COOLING OF JARRED CHEESE SPREADS

An overall heat balance and a model for the cooling of an individual jar are derived for glass jars of cheese spread on Kraft's production line. Good agreement is found between the model predictions and temperature data collected by Kraft. A possible cause of boiling in the cheese is proposed and steps for its prevention are suggested.

1. Introduction

Kraft produce large amounts of cheese spread both for the domestic market and for export. The cheese is marketed in various glass jar sizes ranging from 150 g to 680 g. A vital part of the production is the cooling of the spread after the jars are filled with cheese at about 80–85°C. These jars are placed on a belt and pass through a cooler taking about one hour to pass right through.

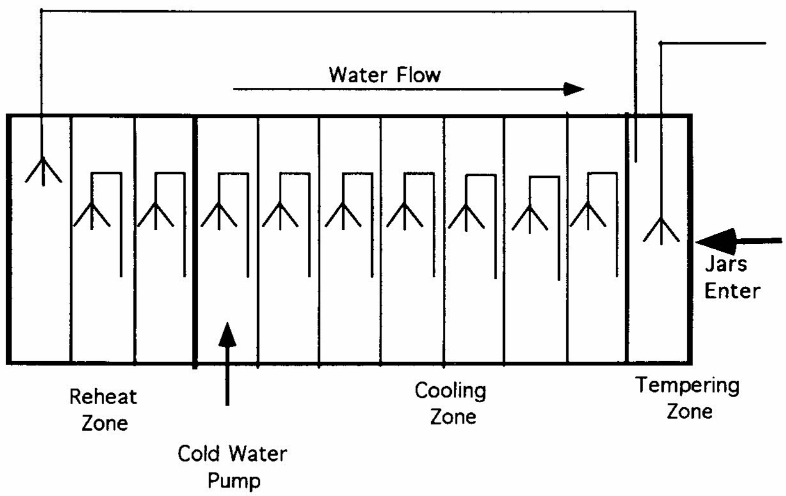

Figure 1: The Kraft 11–zone jar cooler.

The cooler consists of 11 zones as shown in Figure 1. The jars first experience a relatively warm water zone, a tempering zone, in order to prevent cracking of the glass which cannot withstand the temperature jump of more than about 42°C across its width. The next seven zones act as a heat exchanger with the jars moving in one direction and refrigerated water flowing in the other, with sprays facilitating the cooling of the jars. The final three zones are slightly warmer and are designed to bring the exterior temperature of the glass to above the dew point to prevent any surface condensation.

The aim of the cooling process is to bring the temperature of the cheese as rapidly as possible to 25°C. When this does not occur, the cheese may spoil and have to be recalled. Kraft wanted the Study Group to produce a cooling model and to look at optimising the process.

2. Overall heat balance in the cooler

The cheese, glass and belt go into the cooler and in the cooling process heat is exchanged with refrigerated water. We obtain the overall heat balance

$$q_w c_w \Delta T_w = (q_c c_c + q_g c_g)\,\Delta T_p,$$

where q_w, q_c and q_g are the mass flow rates of water, cheese and glass, respectively, c_w, c_c and c_g are the respective specific heats of water, cheese and glass, and ΔT_w and ΔT_p are the temperature changes in the water and "product" (cheese and glass). Estimates of the cooling required for the belt showed that it was negligible. We use data appropriate to the cooling of 500 g jars. The flow rate of refrigerated water into the cooler is about 18 m^3/h which is 18000 kg/h with cheese mass flow rate of 6000 kg/h and a glass mass flow rate of 3000 kg/h. With $c_w = 4.2$ kJ/kg.K, $c_c = 3.19$ kJ/kg.K and $c_g = 0.75$ kJ/kg.K, then

$$\Delta T_w \approx 0.28 \Delta T_p.$$

Since we want to cool the cheese to 25°C and the initial cheese temperature is 81°C, say, then $\Delta T_p = 81 - 25$°C and $\Delta T_w \approx 15$°C. The inlet water temperature of 2°C from the refrigeration system gives a predicted final water temperature of $2 + 15 = 17$°C, which is in good agreement with the data from Kraft. The rate at which heat must be removed from the cooler is $q_w c_w \Delta T_w/3600 \approx 315$ kW. The limit on refrigeration is currently 300 kW.

