SCHEDULING TIRAGE CHAMPAGNE PRODUCTION

Linear programming models for the scheduling of tirage champagne production are presented. The basic model demonstrates that cyclic schedules which reduce the average holdings of both maturation and finished product stock can be determined. Reduced costs associated with tirage bottle rewashing indicate that this is not an economic strategy. A common maturation stock model is presented as an option for future potential earnings.

1. Introduction

Two principal methods for champagne production are used in Australia, namely, *Methode Champenoise* in which maturation takes place entirely in the one bottle, and tirage champagne or transfer method champagne, where two bottles are used. The following is a brief description of the production process in this case (see Figure 1).

Initially, finished stabilized wine is prepared in the cellar for tirage bottling, with the addition of 20 g/l of sweetener and actively fermenting yeast to form the champagne base wine. The champagne base wine is then bottled on the filling/transfer line in tirage bottles which are capped with a crown seal and packed in wooden bins. These bins are then moved to a warehouse where secondary fermentation takes place in the bottle. Fermentation is completed in 4-6 weeks when the product enters its maturation period. The minimum fermentation and maturation period is set out under the Food and Drugs Act as 6 months.

On completion of the maturation period (depending on the product, this may vary between 6, 7, and 9 months), the tirage bottles are taken to the transfer line. Here the transfer machine pierces the crown seals and the wine is transferred under counter pressure (to preserve the wine's natural fermentation gas), to a pressure tank in the cellar. The empty tirage bottles are then decrowned and sent to the twist rinser which inverts and rinses them before they are transported back to the tirage filler. The wine in the pressure tank is cooled, centrifuged and filtered, and additions are made to conform to finished goods product standards. This process takes around 2 weeks, after which the wine is bottled on the expedition line in new bottles which are dressed and packed in accordance to set specifications. The finished product is then stored in the warehouse for 6 weeks. This period of time is essential to settle the wine and for ease of cork extraction.

CELLAR   BOTTLING   WAREHOUSING

PRIMARY FERMENT

FINISHED STAB.
WINE

TIRAGE TRANSFER
& FILLING LINE

SUGAR AND
YEAST
ADDITIONS   TIRAGE BOTTLING   TO INVENTORY FOR
                                                   SECONDARY FERMENT
                            GLASS RINSED   & MATURATION
                                                   (MINIMUM 6 MONTHS)

SPARKLING WINE
TO PRESSURE   TIRAGE TRANSFER
TANK

CENTRIFUGED,   TO EXPEDITION   BOTTLE MATURATION
FILTERED,   BOTTLING LINE   (6 WEEKS)
LIQUER ADDITIONS   (FINISHED PRODUCT)   AND WAREHOUSING
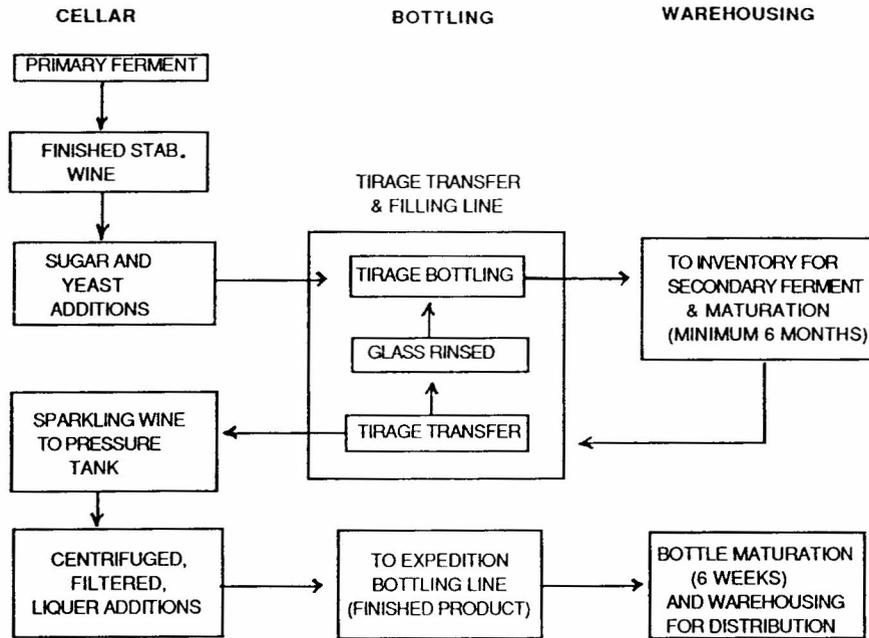   FOR DISTRIBUTION

