MECHANICAL CHARACTERISTICS OF CARBON FIBER YACHT MASTS

Abstract

This paper provides a preliminary stress analysis of a carbon reinforced layered cylinder such as would be found in a yacht mast. The cylinder is subjected to a compressive load and both an analytical and numerical analysis of the resulting stress fields is obtained. Some conclusions are obtained regarding the failure mode for particular examples of such cylinders.

1. Introduction

Fiber reinforced materials are being increasingly used in specialist structures where high stiffness and low weight are of considerable importance. Examples of such specialist structures are turbine blades in jet engines and racing yacht masts. Carbon fiber reinforced materials provide many of the desirable characteristics required in such applications, but they also often display some highly undesirable and sometimes unexpected properties that cause problems in their use in specialist structures. For example, although carbon reinforced turbine blades may exhibit highly desirable characteristics in the radial direction, their properties in the transverse direction may limit their usefulness in this application. Similarly, in the area of racing yacht masts, it is possible to produce a carbon reinforced cylindrical mast which has the highly desirable properties of being relatively light and also has high stiffness in the axial direction along which the fibers are aligned. Unfortunately, when subjected to standard compressive loads in the axial direction, such masts are prone to fracture and splinter as the fibers 'pop out' of the resin which binds them together to form the reinforced composite. This undesirable property needs to be suitably countered if such masts are to be useful in racing yachts. One method of overcoming the problem is to wind layers of the fiber reinforced material around the mast both on the inside and outside in order to stop the axial fibers moving out of line. These inner and outer layers do not significantly improve the strength of the material in the axial direction and of course add to the weight of the mast. It is therefore important to minimize the inner and outer layers and to include no more material than is absolutely necessary to ensure the fibers in the central layer remain in line. The purpose of the preliminary analysis in this paper is to examine the relevant stress fields with a view to determining the minimum inner and outer reinforcing layers. To this end, the stress field in a layered fiber reinforced cylinder under a compressive load is considered. Both analytical and numerical

solutions to the problem are obtained. Both solutions have their limitations, but nevertheless provide some insight into the important features of the problem and also provide information regarding the likely failure mode for cylinders of this type.

2. Statement of the problem

This analysis is specific to a three-layer tubular mast structure in the form of a circular cylinder which has thin walls relative to the mast radius. The mast is fabricated from a fiber-reinforced material consisting of carbon fibers set in an epoxy resin. The middle lamina is oriented with the fibres in the mast direction, and the fibres in the inner and outer layers lie at 90° to the mast axis (i.e. they are purely circumferential). The load is taken to be axial, so no moment is supported and naturally the mast is straight. The fully anisotropic behaviour of the materials is incorporated *in the linear regime*, so the important consequences of these properties are fully evident in the final expressions. The ratio of the stresses in all members to the applied load is obtained for all the principal directions. How these ratios vary with the design of the laminae and the elastic properties of the material is then discussed.

3. Notation

The following notation will be used throughout the analysis

$$a \;=\; \text{width of inner layer}$$

$$b \;=\; \text{width of middle layer}$$

$$c \;=\; \text{width of outer layer}$$

$$r \;=\; \text{radius of tube}$$

$$E_t \;=\; \text{Young's modulus } \textit{transverse} \text{ to the fibres}$$

$$E_l \;=\; \text{Young's modulus in } \textit{longitudinal} \text{ direction (i.e. along the fibres)}$$

$$\nu_{lt} \;=\; \text{Poisson's ratio coupling longitudinal strain to transverse strain}$$

$$\nu_{tl} \;=\; \text{Poisson's ratio coupling transverse to longitudinal strain}$$

$$\sigma_z \;=\; \text{normal stress in plane } \perp \text{ to mast axis (compressive taken as positive)}$$

$$\sigma_\theta \;=\; \text{circumferential stress (tensile taken as positive)}$$

$$\epsilon_r \;=\; \text{strain in radial direction}$$

$$\epsilon_l \;=\; \text{strain in longitudinal direction}$$

$$\text{superscript}^{(o,m,i)} \;=\; \text{refers to the outer, middle or inner layers}$$

The conventions for stress are not totally consistent with those used in normal engineering practice, but the final formulae are quite simple and the sense of the stress is obvious in each particular case. The final design formulae are convention-independent.

