SLURRY BEHAVIOUR IN SEPARATION DEVICES

Four different modelling approaches for slurry behaviour in cyclones are considered: bulk flow models, a simple continuum model, hydrodynamic models and so-called wet granular flow models. The aim is to consider the utility of each approach for design purposes. It is concluded that bulk flow models and the simple continuum model are inadequate for such use, principally because they are unable to account for geometrical effects which affect the flow. However, both the hydrodynamic models and the wet granular flow approach show considerable promise for the task, due to recent increases in computer power and improved models and algorithms. Of these two approaches, the hydrodynamic models are the more mature, but the wet granular flow approach has the advantage of being based explicitly on particle motion.

1. Introduction

This problem was presented to the 1992 Mathematics-in-Industry Study Group at Macquarie University by Oakbridge Limited. The company has a coal mining operation in the Hunter Valley of New South Wales. This report considers various approaches to mathematical modelling of the behaviour of slurries in particle separation devices. A slurry is defined as a mixture of a liquid and solid particles. Slurries are commonly used in transporting and processing many of the minerals produced by the Australian mining industry. For example, the coal industry extracts approximately 2.5 gigatonnes of earth to produce about 150 megatonnes of coal annually. Much of this material is processed in slurry form.

A common practice in the minerals industry is to attempt to separate particles with different physical properties. Coal, for instance, is a mixture of organic material and rock. It may be desirable to separate the particles according to their density difference, or according to their size difference.

The most commonly used separation device is called a **cyclone**. It is a cylindrical shaped device, generally with convergent sides at the bottom (see figure 1). A slurry is fed tangentially into the top of the cyclone and a rapidly swirling flow is set up. The swirling flow generates centrifugal force on the particles, and they move radially outwards. Their outward motion is resisted by drag and other forces, and consequently there is a separation of particles based on some combination of density and size. The larger and more dense particles exit at the bottom of the cyclone, while the smaller or less dense particles are carried out the top.

Another separation device used in minerals processing is the **spiral separator**. A slurry flows down a helical coil shaped device, and the particles are separated by the

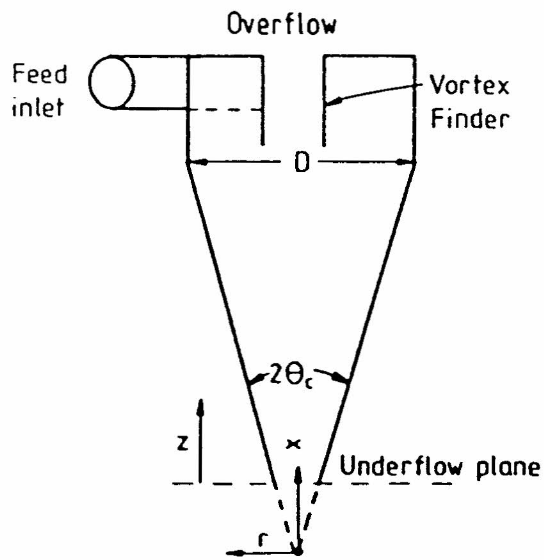

Figure 1: Schematic of a hydrocyclone (from Davidson, 1988).

effects of secondary flow currents set up by the helical motion and by particle sedimentation. The result is that the larger, more dense particles collect near the centre of the spiral and the smaller, less dense particles collect near the outside.

In order to focus the deliberations during the Study Group, it was decided to concentrate on the behaviour of slurries in a cyclone. However, many of the principles involved also relate to behaviour in a spiral separator.

Although the basic mechanisms of separation are reasonably well understood, it is extremely difficult, in practice, to use this knowledge in a predictive sense, as might be required when optimising operations or testing a new design. Consequently, most cyclones used today are of a standard design, and much of the design of new plants is done using correlation models based on previous experience.

The aim here is to examine various modelling approaches for slurry behaviour in cyclones, with a view to developing an approach that could be used as a predictive tool for improved or novel cyclone design. Hopefully such an approach would also be applicable to other separation devices, such as spiral separators. A subsidiary outcome of the modelling approach ought to be an improved understanding of existing cyclone operations, and subsequently improvement in any such operations.

There are four different modelling approaches that were considered at the Study Group. They are

1. Bulk flow models
2. Simple continuum models
3. Hydrodynamic models
4. Wet granular flow model

Each approach is discussed in the following sections. They are, of necessity, described in rather general terms, because of the nature of the Study Group process itself, and the inherent complexity of the slurry behaviour. It would be necessary to do a good deal of model development before the approaches suggested here could be deemed successful. Such development is outside the scope of this report.