Figure 1: The transfer method for champagne production.

It should be emphasised that the transference and refilling of tirage bottles is an integrated process and is performed on the one line, described as the tirage transfer and filling line. The separation of the processes of tirage transference and filling would entail the extra processing steps of removing, storing and reloading the bottles onto the line. In addition to the extra glass handling, the tirage bottles would require to be caustic washed. A caustic washing machine is currently not available, however such a plant could well be justified depending on trade-offs elsewhere in the storage of maturation or finished product stock. It is also worth noting that the bottles used in the tirage process are a dedicated bottle, chosen in preference to expedition glass for cost reasons.

The Manager of Packaging Operations for Penfolds Wines at Nuriootpa (South Australia), Mr Herbert Hruby, was concerned about the levels of stock held in tirage or transfer method champagne. Although company policy allowed a significant component of this product to be released at a maturation age of 9 months, seasonal demand patterns and constraints on bottling capacity made it difficult to manage stock holdings at a level below 12 months. The task of the Study Group was to create a model for scheduling tirage champagne production which allowed a reduction in stock holdings for this product to as close to 9
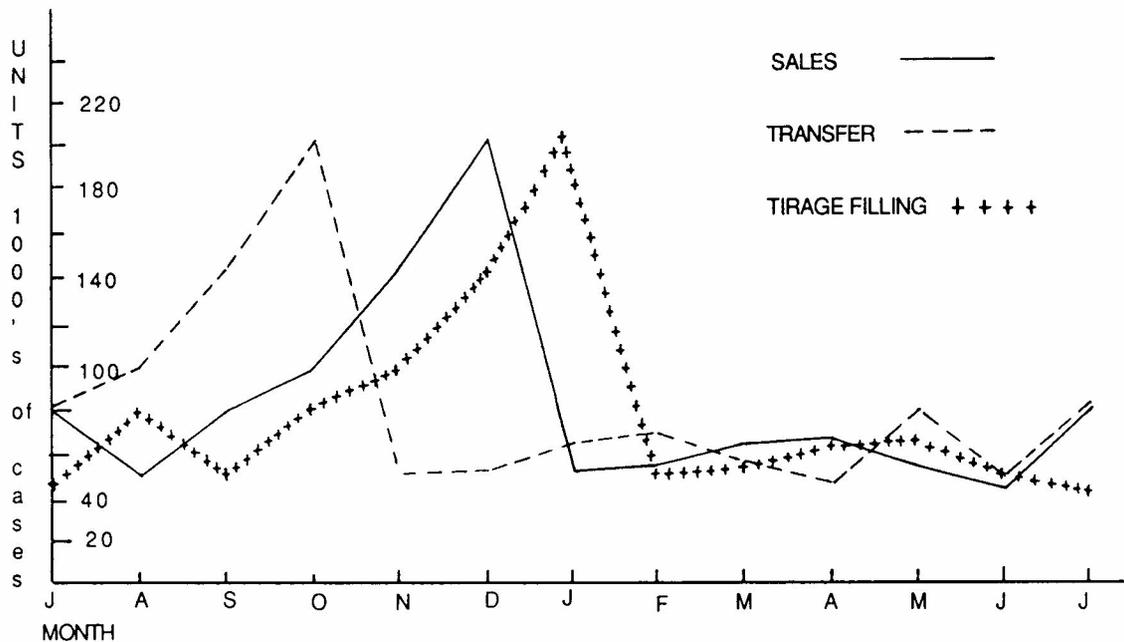
months as possible.

