TRANSIENT MODEL OF A GAS-FIRED FURNACE

1. Introduction

The Selas furnace is part of the metallic coating process used by BHP Coated Products Division. Sheet steel, typically $\sim 1 \text{ mm}$ thick and $\sim 1 \text{ m}$ wide is passed continuously through the furnace and thereby annealed to its required temperature and cleaned of surface contaminants. A typical strip speed is 100m/min. Gas-fired burners inset into the furnace walls produce hot combustion products, which transfer heat to the strip via radiation and forced convection. The heat transfer can be controlled by varying the fuel rate supplied to the burners, as well as by turning burners on or off. A schematic of the furnace is shown in figure 1.

CPD have developed a steady state model of the furnace operation, which can be used to determine optimum operating conditions. However, there is a significant amount of time when a furnace is not operating in steady state, due to a change of product, or to some problem further downstream which may cause the strip speed to change. Hence it is desirable to develop a transient model of furnace operation. The aim is to use such a model to formulate control strategies that will minimise wastage due to incorrect heating of the strip when a transient occurs.

During the MISG this problem was examined in a number of ways including

- 1. Is the steady-state model appropriate?
- 2. What are the appropriate transient model equations?
- 3. What are the dominant effects (terms in the equations)?
- 4. How long does it take for an introduced transient to disappear?
- 5. What understanding can be gained from a linearised model?
- 6. Is heat conduction through the furnace walls significant?
- 7. Are there simpler models of radiation?
- 8. What type of numerical methods are appropriate for solutions?

This report is a summary of the attempts to answer these questions.



Figure 1: Cross-section through a Selas furnace.

2. Steady-state model

The steady-state model is based on consideration of heat transfer in the furnace. It assumes that the gas flow is upwards and is turbulent, so that it is well mixed across the furnace. Then all gas properties are dependent only upon the height variable x. All longitudinal radiation is ignored, so radiation is considered to be in a direction normal to the strip. The furnace walls are assumed to be adiabatic, so any heat absorbed by the wall is radiated back into the furnace. The combustion products of the burners are calculated to have a density and composition corresponding to the adiabatic flame temperature of the combustion process. Thermal conduction longitudinally in the strip is neglected and conduction through the strip thickness is ignored, because it is so thin. The

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result of these assumptions is the following set of equations,

$$\frac{dT_s}{dx} = \frac{Q_{gs}}{\dot{m}_s C_{ps}}$$
(1)

$$\frac{dT_g}{dx} = \frac{\dot{M}(h_{ad} - h_g) - Q_{gs}}{\dot{m}_g C_{pg}}$$
(2)

$$\frac{dm_g}{dx} = \dot{M}(x) \tag{3}$$

$$B_w - B_{gw} - q_{wc} = 0 \tag{4}$$

Equations (1-2) represent heat balances in the strip and gas respectively and equation (3) is a mass balance on the gas. Equation (4) is the wall heat balance. T_s and T_g are the strip and gas temperatures at height x in the furnace, \dot{m}_g is the total mass flow rate of gas and $\dot{M}(x)$ is the input gas flow rate; Q_{gs} is the rate at which heat is absorbed by the strip, due to both convection and radiation, \dot{m}_s is the downward mass flow rate of the strip and C_{ps} is its specific heat; h_{ad} is the enthalpy of the gas at its adiabatic flame temperature and h_g is the enthalpy of the gas in the furnace; C_{pg} is the gas specific heat; B_w is the wall "radiosity", *i.e.* the radiative flux emitted by the wall, B_{gw} is the radiative flux transferred from the gas to the wall and q_{wc} is the heat flux transferred to the wall by forced convection. Equation (4) is a fourth order polynomial in T_s , T_g and the wall temperature T_w .

The above set of equations comprises three o.d.e.'s and an algebraic equation for the 4 unknowns T_s , T_g , \dot{m}_g and T_w (which is related to T_s and T_g by the adiabatic condition (4)). They are nonlinear, due to the radiation terms. Three boundary conditions are required to complete the model specification. The boundary conditions chosen by CPD were

$$T_s(l) = T_{sl} \tag{5}$$

$$T_s(0) = T_g(0) \tag{6}$$

$$\dot{m}_g(0) = \dot{m}_{g0} \tag{7}$$

Here, T_{sl} and \dot{m}_{g0} are known constants, and represent the strip temperature at the top of the furnace (x = l) and a leakage flow of gas into the bottom of the furnace. The boundary condition (6) is based on the assumption that there is only a small amount of gas at the bottom of the furnace, which will equilibrate with the strip temperature.