Now for a material exhibiting transverse anisotropy (i.e. rotational symmetry about the fibre axis), the stiffness matrix has only five unknowns and hence the relation

$$\frac{\nu_{tl}}{\nu_{lt}} = \frac{E_t}{E_l} \tag{1}$$

is easy to obtain (see for example Saada, 1974). The material properties relevant to this calculation are

$$
\begin{aligned}
E_t &= 8 \text{ GPa} \\
E_l &= 125 \text{ GPa} \\
\sigma_l^{\text{tensile strength}} &= 2090 \text{ MPa} \\
\sigma_t^{\text{tensile strength}} &= 64 \text{MPa} \\
\sigma_l^{\text{compressive strength}} &= 1717 \text{ MPa} \\
\sigma_t^{\text{compressive strength}} &= 210 \text{ MPa} \\
\nu_{lt} &\approx 0.35 \\
\nu_{tl} &\approx 0.02\text{: very little Poisson coupling in this direction.}
\end{aligned}
$$

Note that, typically, $(a + b + c)/r \approx 0.02$, so the tube is truly thin and use of the thin walled approximation in this analysis will only cause errors of the order of 1%. Also note that $E_l/E_t \approx 15$, so the tube is much stiffer longitudinally than transversely.

4. Theory for a simple longitudinal load applied to a straight mast, prior to bending in any way.

If the composite is to deform as a whole under an applied load, then the strains of each component lamina must be the same in the radial and longitudinal directions. This is the central idea in the *method of consistent deformations*, also called the *force method* (see for example Popov, 1978). Now because of the symmetry of the stress field, the tube obviously remains circular, so any strain in the circumferential direction causes an identical strain in the radial direction. Thus, for example, we can write the total radial strain of the inner layer as

$$\epsilon_r^{(i)} = \nu_{tl} \frac{\sigma_z^{(i)}}{E_t} + \frac{\sigma_\theta^{(i)}}{E_l}. \tag{2}$$

The first term is the coupling of the strain $(\sigma_z^{(i)}/E_t)$ caused by the normal load to the circumferential direction via ν_{tl}, and the second term is the direct circumferential strain caused by the circumferential stress $\sigma_\theta^{(i)}/E_l$. For the other two layers, the equivalent equations are

$$\epsilon_r^{(m)} = \nu_{lt}\frac{\sigma_z^{(m)}}{E_l} + \frac{\sigma_\theta^{(m)}}{E_t} \tag{3}$$

$$\epsilon_r^{(o)} = \nu_{tl}\frac{\sigma_z^{(o)}}{E_t} + \frac{\sigma_\theta^{(o)}}{E_l}. \tag{4}$$

Now if we consider a small section of the complete laminate, subtended by a small angle $d\theta$, then there is no net radial force on such a section (such as might come from internal pressure in boiler theory). Hence the radial projection of the circumferential forces must be zero, which for unit length gives

$$(b\sigma_\theta^{(m)} + a\sigma_\theta^{(i)} + c\sigma_\theta^{(o)})\sin d\theta = 0$$

$$\Rightarrow \quad b\sigma_\theta^{(m)} + a\sigma_\theta^{(i)} + c\sigma_\theta^{(o)} = 0. \tag{5}$$

Hence, the laminae cannot *all* be in either tension or compression; it will soon become clear that the stress on the middle sheet is of the opposite sign to the stress on the inner and outer strapping.