3. Temperature distribution in a single jar

In the first instance, we consider heat transfer for a single jar of cheese spread. We ignore effects due to the packing together of jars in the cooling unit, so that the temperature distribution in the circular cylindrical jar is axisymmetric. In addition, we neglect any cooling from the top or bottom of the jar. The bottom of the jar is sheltered from the water sprays, and the top of the jar is insulated by the air trapped below the lid, so this should be a good approximation. Then, the temperature, T, of the jar and contents can be written

$$T = T(r, t)$$

where r is the radial coordinate and t is time. The problem is then of one-dimensional heat conduction through the spread and the glass jar, with forced convection of heat by the water sprays surrounding the jar. The heat conduction problem can be written as follows.

In the spread,

$$\frac{\partial T}{\partial t} = \frac{\alpha_1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{1}$$

where α_1 is the thermal diffusivity of the cheese. Similarly, in the glass,

$$\frac{\partial T}{\partial t} = \frac{\alpha_2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{2}$$

where α_2 is the thermal diffusivity of the jar. Initially, assume that the cheese and jar are at a uniform temperature,

$$T(r,0) = T_c. \tag{3}$$

At the centreline ($r = 0$),

$$\frac{\partial T}{\partial r} = 0 \tag{4}$$

and at the spread/glass interface ($r = R_1$), assuming perfect thermal contact,

$$k_1 \left.\frac{\partial T}{\partial r}\right|_{R_1^-} = k_2 \left.\frac{\partial T}{\partial r}\right|_{R_1^+} \tag{5}$$

where k_1 and k_2 are the thermal conductivity of the cheese and the jar, respectively. At the outside edge of the jar ($r = R_2$), a heat flux balance gives

$$-k_2\frac{\partial T}{\partial r} = h(T - T_w(t)), \tag{6}$$

where h is the heat transfer coefficient.

Equations (1–6) can be solved in various ways, but it is instructive to first consider the timescales of heat transfer associated with each medium. In the spread, the thermal timescale is given by

$$\tau_1 = \frac{R_1^2}{\alpha_1}$$

and in the glass, the thermal timescale is

$$\tau_2 = \frac{(R_2 - R_1)^2}{\alpha_2}.$$

Typical properties for cheese spread give $\alpha_1 \approx 1.3 \times 10^{-7}$ m^2/s, for glass give $\alpha_2 \approx 1.9 \times 10^{-6}$ m^2/s and for a 310 g jar, $R_1 = 34.35$ mm and $(R_2 - R_1) = 2.5$ mm. This gives

$$\tau_1 \approx 9000\,\mathrm{s} \qquad \text{and} \qquad \tau_2 \approx 3\,\mathrm{s}$$

which indicates that the temperature in the glass equilibrates much more quickly than that in the spread. Therefore, we use a quasi-steady approximation in the glass, effectively reducing equation (2) to

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \tag{7}$$

which has the general solution

$$T = A\ln(r/R_1) + B.$$

Substituting this into the boundary condition (6) and using (5) we eliminate the coefficients A and B to give a modified boundary condition at R_1 for the heat conduction in the jar of

$$\frac{\partial T}{\partial r} + \mu T = \mu T_w \tag{8}$$

where

$$\mu = \frac{k_2}{R_1 k_1\left[\frac{k_2}{hR_2} + \ln(\frac{R_2}{R_1})\right]}.$$

Thus we have eliminated the need to solve in both the spread and the jar simultaneously, by incorporating the glass solution in the boundary condition (8). Equation (1), with boundary conditions (4) and (8) and initial condition (3) has an exact solution of the form (see, for example, Hill and DeWynne (1987))

$$T(r,t) = u(r,t) + v(r,t) \tag{9}$$

where

$$u(r,t) = \sum_{n=1}^{\infty} a_n e^{-\alpha_1\lambda_n^2 t} J_0(\lambda_n r) \tag{10}$$

satisfies the initial condition and zero boundary condition at R_1 and

$$v(r,t) = \frac{2\alpha_1\mu}{R_1}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0(\lambda_n r)}{(\lambda_n^2+\mu^2)J_0(\lambda_n R_1)}F_n(t) \tag{11}$$