Figure 2: Monthly distribution for sales, transfer and tirage filling based on 9 months tirage maturation.

2. Current scheduling practices

Figure 2 displays the month by month distribution of sales for tirage champagne, together with quantities transferred to finished product stock and refilled for fresh maturation stock. Note that all data presented in this paper are hypothetical and do not represent actual quantities produced. Nevertheless the proportional relationships between related data sets have been maintained. The graphs in Figure 2 clearly shows why it is easy to maintain 12 month stock levels. To maintain a precise maturation period of 9 months, and allowing for a lead time of 2 months for the settling of expedition stock, the peak in tirage filling to create fresh maturation stock must occur exactly 11 months ahead of the peak in demand. However, at this time (January on the graph) there is a considerable shortfall in the availability of tirage bottles if these are to be recycled immediately after the transfer of matured stock. If however the peak in transferred product were pushed forward to coincide with the peak in tirage filling, then bottle recycling would present no problems. Such a schedule however

corresponds to a maturation period of 12 months. In addition to tirage bottle recycling problems, the peak demand in December and the subsequent transfer requirement in October exceeds the capacity of the transfer and refilling line by almost 46%, so that a certain amount of stockpiling is necessary in preceding months even if a 12 month inventory strategy is used.

Monthly Demand		210	51	53	63	66	57	48	80	53	81	97	142	
Month	N	D	J	F	M	A	M	J	J	A	S	O	N	
units transferred		53	63	66	57	48	80	53	81	97	142	210	51	
	51	53	63	66	57	48	80	53	81	97	142	210	51	1
	210	51	53	63	66	57	48	80	53	81	97	142	210	2
	142	210	51	53	63	66	57	48	80	53	81	97	142	3
	97	142	210	51	53	63	66	57	48	80	53	81	97	4
	81	97	142	210	51	53	63	66	57	48	80	53	81	5
	53	81	97	142	210	51	53	63	66	57	48	80	53	6
	29	29	47	78	142	210	51	53	63	66	57	-	29	7
					21	115	210	51	53	63	66			8
					35		192	51	53	39				9
								111	51					10
									14					11

Table 1. 6 month minimum maturation production schedule (1 unit ≡ 1000 cases). The right hand column indicates age in months. Initial holding 663 units corresponding to maximum consecutive 6 month demand. Average stock age 8.6 months

This difficulty is highlighted by the schedule shown in Table 1, which represents a possible "solution" to the problem, offered by Penfolds just prior to the conference. The proposed schedule is for 6 month maturation stock, that is, stock which is ready for expediting once the 6 month maturation period is complete. The initial holding which is the minimum necessary to satisfy the maximum consecutive 6 month demand period, is consistent with quantities transferred, and tirage bottles refilled in the previous 6 month period, working back from November. The number of units of maturation stock transferred to expedition stock each month is based on demand 2 months ahead, allowing for the 2 month settling period. The schedule shown is designed to duplicate itself every 12 months, but is infeasible since the peak production capacity of 144 units is exceeded in October. Even if this production capacity shortfall could be made up (for example through using overtime), it is possible that the schedule would be unacceptable to the winemaker since a significant quantity of maturation stock is not transferred until it is 7-12 months in the tirage bottles. This could be seen as undesirable due to inconsistency in quality. In addition it is unlikely that a cyclic schedule of this type could be maintained under changing demand conditions. The next section will contain an outline of linear programming models devised to overcome these difficulties.