Using the same idea of considering a small section subtended by the angle $d\theta$ and applying radial force balance, it is easy to show that the interfacial stress in the radial direction on, say, the outer lamina, is given by

$$\sigma_r^{(\text{mo-interface})} = \frac{-c}{r}\sigma_\theta^{(o)}. \tag{6}$$

Similar relations can be written down for the other interfaces. Most importantly, these stresses are less than the circumferential stresses by approximately a factor of 50 $(c/r, b/r, a/r)$, so it is realistic to neglect the Poisson induced strains in the circumferential and longitudinal directions caused by these stresses. This is the thin shell approximation.

Turning now to the longitudinal strains, we can write these as

$$\epsilon_l^{(o)} = \nu_{lt}\frac{\sigma_\theta^{(o)}}{E_l} + \frac{\sigma_z^{(o)}}{E_t} \tag{7}$$

$$\epsilon_l^{(m)} = \nu_{tl}\frac{\sigma_\theta^{(m)}}{E_t} + \frac{\sigma_z^{(m)}}{E_l} \tag{8}$$

$$\epsilon_l^{(i)} = \nu_{lt}\frac{\sigma_\theta^{(i)}}{E_l} + \frac{\sigma_z^{(i)}}{E_t}. \tag{9}$$

If the laminate is not to fail between the layers, then these strains must be equal. Equating the longitudinal strains (7,8,9) then gives two independent equations, and similarly two further equations are obtained by equating the radial strains (2,3,4). The force balance (5) provides a fifth equation. Now there are six unknown stresses, $\sigma_\theta^{(o)}, \sigma_\theta^{(m)}, \sigma_\theta^{(i)}, \sigma_z^{(o)}, \sigma_z^{(m)}$ and $\sigma_z^{(i)}$, so it is now possible to express all of these stresses in terms of any single one of them. It is appropriate to choose the longitudinal stress $\sigma_z^{(m)}$ in the middle layer as the free parameter, since it bears the bulk of the load, as will be shown shortly. Solving equations (2,3,4,5,7,8,9) for the unknowns $\sigma_\theta^{(o)}, \sigma_\theta^{(m)}, \sigma_\theta^{(i)}, \sigma_z^{(o)}, \sigma_z^{(i)}$ in terms of $\sigma_z^{(m)}$ gives:

$$\frac{\sigma_\theta^{(o)}}{\sigma_z^{(m)}} = \frac{\sigma_\theta^{(i)}}{\sigma_z^{(m)}} = \frac{\nu_{lt} - \nu_{tl}}{1 - \nu_{lt}\nu_{tl} + \dfrac{E_l}{E_t}\dfrac{(a+c)(1-\nu_{tl}^2)}{b}} \tag{10}$$

$$\frac{\sigma_\theta^{(m)}}{\sigma_z^{(m)}} = \frac{-\dfrac{a+c}{b}(\nu_{lt} - \nu_{tl})}{1 - \nu_{lt}\nu_{tl} + \dfrac{E_l}{E_t}\dfrac{(a+c)(1-\nu_{tl}^2)}{b}} \tag{11}$$

for the circumferential stresses, and

$$\frac{\sigma_z^{(o)}}{\sigma_z^{(m)}} = \frac{\sigma_z^{(i)}}{\sigma_z^{(m)}} = \left\{ \frac{1 - \nu_{lt}^2 + \dfrac{E_l}{E_t}\dfrac{(a+c)(1-\nu_{lt}\nu_{tl})}{b}}{1 - \nu_{lt}\nu_{tl} + \dfrac{E_l}{E_t}\dfrac{(a+c)(1-\nu_{tl}^2)}{b}} \right\} \frac{E_t}{E_l} \tag{12}$$

for the longitudinal stresses. Thus, for a compressive load, all the laminae are in compression in the z-direction, and the circumferential stress is tensile in the hoop layers but compressive in the middle layer. This is intuitively very plausible.