satisfies a zero initial condition and the full condition (8). In equations (10) and (11), λ_n are the positive roots of

$$\lambda_n J_0'(\lambda_n R_1) + \mu J_0(\lambda_n R_1) = 0, \tag{12}$$

the coefficients a_n in equation (10) are given by

$$\begin{aligned} a_n &= \frac{T_c \int_0^{R_1} \xi J_0(\lambda_n \xi) d\xi}{\frac{R_1^2}{2}(1+\frac{\mu^2}{\lambda_n^2}) J_0^2(\lambda_n R_1)} \\ &= -\frac{2\lambda_n J_0'(\lambda_n R_1) T_c}{R_1(\lambda_n^2+\mu^2) J_0^2(\lambda_n R_1)} \\ &= \frac{2\mu T_c}{R_1(\lambda_n^2+\mu^2) J_0(\lambda_n R_1)} \end{aligned}$$

using equation (12) to obtain the last, and simplest form, and

$$F_n(t) = \int_0^t e^{-\alpha_1 \lambda_n^2 (t-\tau)} T_w(\tau) d\tau.$$

If we assume that $T_w(t)$, is piecewise constant, representing a different constant water temperature in each of the 11 zones, then

$$F_n(t) = \frac{e^{-\alpha_1 \lambda_n^2 t}}{\alpha_1 \lambda_n^2} \left[T_{w,m} e^{\alpha_1 \lambda_n^2 t} - T_{w,1} - \sum_{l=1}^{m-1} (T_{w,l+1} - T_{w,l}) e^{\alpha_1 \lambda_n^2 t_l} \right]$$

where a jar is located in zone m after time t and $T_{w,l}$ is the cooling water temperature in zone l.

Combining all of the above parts of the solution gives the following form

$$\begin{aligned} T(r,t) &= \frac{2\mu}{R_1} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r) e^{-\alpha_1 \lambda_n^2 t}}{(\lambda_n^2+\mu^2) J_0(\lambda_n R_1)} \Bigg[T_c + T_{w,m} e^{\alpha_1 \lambda_n^2 t} - T_{w,1} \\ &\qquad - \sum_{l=1}^{m-1} (T_{w,l+1} - T_{w,l}) e^{\alpha_1 \lambda_n^2 t_l} \Bigg]. \end{aligned} \tag{13}$$

The exact solution is valuable because it points out the linear dependence of the spread temperature on the water temperatures $T_{w,l}$ in the zones, and because the solution is easy to evaluate, provided the λ_n can be evaluated. In fact, it is necessary to solve equation (12) numerically for the λ_n, but experience has shown that only a few terms (say 10) of the infinite series are required to obtain an accurate solution.

In order to calculate the total heat transfer out of a jar, which is needed for the zonal heat balance described in Section 4, we need to know $T(R_2, t)$, which can be shown to be

$$T(R_2,t) - T_w = \frac{k_1 R_1 \mu}{h R_2} (T(R_1,t) - T_w) \tag{14}$$

and $T(R_1, t)$ can be evaluated from equation (13).

In principle, we now have a solution to the problem of heat conduction in a single isolated jar. In fact, jars are packed together as they move through the cooler, so the assumption of axisymmetry may not be appropriate. In the worst case, the cylindrical jars will be hexagonally close packed, so that a single jar is in contact with its neighbours at 6 points equally spaced around its circumference. At each of those points, there will be no heat transfer out of the jar, whilst in between these points heat transfer will be high due to the water sprays. We can simulate the effect of the packing on heat transfer by making the heat transfer coefficient h dependent on the azimuthal coordinate θ in the jar. As an example, we used

$$h = h_0 sin^2 3\theta$$

and solved the resulting two-dimensional transient heat conduction problem for $T(r, \theta, t)$ numerically, using the computer package *Fastflo*. Figure 2 shows computed isotherms in the jar, which indicate that the azimuthal variation of h causes only small changes in the solution near the contact points, and that the bulk temperature is not much affected by the contact, since the isotherms rapidly become circular away from the outer edge of the cheese. Therefore, we are justified in neglecting the effect of contact with other jars on the heat transfer for a single jar.