The set of equations and boundary conditions that comprise the above steady state model have the problem that if the leakage flow m_{g0} is zero, then the equations are singular at x = 0. Consequently, the numerical solution is difficult. In order to remove the singularity, it was suggested that the numerator in equation (2) be set to zero at x = 0, *i.e.* replace boundary condition (6) with

$$\dot{M}(h_{ad} - h_g) - Q_{gs} = 0$$
 at $x = 0$ (8)

 $\langle \alpha \rangle$

 $\langle \alpha \rangle$

and set $\dot{m}_{g0} = 0$. Equation (8) is actually an algebraic equation relating T_s, T_g and T_w , but T_w can be eliminated using equation (4). A shooting method is required to solve this system, because the gas temperature at the bottom is unknown. A typical numerical solution for T_s and T_g is shown in figure 2. This result was obtained using a modified version of the original boundary conditions. If the original boundary conditions were used, the numerical solution exhibited the expected singular behaviour near x = 0, with the result that solution was difficult to obtain. However, the solution away from x = 0 seems fairly insensitive to the behaviour near x = 0.

It is interesting to note that an attempt was made to linearise the steady state equations, assuming that the gas inflow \dot{M} is a constant, so from equation (3) \dot{m}_g varies linearly with x. The result, obtained by eliminating T_g , is the following 2nd order o.d.e for T_s

$$xT_s'' + (l_1x + k_1)T_s' + (l_1k_1 - l_4k_2)T_s = l_4k_3$$
⁽⁹⁾

where the primes indicate differentiation with respect to x and l_1, l_4, k_1, k_2 and k_3 are constants, which were not evaluated. Equation (9) is an example of Kummer's equation, which has confluent hypergeometric function solutions (Abramowitz & Stegun, 1965). The equation has a regular singularity at x = 0. It has one solution which is finite at x = 0 and one which has a logarithmic singularity at x = 0.

3. Transient model equations

By using similar assumptions to the steady state model, it is fairly straightforward to derive transient model equations for the strip, gas and wall temperatures and the gas mass flow rate. They are

$$\frac{\partial T_s}{\partial t} - V_s \frac{\partial T_s}{\partial x} = \frac{Q_{gs}}{\rho_s w_s d_s C_{ps}} \tag{10}$$

$$\frac{\partial T_g}{\partial t} + V_g \frac{\partial T_g}{\partial x} = \frac{M(h_{ad} - h_g) - Q_{gs} + Q_{gw}}{\rho_g w_g d_g C_{pg}}$$
(11)

$$\frac{\partial m_g}{\partial x} = M - \rho_g w_g \frac{\partial d_g}{\partial t}$$
(12)

In these equations V_s is the strip speed, V_g is the gas speed and is given by

$$V_g(x,t) = \frac{\dot{m}_g}{\rho_g w_g d_g}$$



Figure 2: Strip and gas temperatures from steady state model (solid lines) and linearised model (dashed lines). Note the two different axes.

 ρ , w and d are the density, width and thickness respectively of the strip (subscript s) and the gas (subscript g). The final term in equation (12) allows for the possibility that the strip thickness d_s may vary, and hence the thickness of the gas flue d_g must also vary, since $d_s + d_g$ is constant.