2. Bulk flow models

There is a considerable body of literature relating to slurry behaviour in cyclones, much of it using the bulk flow approach. A recent example is the model of Holland-Batt (1982), who determines the residence time of particles in a cyclone and performs a balance between centrifugal force and drag on the particles, to determine the so-called "cut-size", d_{50} - the size for which 50% of particles exit at the bottom and 50% exit at the top, given by

$$d_{50} = \sqrt{18\left(1+\frac{6}{2^{10k}}\right)\frac{\mu}{(\sigma-\rho)}\frac{u_p}{(1-C)^{14}}} \qquad (1)$$

Here k is a factor to take into account the non-spherical nature of particles in the slurry, μ is the fluid viscosity, ρ its specific gravity and σ is the specific gravity of the particles. The radial velocity of the particles relative to the fluid, u_p, and the acceleration generated in the cyclone, a, are worked out as average quantities over the whole cyclone. The effect of particle concentration is also taken into account in estimating u_p. Because this model takes very little account of the detailed slurry motion, the estimate of cut-size cannot easily be related to the cyclone geometry. This is generally true of such models and is a significant drawback in using them for design purposes. The cyclone geometry and the nature of the flow inside the cyclone have a significant effect on the cyclone performance as a particle separation device. Therefore, the bulk flow models are unlikely to be useful for development of new designs or for predicting the behaviour of a cyclone in an unusual operating regime. A more detailed discussion of bulk flow models is given in the book by Svarovsky (1984).

3. Simple continuum models

The aim of this approach is to develop the simplest possible model of a slurry as an aid in analysis of more complicated models. As a first step, it is useful to consider basic aspects of the motion in a cyclone. The first consideration is the flow regime. Typically, the swirling flow set up in a cyclone has a Reynolds number in the range $10^4 - 10^5$, based on the viscosity of the liquid phase and the dimensions of the cyclone. Thus the flow is expected to be turbulent. However, swirl tends to act to inhibit the onset of turbulence in certain situations, and this is the case in cyclones (see Svarovsky, 1984). Tritton & Davies (1985) discuss the conditions under which rotation acts to stabilise flow, and they define a parameter B, which is a measure of the effect of rotation on flow stability. For the experimental results of Knowles *et al.* (1973), a rough estimate of B shows it to be around 10 in a hydrocyclone. A large, positive value of B such as this indicates that rotational stabilisation is dominant (Tritton & Davies, 1985). The inlet Reynolds number in this experiment is about 28000, so the inlet flow is definitely turbulent. However, although there is no explicit indication from the experiments that the flow remains turbulent throughout the cyclone, it appears that this is the case.

The existence of high particle concentrations in the flow may act to increase the local effective viscosity of the slurry, perhaps by up to an order of magnitude greater than the viscosity of the fluid alone. Some measurements of the viscosity of coal-water mixtures have been made for use in pipeline transport (Heaton & McHale, Kefa *et al.*), and they show such an increase in viscosity at high particle concentrations. An order of magnitude increase in viscosity reduces the effective Reynolds number of the flow by the same factor, bringing it towards the transition regime. Other evidence exists which shows that turbulence may be enhanced by particles, if their motion crosses the fluid trajectories, as happens in cyclones.

Given the above information, it is attractive to assume that the combination of increased effective viscosity and rotational stabilisation causes the flow in the cyclone to be laminar. The fluid viscosity, μ, could then be assumed to be a function (determined experimentally) of the local particle concentration, C, *i.e.* $\mu = \mu(C)$. Of course, a more accurate representation would need to account for the non-Newtonian behaviour of the slurry. Then, the particle motion could be modelled by lumping all the interaction forces into a single equation transport model for the local concentration, C, of particles. Such a system of equations is

$$\frac{d\mathbf{v}}{dt} = f(C)\,(\mathbf{v} - \mathbf{u}) \tag{2}$$

$$\frac{\partial C}{\partial t} + \nabla.(C\mathbf{v}) = 0 \tag{3}$$

Here, $\mathbf{v}$ is the velocity of particles in the slurry and $\mathbf{u}$ is the velocity of the slurry as a whole. The first equation is a momentum balance and the second equation is a mass balance.

A difficulty with this model is that the exact form of the function f is difficult to determine. However, its major advantage is that it is about as simple as can be developed from the hydrodynamical modelling approach. The model equations lend themselves to some simple analysis, if one postulates a form for the function $f(C)$. Another disadvantage of the approach is that it has lumped all the particles in the slurry into a single "phase". This is undesirable, since the purpose of the cyclone is to separate particles with different properties, such as size or density. An extension of the model would be to consider the particles in the slurry to be represented by several different "phases", based on either their size or density, and to write a transport equation for each phase. However, to do this would be to defeat the purpose of such a model, because then, for each phase i, say, having concentration C_i, one would need to determine the function $f_i(C_1, C_2, ...)$. The difficulty in doing this would be comparable to accounting for all the interaction forces between particles, and would lead to a model with a large number of imperfectly known parameters. The result would be a model that would be extremely poor in a predictive sense. The above discussion leads us to wonder whether there is any point in considering such a simple model. Given the fact that we are looking for a modelling approach that will enhance the ability to design new cyclone geometries, it is extremely unlikely that this approach would be useful.