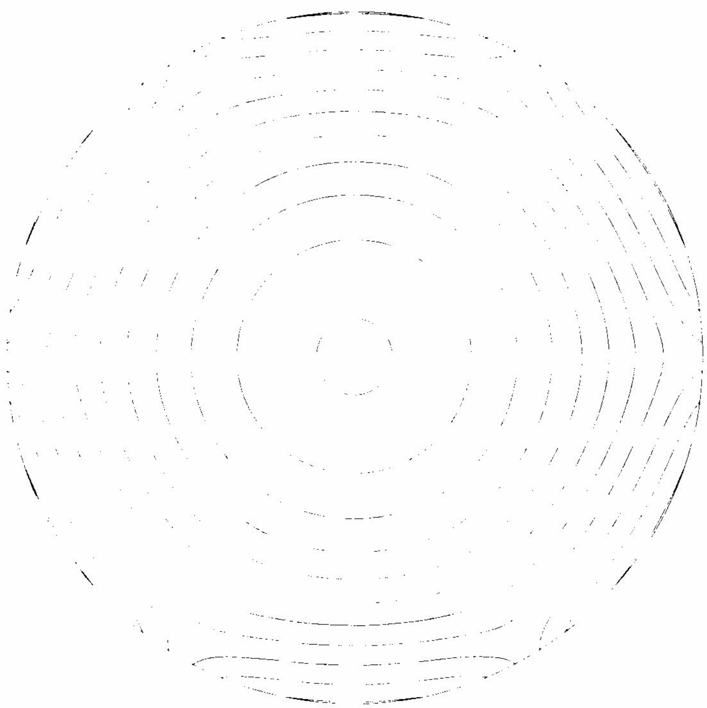

Figure 2: Temperature distributions predicted by a two-dimensional model of the heat conduction.

4. Zonal heat balance

Typically, water sprayed on jars passing through a given zone, call it zone m, comes from the tray beneath a neighbouring zone, call it zone $m+1$. The water then gains heat from the jars upon which it is sprayed, and collects in a tray below zone m, ready for delivery to another zone, or to a heat exchanger. Thus, if we assume that the water temperature in each zone is a constant, $T_{w,m}$, then a heat balance in our typical zone m gives

$$q_w T_{w,m+1} c_w + H_m - q_w T_{w,m} c_w = 0 \tag{15}$$

where H_m is the total amount of heat taken out of the jars by the water per unit time. It is straightforward to see that

$$\begin{aligned} H_m &= -J 2\pi R_2 H h \int_{t_{m-1}}^{t_m} (T_{w,m} - T(R_2,\tau))d\tau \\ &= -J 2\pi H k_1 R_1 \mu \int_{t_{m-1}}^{t_m} (T_{w,m} - T(R_1,\tau))d\tau \\ &= J 2\pi H k_1 R_1 \mu \\ &\quad \left((t_m - t_{m-1})T_{w,m} - \frac{2\mu}{R_1} \sum_{n=1}^{\infty} \frac{1}{\alpha_1 \lambda_n^2 (\lambda_n^2 + \mu^2)} \Big[\alpha_1 \lambda_n^2 (t_m - t_{m-1}) T_{w,m} \right. \\ &\quad \left. + \left(T_{w,1} - T_c + \sum_{l=1}^{m-1} (T_{w,l+1} - T_{w,l}) e^{\alpha_1 \lambda_n^2 t_l}\right)\left(e^{-\alpha_1 \lambda_n^2 t_m} - e^{-\alpha_1 \lambda_n^2 t_{m-1}}\right) \Big] \right) \end{aligned}$$

where, as in Section 3, we have neglected top and bottom heat transfer, and we have used equations (13) and (14) to obtain the final expression for H_m. In summary, we can substitute this expression into equation (15) to obtain an equation that is linear in the T_w's for the zones. In fact, a similar equation may be written for each zone, although there may be some variation due to the plumbing arrangement of each zone. Thus we can generate a set of simultaneous linear equations that can be solved for the water temperatures in each zone. These temperatures can then be re-substituted into the analytical solution for the spread temperature, in order to determine the variation in temperature throughout the cooling unit.