It is possible that heat conduction through the furnace walls is not negligible during a transient phase, so an additional term, Q_{gw} where

$$Q_{gw} = P_w(B_w - B_{gw} - q_{wc})$$

has been included in equation (11), which represents the heat flux from the wall to the gas via conduction. P_w is the perimeter (*i.e.* surface area) of the

walls. Conduction in the walls may be assumed to be one dimensional, neglecting longitudinal conduction, so the appropriate equation is

$$\kappa_w \frac{\partial^2 T_w}{\partial y^2} = \frac{\partial T_w}{\partial t} \tag{13}$$

with the boundary condition at the inside of the wall (y = 0, say) being

$$-k_w \frac{\partial T_w}{\partial y} = B_{gw} - B_w + q_{wc} \tag{14}$$

Here, κ_w is the thermal diffusivity of the wall and k_w is its thermal conductivity. It is also necessary to have an equation of state, for which the ideal gas law, with constant pressure, is appropriate, namely

$$\rho_g T_g = \text{const}$$
(15)

The above model results in a set of coupled partial differential equations for the strip, gas and wall temperatures and the gas mass flow rate. The inclusion of wall heat conduction and time variation has led to a considerably more complicated set of equations that require solution. The remainder of this report, and indeed the majority of the activity at the Study Group, involves examining these equations in order to understand the important features of the equations and to ultimately develop efficient methods of solution.

4. Scaling

The following scalings are appropriate: $T_s - T_{sl}$ with Δ , the change in strip temperature from top to bottom in the furnace, $T_g - T_{sl}$ with Δ , x with l, the furnace length, and t with $\tau = l/V_{s0}$ where V_{s0} is a typical speed of the strip. Then τ is the transit time any point on the strip spends in the furnace. The scaled equations then become

$$\frac{\partial T_s}{\partial t} - \frac{V_s}{V_{s0}} \frac{\partial T_s}{\partial x} = \frac{\alpha l Q_s}{V_{s0}} \tag{16}$$

$$\frac{\partial T_g}{\partial t} + \frac{V_g}{V_{s0}} \frac{\partial T_g}{\partial x} = \frac{\beta l Q_g}{V_{s0}} \tag{17}$$

Here the nonlinear right hand sides of the equations for T_s and T_g have been incorporated into the single terms, Q_s and Q_g respectively. They have then been scaled with as yet unknown constants $\alpha\Delta$ and $\beta\Delta$. The idea of the above scalings is that the most obvious time scale for a transient to disappear is τ , which is the time it takes for a piece of strip to pass through the furnace. We then want to know if the heating process, as described by the right hand sides of the equations, has much effect on this time scale. Typical values of the constants in the above scalings are $\Delta = 600K$, l = 10m and $V_{s0} = 1 \text{ms}^{-1}$.

5. Linearised equations

One approach to estimating the timescale for transients is to consider a linear model, which exhibits most of the features of the actual transient model. This has the advantage that actual solutions may be obtained. Consider, for example, a model problem where all the hot gas enters at the bottom of the furnace, as shown in figure 3. Also, assume that no heat is lost through the furnace walls and that the heat transfer between the strip and the gas takes the form of Newton's law of cooling. Then the appropriate linear model equations, in dimensionless form, are

$$\frac{\partial T_s}{\partial t} - \frac{\partial T_s}{\partial x} = \alpha (T_g - T_s) \tag{18}$$

$$\frac{\partial T_g}{\partial t} + b \frac{\partial T_g}{\partial x} = -\beta (T_g - T_s) \tag{19}$$

Both the strip and gas velocities have been taken to be constants, namely 1 and b respectively. A value of b roughly comparable with the conditions in a real furnace is b = 10. It is possible to obtain estimates of appropriate values for α and β by "fitting" a steady state solution of the linear model equations to the sample numerical result of the actual nonlinear steady state model. This exercise yields the approximate values $\alpha = 0.5$ and $\beta = 2$. The "fitted" linear solutions for T_s and T_g are superimposed on the numerical solutions shown in figure 2, and the comparison between the graphs is not unreasonable.