3. Linear programming models

3.1 Model evolution

Initial investigation by the problem moderator revealed no previous published material relating to this problem, although several references (see *e.g.* Wagner (1975)) pointed to linear programming as an effective tool for production scheduling. The moderator presented such a preliminary model to the group, and it was agreed that this approach should be pursued. The usual modelling evolutionary process unfolded in the following days of discussion in which attempts were made to reduce the size of the initial model, and then further terms were reintroduced when it was realized that information had been lost in the process. The principal points raised during discussion can be summarised as follows:

- **Costs**

  There was much debate on how costs should be assigned in the objective. It was recognised that holding costs for both maturation stock and expedition stock were significant, as well as the cost of any glass handling required if tirage bottles were not immediately refilled. The consensus of opinion was that holding costs should be regarded as opportunity costs for alternative investment (see *e.g.* Love, 1979) and that these should be included as compounding factors in the objective.

- **Glass handling**

  Although Penfolds' current practice is to recycle tirage glass immediately at the transfer stage, the Group decided to consider options for setting aside tirage glass and rewashing it at a later stage.

- **Common maturation stock**

  Current practice involves the production of 3 major products with maturation ages of 6, 7 and 9 months respectively, using different base wines. It was suggested that one way of cushioning the overrun of maturation stock beyond the specified age would be to combine their production using the same base wine. Significant differences in the quality of base wine used may rule this strategy out however.

The following subsections contain a description of 3 models developed on the basis of Group discussions.

3.2 The basic model

The model described is for the scheduling of nine month maturation stock, but can be easily generalized for maturation stock of any age (≥ 6). Note that in the variable definitions, t subscripts represents points of time denoting the commencement of a period. All periods can be assumed to be months for the purposes of this discussion. The following notation is used (with all decision variables non-negative):

$$D_t \quad = \quad \text{demand for finished product in period } t+1$$
$$T_{tj} \quad = \quad \text{units of maturation stock transferred in period } t+1 \text{ of}$$
$$\text{maturation age } j$$
$$V_t \quad = \quad \text{total units of maturation stock transferred in period } t+1$$
$$M_{tj} \quad = \quad \text{units of maturation stock held at time } t \text{ of age } j$$
$$El_t \quad = \quad \text{units of 1 period old expedition stock held at time } t$$
$$A_t \quad = \quad \text{units of 2 period old or greater, expedition stock held at}$$
$$\text{time } t \text{ (available finished product)}$$
$$r \quad = \quad \text{interest rate}$$

The model objective is to minimise variable production costs. Since company policy requires maturation stock to develop to 9 months, value added costs are only considered beyond this time. In this model we assume that stock cannot be held beyond 12 months, with a progressively higher penalty incurred for stock held in months beyond the ninth. Likewise, expedition stock requires a 2 month settling period, and only stocks held beyond this time are costed in the objective. In order to simplify the model it is assumed that all excess expedition stock of age 2 months or older is costed at the same rate. A factor of 1 is used to represent the holding cost/unit/month for maturation stock, and a factor of 2 for expedition stock, reflecting the relative value added components of the finished product. Thus the objective may be stated as

$$\text{minimize} \quad \sum_t \{ \sum_{j=9}^{12} (1+r)^{j-9} \, M_{tj} + 2(A_t - D_t) \} \tag{1}$$

where summation is taken over the period of the schedule $(0, \ldots, s)$. In formulating the constraints it is worth noting that once initial maturation stock levels are specified the convertible maturation holdings of age 9 months are automatically determined for the remainder of the schedule. Consequently, the movement of maturation stock from one month to the next, and variables associated with these transitions can be ignored, thus reducing the size of the problem. Production constraints are as follows:

$$V_t = \sum_{j=9}^{12} T_{tj}, \quad t = 0, \ldots, s \tag{2}$$

$$V_t \leq 144, \quad t = 0, \ldots, s \tag{3}$$

The latter represents production capacity.

