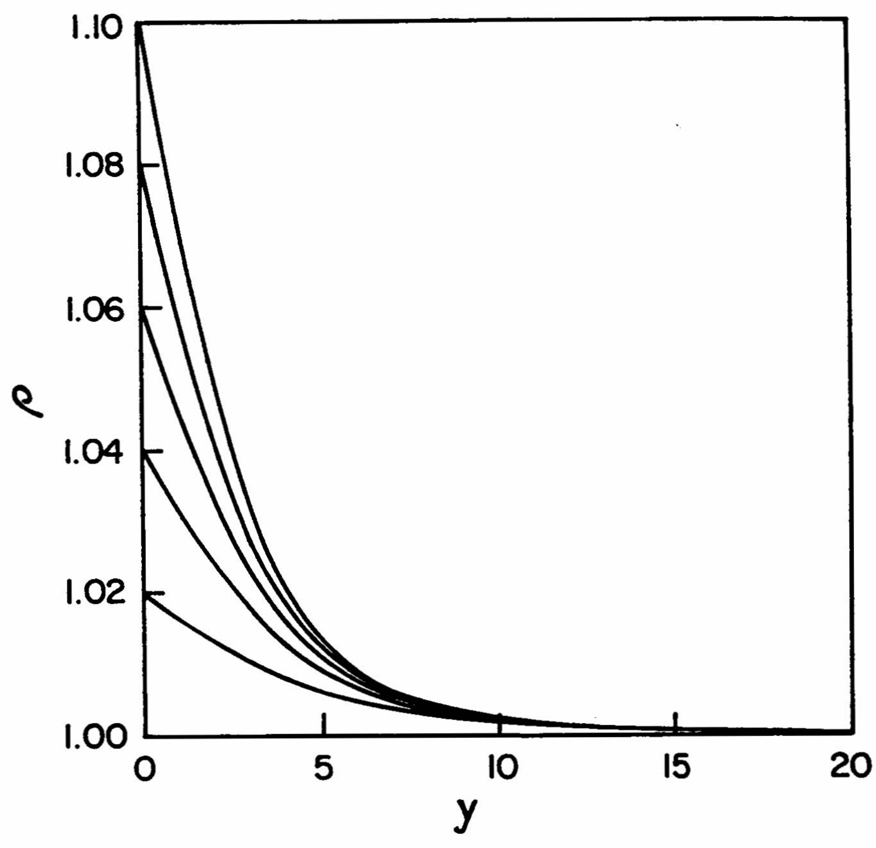

Figure 1: Solution of (7) with $\rho_\infty = 1$.

5. Boundary Conditions and Compressor Stations

Let us consider the scaled equation (6) and take $Z = 1$ in (3). Furthermore let us consider a compressor station at $y = y_c$. Since pressure is directly proportional to density, we may rewrite (6) as

$$\frac{\partial p^*}{\partial s} = \frac{\partial}{\partial y}\left(\frac{(p^*)^{1/2}}{|p_y^*|^{1/2}} p_y^*\right) \tag{8}$$

where $p = p^*\bar{p}$ and $\bar{p}$ is a typical pressure. In practice, the input pressure at Moomba ($y = 0$) is held fixed and it is therefore appropriate to specify

$$p^*(0) = p_0^* \tag{9}$$

At Sydney ($y = 1$) we can predict the required output flux

$$p^* p_y^* = F(s) \text{ at } y = 1 \tag{10}$$

where $F(s)$ is known.

However the presence of a compressor at $y = y_c$ poses some problems. Since the flux is continuous,

$$p^* p_y^* \text{ is continuous at } y = y_c \tag{11}$$

However a further boundary condition is required and a possible candidate is to specify a relative pressure jump at $y = y_c$. That is,

$$\lim_{y \to y_c^+} p^* = \alpha(s) \lim_{y \to y_c^-} p^* \tag{12}$$

where $1 < \alpha_L < \alpha < \alpha_U$ or $\alpha = 1$.

We therefore solve equation (8) subject to conditions (9), (10), (11) and (12). Since it is not possible to do this analytically, a numerical solution is necessary. However the boundary condition (12) imposes an unsatisfactory feature on the solution. When the compressor is turned on, the flux will initially be very large. This is evident from the similarity solution discussed previously. However, large fluxes violate constraints imposed on compressor operation. This difficulty led to considerable discussion at the study group although little progress was made due to inadequate knowledge of compressors and their operation. However, the consensus of opinion was that the time scale of the transient in question was small and could be ignored.

6. Time Series Analysis

An alternative to prediction via modelling is to use time series applied to pipeline data. Some preliminary analysis applied to two weeks of data when the compressor at Bulla Park was not operational was performed after the study group (the computer in Melbourne was down during the study group) and the results were quite encouraging.

Cross-correlation analysis of the flow at Moomba and the pressure at Sydney found that the pressure lagged the flow at Moomba by 77 hours. Figure 2 shows two weeks of flow data adjusted for the time delay. From the plot it can be seen that the peaks are aligned, though they are sharper in the pressure data than in the flow data.