This is a significant and very useful result, because it allows the prediction of the temperature variation throughout the cooling unit, and it can be used to analyse the current cooler configuration. In can also be used to investigate modifications to the configuration, in order to determine whether improvements to the cooling capacity are possible. Finally, it would allow investigation of the ability of the cooling unit to handle different product design, such as larger jars or a filling that has different thermal properties. It would be relatively straightforward to implement this formulation in software as a tool for understanding the cooling unit.

4.1 Comparison with experiment

Although we have justified the assumptions that led to the development of a one-dimensional model for heat transfer in the jars, and the subsequent linear dependence of the spread temperature upon the water temperature, the accuracy of the solution can only be determined by comparison with experiment.

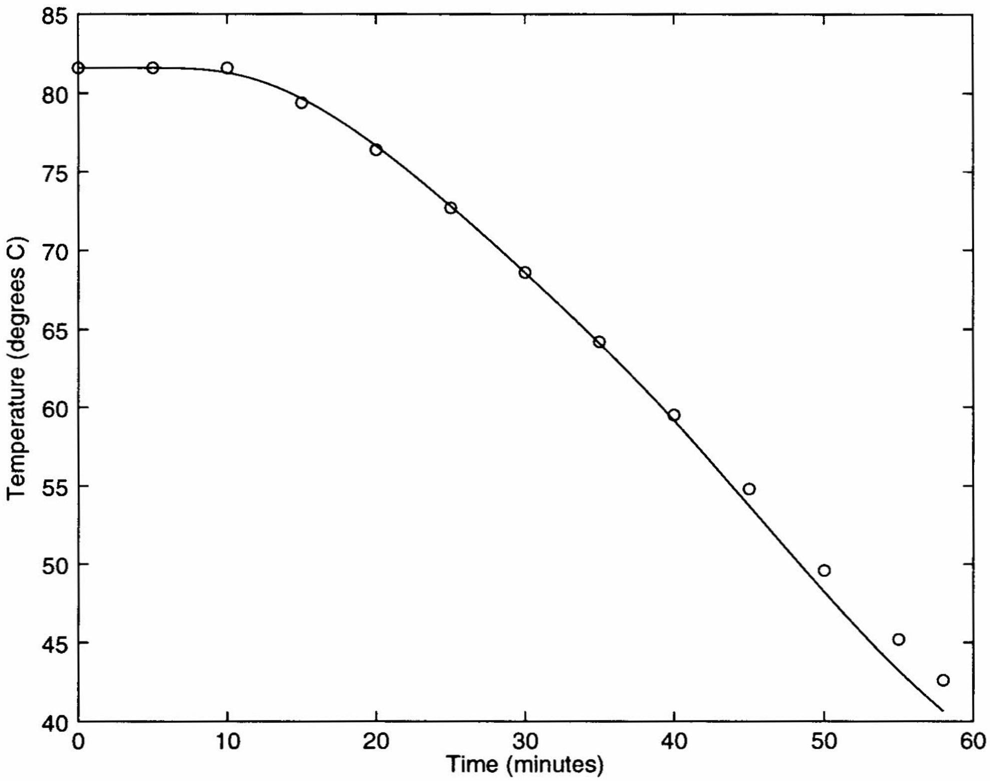

Figure 3: The predicted core temperature of the cheese spread in the jar as it moves through the cooler using the water cooler zone temperatures supplied by Kraft. The circles are the experimentally determined temperatures found by Kraft.

Figure 3 shows the result of evaluation of the spread temperature analytical solution for given water temperatures in each zone. These water temperatures were measured by Kraft, and Kraft also measured the temperature at the centre of jars as they reached the end of each zone. The measured centre temperatures are also recorded on the figure, and the agreement between measurement and theory is remarkably good. This gives us confidence that the one-dimensional model is a very good approximation. Figure 4 shows the calculated radial variation of temperature in a jar at the end of each zone. It shows that there is an initial steep gradient near the outside of the jar, and only a small change in the centre temperature for the first two zones. This is important to the formation of bubbles, as described in Section 5. The overall temperature drops, and the warming zones at the end tend to even out the variation throughout the jar.