If G_j $(j = 0, \ldots, 12)$ are given initial maturation stock levels of age 0 to 12 months, then convertible maturation stock levels of age 9 months for each period are given by

$$M_{t+19} = \left\{ \begin{array}{ll} G_{8-t}, & t = 0, \ldots, 8 \\ V_{t-9}, & t = 9, \ldots, s \end{array} \right. \tag{4}$$

$$\text{and} \quad M_{oj} = G_j \quad j = 9, \ldots, 12 \tag{5}$$

Subsequent maturation stock levels are then given by

$$M_{t+1j} = M_{tj-1} - T_{tj-1}, \quad j = 10, \ldots, 12, \quad t = 0, \ldots, s \tag{6}$$

$$\text{with} \quad T_{tj} \leq M_{tj}, \quad j = 10, \ldots, 12, \quad t = 0, \ldots s \tag{7}$$

The remaining constraints relate to expedition stock handling and demand requirements :

$$E1_{t+1} = V_t, \ t = 0, \ldots, s \tag{8}$$

$$A_{t+1} = A_t - D_t + E1_t \tag{9}$$

$$A_t \geq D_t, \ t = 0, \ldots, s \tag{10}$$

Note that if a periodic solution is required with period s (for example of 12 month cycle in which $s = 12$), then the additional constraints

$$\begin{array}{ll} A_s &= A_0, \quad V_s = V_0, \quad E1_s = E1_0 \\ M_{sj} &= M_{0j} \quad \text{and} \quad T_{sj} = T_{0j} \quad j = 10, \ldots, 12 \end{array} \tag{11}$$

are required.

3.3 The glass handling model

We now allow the option of not refilling all tirage bottles at the transfer stage, setting them aside, and re-using them if necessary at a later stage. Two additional costs are incurred, namely, the cost of rewashing and the additional

handling of the old glass, and the cost of storing this glass. Based on data supplied, refilling a unit of tirage bottles has a cost an order of magnitude higher than the monthly storage cost of a unit of maturation stock, whilst the storage of bottles has a monthly cost an order of magnitude lower.

Additional non negative variables are defined as follows :

$$S_t \quad = \quad \text{units of tirage bottles not refilled in period } t+1$$
$$BA_t \quad = \quad \text{units of tirage bottles available for filling in } t+1$$
$$R_t \quad = \quad \text{units of old tirage bottles refilled in } t+1$$

Then (1) is replaced by

$$\text{minimize} \quad \sum_t \{ \sum_{j=9}^{12} (1+r)^{j-9} M_{tj} + 2(A_t - D_t) + 10R_t + .1BA_t \} \tag{12}$$

Changes to the basic constraints require (3) to be replaced by

$$V_t + R_t \le 144, \quad t = 0, \ldots, s \tag{13}$$

and (4) to be replaced by

$$M_{t+19} = \begin{cases} G_{8-t}, & t = 0, \ldots, 8 \\ V_{t-9} - S_{t-9} + R_{t-9}, & t = 9, \ldots, s \end{cases} \tag{14}$$

Management of tirage bottles is described by

$$BA_{t+1} = BA_t + S_t - R_t, \quad t = 0, \ldots, s \tag{15}$$

All other constraints from the basic model are unchanged, and hence (12, 2, 13, 14, 5-10, 15) define the glass handling model. Additional periodic conditions may be imposed if necessary.

3.4 The common maturation stock model

Although current company policy may prohibit this strategy, it is an interesting option for future consideration and may demonstrate significant potential for saving.

In this model we will assume 3 varieties of champagne are each produced from a common base wine. Variety 1 is transferred at 6 months with no overrun

allowed, variety 2 can be transferred either at 7 months or 8, with a preference for 7 months, whilst variety 3 may be transferred either at 9, 10 months, with a high penalty for any overrun beyond this time.

Glass handling is omitted here but could be included if desired. A new set of non negative variables is required :

$$
\begin{array}{lcl}
D_{ti} & = & \text{demand for variety } i \text{ in period } t+1 \\
TD_t & = & \text{total demand in period } t+1 \\
T_{tj} & = & \text{units of maturation stock of age } j \text{ transferred in } t+1 \\
V_{ti} & = & \text{units of variety } i \text{ transferred in } t+1 \\
M_{tj} & = & \text{units of maturation stock of age } j \text{ held at time } t \\
E1_{ti} & = & \text{units of 1 month old expedition stock of variety } i \text{ held at } t \\
A_{ti} & = & \text{units of 2 month old expedition stock of variety } i \text{ held at } t \\
AT_t & = & \text{total available expedition stock held at } t \\
TOT_t & = & \text{total units transferred in } t+1 \\
\text{overun}_t & = & \text{maturation stock of age greater than 10 months}
\end{array}
$$