The aim of the linear model approach is to introduce a perturbation to the strip temperature at t = 0 and determine the time required for this perturbation to disappear. That is, we set

$$T_s(x,t) = T_{s0}(x) + T'_s(x,t)$$
(20)

$$T_g(x,t) = T_{g0}(x) + T'_g(x,t)$$
(21)

where the subscript 0 indicates a steady state solution and the primed variables represent perturbations. The initial perturbation is defined by

$$T_s(x,0) = T_{s0}(x) + 1 \tag{22}$$

Now the equations for the perturbed variables are identical to equations (18-19), so for the remainder of this section we will drop the primes for convenience. The appropriate boundary conditions for the perturbation variables are

$$T_s(x=1) = 0$$
$$T_g(x=0) = 0$$



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Figure 3: Linearised model.

and solution of this system will give information about the strip temperature at the bottom of the furnace $(T_s(x=0))$. Rather than attempt solution of this system we integrate (18-19) with respect to time, t and write

$$\hat{T}_s(x) = \int_0^\infty T_s(x,t)dt$$
$$\hat{T}_g(x) = \int_0^\infty T_g(x,t)dt$$

with the resultant equations for \hat{T}_s and \hat{T}_g being

$$\frac{d\hat{T}_s}{dx} = \alpha(\hat{T}_g - \hat{T}_s) + 1 \tag{23}$$

$$\frac{d\hat{T}_g}{dx} = \beta/b(\hat{T}_g - \hat{T}_s) \tag{24}$$

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Solution of (23-24) and evaluation of the boundary conditions yields the following result

$$\hat{T}_{s}(0) = \frac{1 + \frac{\alpha}{\lambda} (\frac{e^{\lambda} - 1}{\lambda} - 1)}{1 + \frac{\alpha}{\lambda} (e^{\lambda} - 1)}$$
(25)

$$\simeq \frac{1+\frac{\alpha}{2}}{1+\alpha} \tag{26}$$

where $\lambda = \alpha - \beta/b$ is small for the values of α , β and b given here, which leads to the approximate relation given above. Now if we expect that the strip temperature at the bottom of the furnace will decay roughly exponentially, *i.e.*

$$T_s(0,t) \sim e^{-t/t_0}$$

then

$$\hat{T}_s = \int_0^\infty T_s(0,t) dt \sim t_0$$

so the result given by equation (26) above gives a time scale for the decay of a perturbation in the furnace, i.e.

$$t_0 \sim \frac{1 + \frac{\alpha}{2}}{1 + \alpha} = O(1)$$

This is a particularly interesting result. Firstly, it says that any perturbation to a steady state will disappear in about the same time as it takes a piece of strip to pass through the furnace. Secondly, this decay time depends only upon α , to first order. This means that the rate of heat transfer to the strip is the rate limiting mechanism, rather than the rate at which heat is supplied by the gas. This is sensible when we know from the steady state model that only about 30% of the heat available in the gas is transferred to the strip, with the rest going out the top of the furnace. Finally, it is worth noting that $t_0 < 1$, indicating that the term on the right hand side contributes positively to the damping out of a perturbation, so that t_0 is less than the characteristic residence time of the strip. This also is consistent with the fact that there is an excess of heat available in the gas, so that the heat transfer to the strip can be adjusted to attain a steady state more rapidly.

There are other methods of obtaining the same time scale as calculated here. They include considering the Laplace transform of the linear equations or examining the appropriate Green's function solution to the equation. Attempts were made to do this during the Study Group, and it became clear that all 3 methods led to equivalent timescales.

The importance of this result is that it appears that the linear model, if perturbed, will achieve a new steady state in a time comparable to the residence time of the strip. Whilst the above linear model is a gross simplification of the actual nonlinear model, it does maintain sufficient of the physics to allow some relevance to be attached to the result calculated here. In other words, it is to be expected that in the real furnace a change to a new steady state will occur in a time comparable with the strip residence time, which is about 10 seconds for the BHP furnaces.

6. Wall heat conduction

The walls of the furnace consist almost entirely of refractory material, which is a poor conductor of heat. In the steady state model, the thermal conductivity is, in fact assumed to be zero, so that no heat is conducted through the wall. It is not clear whether the same assumption is appropriate in the transient case. At first thought, it would seem that the wall conduction would not affect the strip temperature in any way, unless it was a very large effect. This is because there is, in general, a large surplus of heat in the gas, *i.e.* the majority of the heat added to the furnace via the gas actually exits the furnace at the top. The results of the steady state calculation show this. So the energy lost to wall heat conduction would need to be comparable to that lost to the strip for it to be significant. The following calculation provides some insight into the relative effect of wall heat conduction. If the temperature at the wall (y = 0) changes by an amount ΔT , then the flux of heat conducted into the wall in t seconds is approximately