The relation (12) is most interesting. Clearly if we make the material isotropic by putting $E_t = E_l$ and thus $\nu_{lt} = \nu_{tl}$, then the right hand side of (12) becomes 1, as it should, and the layers would then be indistinguishable. Alternatively, if the mast was made of three *loose* concentric layers (i.e. with no stress coupling between the layers), then we would expect the above ratio to be

$$\frac{\sigma_z^{(o)}}{\sigma_z^{(m)}} = \frac{\sigma_z^{(i)}}{\sigma_z^{(m)}} = \frac{E_t}{E_l}, \tag{13}$$

as is well known. This in fact is obtainable from (12) by sending $\nu_{lt}, \nu_{tl} \to 0$ to simulate 'loose coupling'. However when the layers are glued, the tension in the outer lamina (for a compressive load) 'pulls' these layers in along the mast axis, thereby increasing the fraction of the load taken by the middle lamina rather

slightly. For example, with the current mast proportions of $b = 2a = 2c$, the factor in curly braces in (12) is about 0.986. However, for any realistic values of a, b and c, the curly brace factor is not going to differ appreciably from 1, so this is not a significant fact.

What is interesting is that these stress ratio formulae are symmetric in a and c. More precisely they are functions only of the sum $a + c$. This is probably due to the thin wall approximation — the basic algebraic system that has been set up is unable to distinguish inside from outside. This may not be such an unrealistic feature as might first be supposed.

It is useful to show a simplified form of the stress ratios for the special case $a = c$ (equal thickness inner and outer hoop layers). In this case the ratios are functions of only one free parameter, a/b, and may be written in the simple forms

$$\frac{\sigma_\theta^{(o)}}{\sigma_z^{(m)}} = \frac{\sigma_\theta^{(i)}}{\sigma_z^{(m)}} = \frac{0.33}{1 + 31.5a/b} \le 0.33 \tag{14}$$

$$-\frac{\sigma_\theta^{(m)}}{\sigma_z^{(m)}} = \frac{0.66a/b}{1 + 31.5a/b} \le 0.021 \tag{15}$$

$$\frac{\sigma_z^{(o)}}{\sigma_z^{(m)}} = \frac{\sigma_z^{(i)}}{\sigma_z^{(m)}} = 0.0636 \left(\frac{a/b + 0.0283}{a/b + 0.0318}\right) \le 0.0636 \tag{16}$$

with the obvious inequalities on the right. Now if we compare these expressions to the ratios

$$\frac{\sigma_l^{\text{tensile strength}}}{\sigma_l^{\text{compressive strength}}} = 1.217, \tag{17}$$

$$\frac{\sigma_t^{\text{compressive strength}}}{\sigma_l^{\text{compressive strength}}} = 0.122, \tag{18}$$

it is immediately clear that, *for all* a/b, the inner and outer layers will not fail in longitudinal tension, nor in transverse compression, before the inner lamina fails in longitudinal compression. Similarly the middle layer will not fail in transverse compression before it fails in longitudinal compression. Interestingly, these formulae indicate that even very thin hoop layers ($a/b \approx 0$) are at about 27% of their tensile strength, and, at worst, 52% of their transverse compressive strength, at the point where the inner layer fails in longitudinal compression.

5. Numerical analysis

A numerical analysis relevant to the problem specified in Section 2 was carried out using using the finite element package MSC/NASTRAN with bi-linear quad shell elements (membrane plus bending). The analysis was based on classical bending theory. A linear static analysis was considered to examine material failure prior to buckling. Also a linear buckling analysis (eigenvalue) and a geometric non-linear analysis (without imperfections) for local buckling of a thin shell was considered to determine the failure load due to buckling (see figures 2, 3 and 4).