The model is now

$$
\text{minimize} \quad \sum_t \{T_{t8} + T_{t10} + 2(AT_t - TD_t) + 10^3 \text{overun}_t\} \tag{16}
$$

subject to

$$
V_{t1} = T_{t6} \tag{17}
$$

$$
V_{t2} = T_{t7} + T_{t8} \tag{18}
$$

$$
V_{t3} = T_{t9} + T_{t10} \tag{19}
$$

$$
TOT_t = \sum_{j=6}^{10} T_{tj} \tag{20}
$$

$$
TOT_t \leq 144 \tag{21}
$$

$$
M_{t+1\,6} = \begin{cases} G_{5-t}, & t = 0,\ldots,5 \\ V_{t-6}, & t = 6,\ldots,s \end{cases} \tag{22}
$$

$$
M_{oj} = G_j, \ j = 6,\ldots,10 \tag{23}
$$

$$
TD_t = \sum_{i=1}^{3} D_{ti} \tag{24}
$$

$$
T_{tj} \leq M_{tj} \tag{25}
$$

$$
M_{t+1j} = M_{tj-1} - T_{tj-1}, \ j = 7,\ldots,10 \tag{26}
$$

$$
\text{overun}_t = M_{t10} - T_{t10} \tag{27}
$$

$$
E1_{t+1} = V_{ti}, \ i = 1,2,3 \tag{28}
$$

$$A_{t+1} = A_{ti} - D_{ti} + E1_{ti}, \ i = 1, 2, 3 \tag{29}$$

$$A_{ti} \geq D_{ti}, \ i = 1, 2, 3 \tag{30}$$

$$AT_t = \sum_{i=1}^{3} A_{ti} \tag{31}$$

where all t subscripts are taken over the length of the schedule, $t = 0, \ldots, s$, and additional periodic conditions may be imposed if necessary.

4. Results

The modelling language LINGO was used to run experiments with each of the models described in section 3. Optimization within LINGO is carried out via the Linear Programming optimization software LINDO. LINGO enables large models to be easily generated, and for model parameter changes to be readily made within the LINGO code.

A 12 month periodic production schedule generated using the basic model is shown in Table 2. If the periodic requirement is placed on the schedule, a feasible solution is not guaranteed unless the initial conditions are carefully chosen. An optimal set of initial conditions can be obtained by first running the model with only the demands specified. The values used for V_0, M_{0j}, $E1_0$ and A_0 shown in Table 2 were generated in this manner. These initial values will not be unique in general, and alternative feasible but suboptimal solutions will exist for other sets of initial conditions. The schedule shown in Table 2 demonstrates the trade-off which occurs between maturation stock held beyond 9 months, and excess expedition stock.

Period t		0	1	2	3	4	5	6	7	8	9	10	11	12
Demand D_t		80	53	81	97	142	210	51	53	63	66	57	48	80
Transferred V_t		144	80	144	144	51	53	63	66	57	48	80	71	144
Convertible Stock Age	9	144	7	144	144	51	53	63	66	57	48	80	144	144
	10	73	73											73
	11													
	12													
El_t		71	144	80	144	144	51	53	63	66	57	48	80	71
A_t		80	71	162	161	208	210	51	53	63	66	57	48	80
Excess		-	18	81	64	66								

Table 2. Tirage production schedule generated by the basic model. Objective value = 1688

A non periodic 12 month schedule generated by the glass handling model is shown in Table 3. Initial maturation and expedition stock values are identical

to those used in the basic model. If glass rewashing costs are left at 10 as given in the objective, the solution obtained involves no setting aside and rewashing, and is essentially the same as in Table 2. Reduced costs associated with the rewashing variable R_t suggest that a reduction in these costs of the order of 95% would be required to make this option economic. The schedule shown in Table 3 corresponds to the case where these costs have been reduced by 95%. It shows that 40 units of glass are not refilled in the second month, and 144 in the third. These 184 units of glass are eventually rewashed in the fourth and fifth months.