$$\frac{\sqrt{\kappa}\rho C_p \Delta T}{\sqrt{t}}$$

where κ is the thermal diffusivity of the wall material, ρ is its density and C_p its specific heat. If we take $\kappa = 10^{-6} \text{m}^2/\text{s}$ and $\rho C_p = 10^6 \text{J/kg.K}$, then the rate of wall heat conduction is

$$\frac{1000\Delta T}{\sqrt{t}}$$
W/m²

Now if the appropriate timescale for a transient is of the order of the time for the strip to pass through the furnace, then we should set t = 10s, and so the rate of heat conduction is

$$H_w \simeq 7 * 10^5 \mathrm{W}$$

for $\Delta T = 100K$ and a wall surface area of $20m^2$. This can be compared with a typical rate of energy input from the gas, which can be obtained from the steady state model to be approximately

$$H_a \simeq 7 * 10^6 W$$

Hence the wall heat conduction is of the order of 10% of the heat input from the gas. This is small, but it is a comparable amount to the amount of heat

transferred to the strip via radiation and conduction. Thus it is not necessarily true that the wall heat conduction is a negligible effect. In any case, it would be sensible to attempt a solution of the transient model equations in the absence of heat conduction initially, because later inclusion in the solution procedure should not require a great deal of extra work.

7. Numerical solution methods

The solution of the complete set of equations for the transient model of the Selas furnace, as described in section 3, needs to be done numerically. There are at least three possible methods that could be used, namely (a) a shooting method (b) a backward method of characteristics and (c) a Laplace transform method. A brief description of each method follows:

The shooting method

The equations for the strip and gas temperatures (10-11) are discretised in time only, using a fully implicit, backward Euler scheme *i.e.*

$$\left(\frac{\partial T_s}{\partial x}\right)^{(n+1)} = \frac{1}{V_s^{(n+1)}} \left[\frac{T_s^{(n+1)} - T_s^{(n)}}{\Delta t} - R_s^{(n+1)}\right]$$
(27)

$$\left(\frac{\partial T_g}{\partial x}\right)^{(n+1)} = \frac{1}{V_g^{(n+1)}} \left[R_g^{(n+1)} - \frac{T_g^{(n+1)} - T_g^{(n)}}{\Delta t} \right]$$
(28)

$$\{B_w - B_{gw} - q_{wc}\}^{(n+1)} = 0$$
⁽²⁹⁾

where R_s and R_g represent the terms on the right hand sides of equations (10-11), and the superscripts (n) and (n + 1) denote solutions at times t_n and t_{n+1} respectively. The above system of equations is now in the same form as the steady state problem, so the same shooting method used there can be used here to calculate $T_s^{(n+1)}$ and $T_g^{(n+1)}$. This method has the advantage that it can be implemented by modifying currently existing software, so the development time should be short. However, there is a disadvantage that if there is a discontinuity in the strip thickness, then there is a corresponding discontinuity in the strip temperature, and the shooting method will fail. In such a case, there is also discontinuity in the wall temperature.

The backward characteristics method

We define the characteristic lines $X_s(\zeta; x, t)$ and $X_g(\zeta; x, t)$, associated with the point (x,t) by

$$\frac{dX_s}{d\zeta} = -V_s(X_s,\zeta), \qquad X_s(\zeta;x,t) = x \tag{30}$$

$$\frac{dX_g}{d\zeta} = V_g(X_g, \zeta), \qquad X_g(\zeta; x, t) = x \tag{31}$$

Then along the characteristic X_s we have

$$\frac{dT_s}{d\zeta} = R_s \tag{32}$$

and along X_g we have

$$\frac{dT_g}{d\zeta} = R_g \tag{33}$$

Then we partition the furnace length into a discrete set $\{x_0 = 0, x_1, \ldots, x_N = l\}$ and suppose that T_s , T_g and T_w are known at these points at time t_n . The positions of the characteristics at time t_n can be found by integrating the characteristic equations (30-31) back from the n + 1 time level to the n time level. Then the temperatures on the characteristics at this time level need to be determined by interpolation from the known temperatures. Finally, equations (32-33) can be integrated forward to the n + 1 time level to provide the solution. This method has the advantage that it can handle a discontinuity in the strip thickness, and hence the strip temperature.