4. Hydrodynamic models

There have been several models of cyclones that take the hydrodynamic approach. These include Bloor & Ingham (1987), Pericleous & Rhodes (1986), Hargreaves & Silvester (1990) and Davidson (1988). These models consider the detailed flow dynamics of the liquid in the cyclone. The result is typically an axisymmetric model of turbulent flow. For example, Davidson (1988) solves the turbulent Navier-Stokes equations in cylindrical coordinates (x, r, θ)

$$\rho\frac{\partial}{\partial x}(ru\phi)+\rho\frac{\partial}{\partial r}(rv\phi) = \frac{\partial}{\partial x}(r\mu_{eff}\frac{\partial\phi}{\partial x})+\frac{\partial}{\partial r}(r\mu_{eff}\frac{\partial\phi}{\partial r})+S_\phi \tag{4}$$

where $\phi = (u, v, rw)$ and (u, v, w) are the velocity components. Also,

$$\begin{aligned} S_u &= -r\frac{\partial p}{\partial x} \\ S_v &= -r\frac{\partial p}{\partial r}+\rho w^2-\mu_{eff}\frac{v}{r} \\ S_w &= 0 \end{aligned}$$

The fluid density ρ is a constant in the absence of particles but the viscosity μ_{eff} is a variable, in order to account for turbulence effects. The solution of these equations requires extensive computations just to determine the flow dynamics in the absence of particles. Inclusion of particles is often attempted by considering each particle size (or density) fraction as a separate, spatially continuous phase. For instance, Davidson solves an equation for the local particle concentration, C, as

$$\rho\frac{\partial}{\partial x}(ru_pC)+\rho\frac{\partial}{\partial r}(rv_pC)=\frac{\partial}{\partial x}(r\Gamma_{eff}\frac{\partial C}{\partial x})+\frac{\partial}{\partial r}(r\Gamma_{eff}\frac{\partial C}{\partial r}) \quad (5)$$

where (u_p, v_p) are the (x, r) components of particle velocity and Γ_{eff}/ρ is the effective turbulent diffusion coefficient of the particles. This equation assumes that C is small, so that ρ can be taken as uniform and the particle transport uncouples from the underlying fluid flow.

For dense slurries, as are likely to exist in cyclones, this model is inadequate. It is necessary to solve a multiphase flow dynamics problem in this situation, which is more difficult than a single phase problem. In addition to the difficulty of solving such problems, there is uncertainty about the appropriate structure of the model equations. This arises because of the large number of interactions that are occurring between the particles, and between the fluid and the particles. Shook & Roco (1992) identify the possible forces acting on particles: Brownian diffusion, electrostatic repulsion, mass attraction, drag, lubrication and collision. The particles also affect the fluid behaviour through turbulence modulation and hence by changing the effective turbulent viscosity of the slurry. The computational requirements for solution of such a multiphase system is considerable. Nevertheless, fundamental work in this approach continues, and the rapid increase in available computational power makes this type of modelling possible. Indeed, it appears to be the most mature approach to modelling slurry behaviour and has considerable potential for predicting the behaviour of slurries in cyclones. In addition, the approach has a great deal of relevance to the local context, because of the existing expertise in this area provided by Dr Malcolm Davidson, of the CSIRO Division of Mineral and Process Engineering (see above). Recent development of his approach includes an attempt to model high particle concentrations.

5. Wet granular flow model

The discussion in the previous sections takes the approach that the motion in the cyclone can be described in terms of a continuum – equations are written down for one, two or more continuous phases and attempts are made to solve them. The behaviour of individual particles is impossible to predict in this manner. If we consider that the part of the slurry that is of most interest to the minerals industry is the particles, such as lumps of coal, and we know that the particles are discrete entities, then the possibility of success using a continuum approach seems unlikely. A better approach would be one that focuses on the particle motions, and considers the fluid as a secondary medium.