Period t		0	1	2	3	4	5	6	7	8	9	10	11	12
Demand D_t		80	53	81	97	142	210	51	53	63	66	57	48	80
Transferred V_t		144	80	144	144	51	53	63	66	57	48	80	104	0
Convertible														
Stock Age	9	144	7	144	144	51	53	63	66	57	48	80	104	0
	10	73	73											
	11													
	12													
El_t		71	144	80	144	144	51	53	63	66	57	48	80	104
A_t		80	71	162	161	208	210	51	53	63	66	57	48	80
Excess				63	64	66								
Glass														
Handling	S_t	0	0	40	144	0	0	0	0	0	0	0	0	0
	BA_t	0	0	0	40	184	91	0	0	0	0	0	0	0
	R_t	0	0	0	0	93	91	0	0	0	0	0	0	0

Table 3. Tirage production schedule generated by the glass handling model. Objective value = 1579

Period t		0	1	2	3	4	5	6	7	8	9	10	11	12
Demand D_t	1	20	13	21	25	36	52	11	13	15	16	15	12	20
	2	30	20	30	36	53	79	20	20	24	25	21	18	30
	3	30	20	30	36	53	79	20	20	24	25	21	18	30
Transferred V_t	1	43	62	29	0	11	13	31	0	21	6	20	13	21
	2	30	36	53	79	20	20	24	25	39	0	30	20	30
	3	71	0	62	65	20	20	24	25	21	18	31	98	93
Convertible	6	144	135	128	44	56	58	144	98	144	144	51	53	79
Stock age	7	92	101	73	99	44	45	45	113	98	123	138	31	40
	8	0	62	65	20	20	24	25	21	88	98	123	138	31
	9	71	0	62	65	20	20	24	25	21	49	98	93	118
	10	0	0	0	0	0	0	0	0	0	0	31	98	93
El_t	1	13	43	62	29	0	11	13	31	0	21	6	20	13
	2	20	30	36	53	79	20	20	24	25	39	0	30	20
	3	20	71	0	62	65	20	20	24	25	21	18	31	98
A_t	1	20	13	43	84	88	52	11	13	31	16	21	12	20
	2	30	20	30	36	53	79	20	20	24	25	39	18	30
	3	30	20	71	41	67	79	20	20	24	25	21	18	31
Excess	1			22	59	52						16		6
	2												18	
	3			41	5	14								1

Table 4. Tirage production schedule generated by the common maturation stock model. Objective value = 1344

A 12 month non periodic schedule generated by the common maturation stock model is shown in Table 4. Although the total demands used over the schedule are the same as for the other models, the breakdown of these between the three varieties makes it harder to match initial conditions so as to draw comparisons with other results. It is likely that savings could be achieved if such a policy of using common maturation stock was adopted.

5. An alternative model

Subsequent to the Study Group, an alternative model for the problem, involving an optimal control theory approach, was suggested (Newsam, 1991). The production process was modelled as a system of differential equations and inequalities whose associated variables are continuous functions of time. Theory then predicts that an optimal schedule for the process will consist of a set of extremal strategies (*e.g.* immediate refill of empty tirage bottles) which can be easily identified in terms of well known scheduling principles, such as just-in-time inventory control. Furthermore, each strategy is pursued over a sub-interval of the planning horizon and is followed by an instantaneous switch to the next strategy. Finally these results are used to answer the question raised at the conference, namely, whether it would pay to rewash and store tirage bottles instead of immediately refilling them. A necessary and sufficient condition relating various costs that must be satisfied for rewashing to pay is given for a simplified model, along with the associated best schedule. The results can be extended in part to the complete model. These results confirm that rewashing is not cost effective with the relative costs provided.

6. Acknowledgements

References

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