The Laplace transform method

In this method we consider perturbations to the steady state temperatures, i.e. set

$$T_s = T_{s0} + \delta T_s ;$$
 $T_g = T_{g0} + \delta T_g ;$ $T_w = T_{w0} + \delta T_w ;$

and upon substitution into the governing equations we obtain a linear system of equations of the form

$$A\mathbf{u}_t + B\mathbf{u}_x = C\mathbf{u} + \mathbf{d} \tag{34}$$

where A, B and C are 3×3 matrices whose coefficients depend on x, d is a vector and $\mathbf{u} = [\delta T_s, \ \delta T_g, \ \delta T_w]^T$. Now we take the Laplace transform with respect to time, t, yielding

$$B\bar{\mathbf{u}}_x = (C+pA)\bar{\mathbf{u}} + \mathbf{d}' \tag{35}$$

where $\bar{\mathbf{u}}$ is the Laplace transform of \mathbf{u} , p is the Laplace transform variable and $\mathbf{d'} = \mathbf{d} + A\mathbf{u}(0)$. If we let U be a fundamental matrix of solutions of

$$B\bar{\mathbf{u}}_x = (C + pA)\bar{\mathbf{u}}$$

and let $\mathbf{\bar{u}} = U\mathbf{v}$, then it is easy to show that

$$BU\mathbf{v}_x = \mathbf{d}'$$

or

$$\mathbf{v} = \int (BU)^{-1} \mathbf{d}' dx$$

and so

$$\bar{\mathbf{u}} = U \int (BU)^{-1} \mathbf{d}' dx$$

The solution **u** is found by inverting this expression, which will require finding the singularities in the above expression in the p-plane. This method is appropriate for small perturbations from a steady state.

Of all of the above 3 methods, it would seem that the most appropriate method to implement is the backward characteristics method. Of course, the above descriptions are quite sketchy, so some more work is needed to complete the development of each of the methods. None of the above discussion has included the possibility of needing to calculate wall heat conduction, as described by equations (13-14). If it is decided that this is necessary, then it is not too difficult to include it in any of the above three methods. The wall heat conduction equation is actually decoupled from the remainder of the model equations, so can be solved independently once T_s and T_g are known at any time level. By assuming one dimensional heat conduction on a semi-infinite interval, the solution to the heat conduction equation is straightforward. In particular, since the model only requires T_w at y = 0, then the simplest approach may be to use a boundary integral formulation. The result will be a single nonlinear integral equation for T_w . There should be no significant difficulty in adding this to the model solution, once the problem has been solved without wall heat conduction.

8. Conclusions

This report provides only a brief summary of the most significant discussions at the Study Group. One of the most significant points arising from the work is the result of the linearised model, which says that a disturbance introduced to the strip temperature will disappear in roughly the same time as it takes a piece of strip to pass through the furnace, *i.e.* of the order of 10 seconds, in the case of a typical Selas furnace. This is a most intriguing result. It brings into question the need to develop solutions to the fully nonlinear transient model equations, because it appears from discussions that changes in furnace operations can often take a few minutes to return to a steady state. It would appear, then, that the reason for the discrepancy could be due to factors that cannot be easily incorporated into a mathematical model, such as operator intervention or a time delay due to turning burners on/off. If the true transient time is only ~ 10 seconds, then an appropriate strategy for minimising wastage would be to set the operating conditions to those appropriate for the expected new steady state as soon as a change in input is detected. Only the steady state model would be needed for such a strategy.

However, the above analysis relies on the assumption that wall heat conduction has negligible effect on the transient solution. If it does indeed have an effect, then it is likely to increase the time needed to achieve a new steady state. It is difficult to envisage that any such time delay would be very significant.

A final matter that was discussed at the Study Group was whether the method used in the steady state model for calculation of the radiative transfer between the gas, the strip and the wall was appropriate. Because radiative transfer is the dominant heat transfer mechanism in the furnace, it is necessary to have as accurate a model as possible. A suggested reference for reviewing the currently used method is Rohsenow *et al.*

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