Until quite recently, such a particle based approach has been considered to be too difficult to handle computationally, because of the huge number of particles involved in a real slurry system. However, recent developments in the size and speed of computers and improvements in algorithm design have led to the solution of problems in granular flow consisting of up to one million particles. The recent successes in granular flow modelling are based on writing a single vector equation of motion for each particle in the system and then calculating its trajectory, taking into account all the local effects on the particle, such as collisions and attractive or repulsive forces. The algorithmic advances have been in the area of reducing the time taken to calculate all the collisional interactions that a particle may have with other particles, in a system of N particles, to $O(N)$ rather than $O(N^2)$. These methods dramatically reduce the computational time requirements for solutions involving large numbers of particles. At the present time, simulations involving of the order of 10000 particles would require about 1 day of CPU time on a modern workstation computer. It is clear that the rapid increase in speed of computers will continue to reduce the computational cost. Dr Paul Cleary of the Division of Mathematics and Statistics (**dms**) has developed models of landslides and of particle motion in a pin mill (see figure 2) using these techniques (Cleary, 1991).

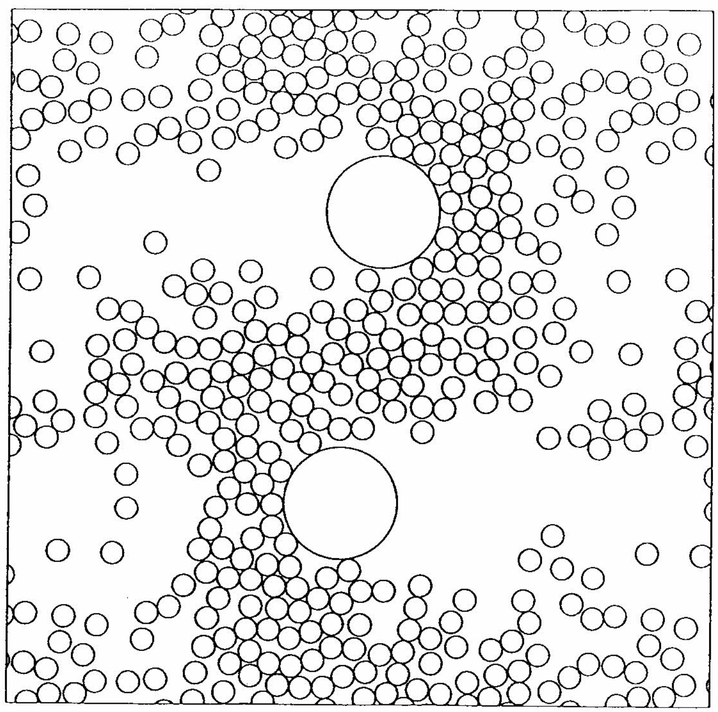

Figure 2: Granular flow in a pin mill. The two large circles represent the pins, which are moving in opposite directions. The picture shows the calculated positions of 292 particles at a particular instant in time (from Cleary, 1991).

It appears feasible to extend the granular flow techniques to include the effects of hydrodynamic forces on the particles, as a model of a slurry. Typically, the equation of motion for a particle i in a wet granular flow model would look like

$$\frac{d\mathbf{v}_i}{dt} = \sum_{j \neq i} \mathbf{F}_{ij} + \mathbf{F}_I + \mathbf{F}_c \tag{6}$$

where $\mathbf{F}_{ij}$ is the force due to collisions, modelled as a spring and dashpot, $\mathbf{F}_I$ is the total interaction force between the particle and the fluid (such as drag) and $\mathbf{F}_c$ is the body force on the particle, such as the centrifugal force due to the swirling motion in the cyclone.

Obviously, the focus of such a "wet granular flow" modelling technique is the particles rather than the fluid - exactly what is required when modelling particle separation devices. Consequently, such simulations may well be practical for design purposes. A considerable amount of effort is required to develop such a wet granular flow model for slurry flow, but it appears to be extremely well suited to the type of problems that arise in cyclone design.

6. Conclusion

We have presented descriptions of four possible modelling approaches for design of cyclones - the bulk flow approach, a simple continuum approach, the hydrodynamic modelling approach and the wet granular flow approach. The first of these appears to be unsuitable for design purposes because of its heavy reliance on empirical information, much of which may be inappropriate when considering novel designs. The second approach can yield amenable equations for solution, but is too simplistic to be practically useful. The other two approaches show considerable promise for modelling of dense slurries. The hydrodynamic modelling approach is the more mature and recent models are producing results that have qualitative agreement with the real situation (Davidson, private communication). It is anticipated that it will be a viable predictive tool for cyclone design in the near future. The fourth approach - based on the emerging particle-based methods for modelling granular flows - is very promising. Granular flow codes already exist, as does local expertise (Dr Cleary of **dms**) and there is some work already in progress to incorporate hydrodynamic effects. It is based on more recent research, and therefore less well developed than the hydrodynamic models, but it is certainly worth continued effort in areas such as algorithm development and application to real slurry systems.

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