The results obtained are summarized in figure 1 which provides the failure load for a cylinder under compression with the total width of the cylinder wall being 1 millimeter and the percentage of the total material forming the inner and outer layers given as percentage hoop reinforcment in the graph.
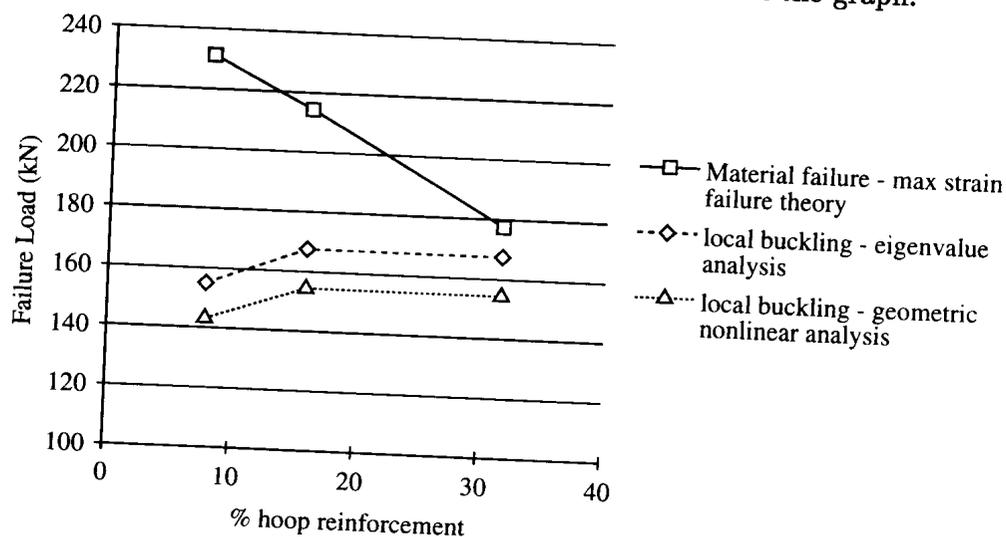
Figure 1: Material failure and local buckling for a carbon/epoxy composite mast. Wall thickness is 1 mm.

The numerical results indicate that buckling of the mast will occur before compressive failure in the middle layer. Furthermore the results show that for a mast of 1 millimeter thickness the load at which buckling occurs is maximised by having at least 15% of the mast distributed equally in the inner and outer layers. Since it is appropriate to have as much as possible of the mast in the middle layer to support the compressive load these results suggest no more than 15% of the mast should be distributed in the inner and outer layers.