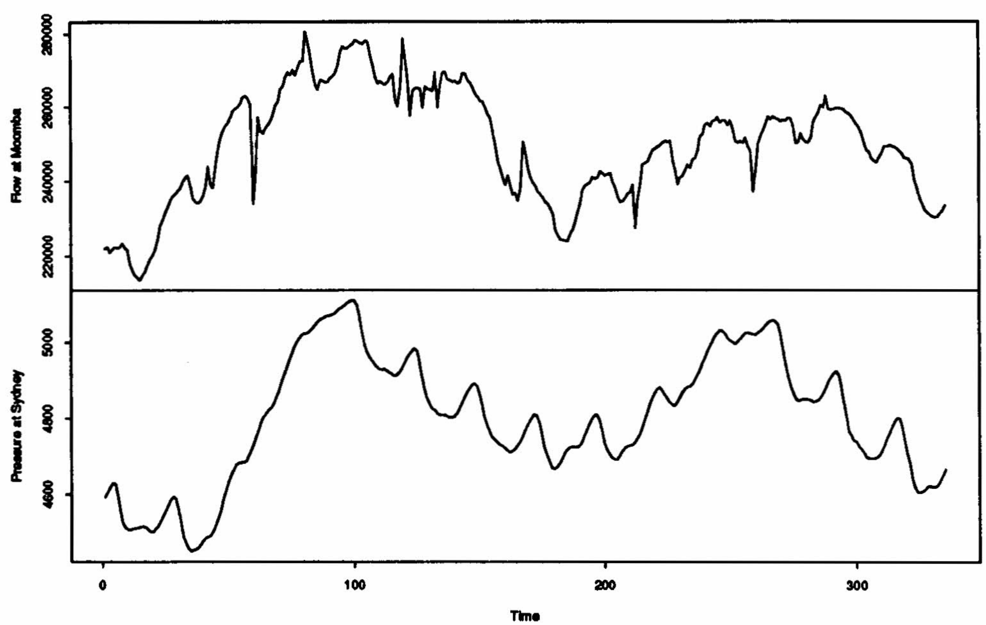

Figure 2: Flow and pressure adjusted by time delay

A regression of Pressure on Flow explains about 60% of the variance, though there remains substantial serial correlation in the residual pressure variation. This appears to be related to the daily and weekly cycles evident in the data. There is negligible correlation between the hour to hour fluctuations in the flow and pressure series. Further analysis suggests that the flow at Sydney is also correlated with the pressure at Sydney (several days later) and this could also be used in a better predictor of pressure at Sydney and thus ultimately in a control system.

7. Optimization of Compressor Operation

The formulation for optimally operating a pipeline is relatively straightforward although quantifying the costs involved is likely to be difficult.

Let the cost associated with starting the compressor be $\$W$ and the cost associated with running the compressor by $\$z$/hour. These costs should reflect not only compressor costs but also costs associated with damage due to pressure transients when starting the compressor. Although we have taken the costs to be constant, there is no reason in principle why they could not be complex functions of compressibility factor etc.

Introducing the function

$$\sigma(t) = \begin{cases} 0 & , \quad \alpha = 1 \\ 1 & , \quad \alpha > 1 \end{cases}$$

we obtain

$$m(t) = \frac{W}{2}\left|\frac{d\sigma}{dt}\right| + z\sigma$$

as the cost rate of operating the compressor.

We also have some constraints. The pressure at the inlet is held constant and the flux at the outlet is prescribed. Furthermore the outlet pressure should not drop below a prescribed minimum and should not exceed a prescribed maximum. With these constraints we need to choose $\alpha(t)$ to minimize

$$M(t) = \int_0^t m(t)dt.$$

The algorithmic details of how this optimization could be performed were not discussed at the study group.

8. Concluding Remarks

The consensus of opinion after the study group meeting was that:

- simple models are useful for modelling flows in gas pipelines when the aim is to obtain flow estimates for control purposes.
- time series analysis is a potentially useful tool for predicting pressure variation. Further work is required for this.
- further work is required to more accurately model compressors.

- the costs associated with the compressor operation and damage to the pipeline need to be quantified. If for example the cost associated with turning a compressor on dominated, the control strategy would be simplified considerably.

Notation

ρ	-	density
ρ_0, ρ_1	-	density at inlet and outlet
U	-	mass flux of gas
p	-	pressure
p_0, p_1	-	pressure at inlet and outlet
x	-	length coordinate
t	-	time
h	-	elevation of pipeline
g	-	gravitational acceleration
f	-	friction factor
D	-	diameter of pipe
L	-	length of pipeline
l_1, l_2	-	length scales
R	-	gas constant
T	-	temperature
Z	-	compressibility factor
M	-	molecular weight of gas
a, b	-	experimental constants in Z
V	-	gas velocity
c	-	isotropic speed of sound
$\bar{V}$	-	typical value of V
ρ^*	-	scaled density
$\bar{\rho}$	-	typical value of ρ
c^*	-	scaled speed of sound
$\bar{c}$	-	typical value of c
s	-	scaled time
τ	-	time scale
y	-	scaled length variable
Q	-	mass flow rate
τ_1	-	travel time for gas
η	-	similarity variable
y_c	-	scaled position of compressor

$F(s)$	-	scaled flow rate
α	-	compressibility factor
α_L, α_U	-	upper and lower limits of α
σ	-	step function
M	-	cost of compressor operation
m	-	cost rate of compressor operation

References

Wilkinson, J.F., Halliday, D.V. and Hannah, K.W.; Transient flow in natural gas transmission systems, American Gas Assoc., Arlington VA., 1964.

Wooding, R.A.; Numerical Calculation of Transient Flow of Natural Gas in Pipeline Networks, Tech. Rep 62, 1977, DSIR, Appl. Math. Div.