L

L/D = 2

L/D = 5

Shell thickness = 1 mm

Diameter D = 47 mm

Hoop reinforcement = 16%

Local buckling is not
sensitive to L/D ratio

Buckle wavelength ≈ radius
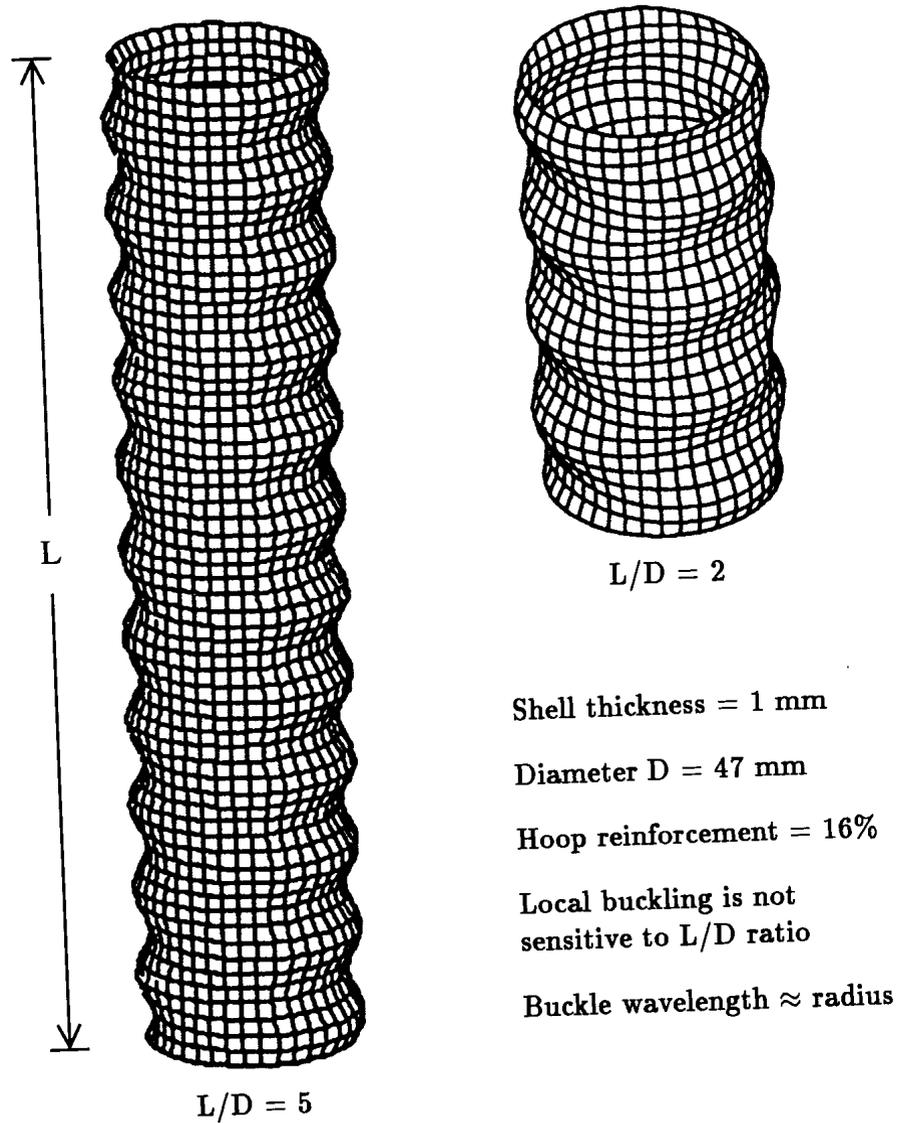
Figure 2: Local buckling of a thin shell
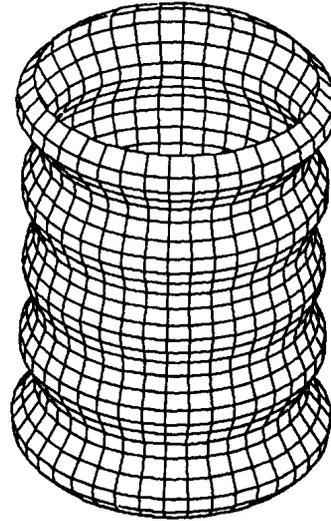
Figure 3: Geometric nonlinear analysis: local buckling mode
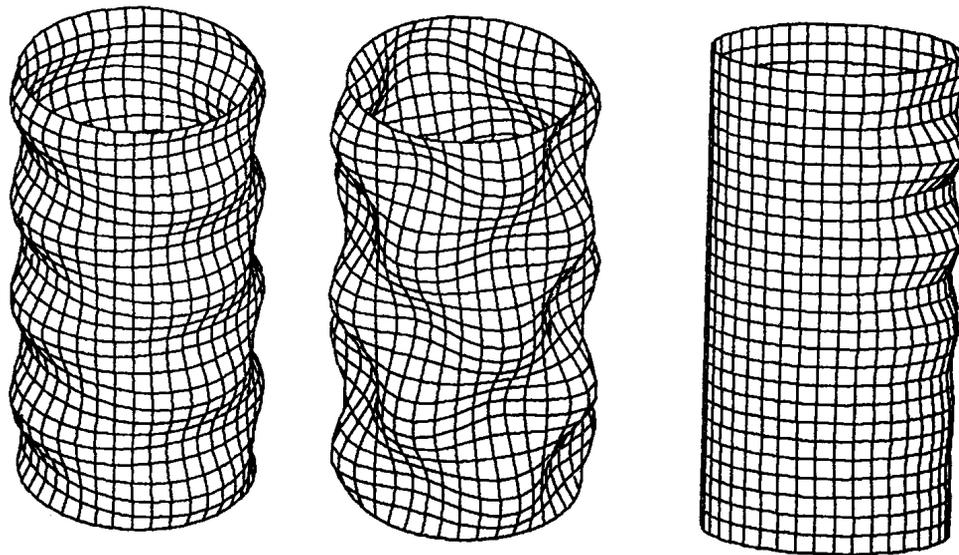
Figure 4: Linear buckling: eigenvalue analysis. (a) Compression, lowest eigenvalue. (b) Compression, higher mode. (c) Compression plus bending, lowest eigenvalue.

6. Some conjectures, speculations, and recommendations

If the possibility of buckling is ignored then the conclusion from the analysis in Sections 1–4 would be to make the inner and outer hoop layers as small

as is convenient. This is because the area of the middle lamina would need to be increased as much as possible in order to reduce the compressive stress for a given load and mast weight. This seems to be in accord with a mast constructor's observation that 'only a layer of sticky tape is needed to keep the layers in'. Accordingly, if a/b is small, it is possible to naively obtain an upper bound for the stress in the middle lamina from equation (16),

$$\left|\sigma_z^{(m)}\right| \leq \frac{|P|}{2\pi r(0.113a + b)}, \tag{19}$$

assuming an applied compressive load P. This can be rearranged to give the restriction

$$0.113a + b > \frac{|P|}{2\pi r \sigma_l^{\text{compressive strength}}}, \tag{20}$$

but this formula is likely to grossly underestimate the required thickness since, as is clear from the numerical analysis, buckling is very likely to occur prior to pure compressive failure.

Unfortunately, there are no simple analytical formulae which give a clear indication as to how the stresses vary in bending or local buckling. However, buckling formulae are usually sensitive only to elastic constants (e.g. the Euler formula or the beer-can formula), so the critical buckling load of this composite is likely to be given by a complicated function of the anisotropic elastic constants only. It is difficult to estimate how this will vary with a/b. Nevertheless, the simple theory given above does give a reasonably accurate idea of how the stresses are distributed prior to failure, and thus may be of some use in the construction of yacht masts.

It seems likely that, given the slenderness ratios and rigging supports used in practice, the material will fail in local buckling or longitudinal compression well before the Euler load. But consider the beer-can formula for an isotropic thin cylinder of thickness h and radius r (see Timoshenko, 1961):

$$\sigma_{\text{cr}} = \frac{Eh}{r\sqrt{3(1 - \nu^2)}}. \tag{21}$$

If $h/r \ll 1$, then the buckling stress σ_{cr} is usually well below the compressive strength. Unfortunately, in the case of these masts, h/r is not especially small, so the local buckling strength is likely to be of the same order as the compressive strength. Another possible problem is that the beer-can failure wavelengths might be quite large, so an experiment or finite-element calculation might have to use a fairly long cylinder to avoid any finite-length dependence of the critical stress. The simple isotropic formulae predict about a 30 millimeter wavelength for a 92 millimeter diameter mast of thickness 1 millimeter.

It seems likely that a full finite element calculation has to be done to satisfactorily analyse the problem. And given the size of the 'parameter space' (the range of possible loads, a/b, etc.) through which such a calculation would have to search, perhaps some restrictive decision would need to be made about the type of loads considered. The direct axial load may be a good starting point, since it can be directly compared with the preceding theory; the expressions given above are probably a good check on the finite element calculation prior to buckling. Secondly, from the simple statics of the rigging etc., it should be possible to find that part of the mast subject to the maximum moment, and use this moment plus the axial load as the load conditions in another finite element computation. Then it would be appropriate to try to maximise the load as a function of a/b, as was attempted at the MISG. Naturally the finite element result would have to be mesh-independent, and free of any finite length artifacts.

Acknowledgments

References

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