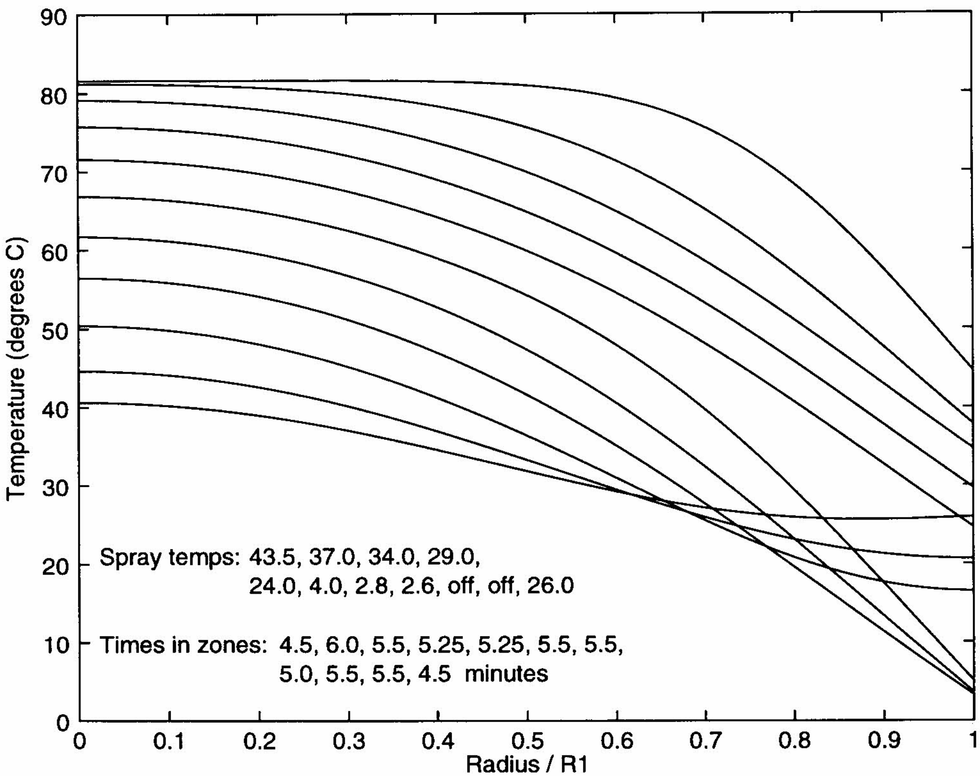

Figure 4: Temperature distribution in the cheese as a function of radius in the 11 cooler zones. The spray temperatures are taken from experiments by Kraft.

5. Bubble formation and boiling

It seemed from the video shown by the Kraft representatives that the bubbles observed in the sample jars do not originate from boiling of the cheese. It appears that bubbles formed during the filling process are transported to the outside of the jar by convective motion resulting from the cheese boiling in the centre.

The boiling of the cheese appears to result from a reduction in the pressure in the gap between the cheese and the lid. This reduced pressure is such that the cheese at the centre of the jar, which is still nearly at the initial temperature, can boil.

Another effect of this pressure drop is conjectured to be the enlargement of the pre-existing entrained air bubbles to a more noticeable size. The pressure reduction in the gap results from three sources:

- increase in the gap volume as the cheese cools and contracts (especially at the edges)
- the steam in the gap condenses on cooling
- the air in the gap cools and contracts.

5.1 Increase in gap volume

The contraction of the cheese is approximated by

$$C = C_0 - \iiint_{cheese} (T_0 - T(r,t))\, c_e \, dV$$

where T_0 is the initial temperature of the cheese which occupies a volume C_0, C and $T(r,t)$ are the volume of the cheese and the radial temperature profile in the cheese, respectively and c_e is the coefficient of thermal expansion of the cheese.

A temperature profile $T(r,t)$ has been determined by the methods outlined in Section 3. However, for simplicity, we assume a parabolic temperature profile. In order to consider the worst case we assume the temperature of the cheese at the centre to be T_0 and the boundaries to be at the temperature of the water spray T_w. Thus we take a temperature profile given by

$$T(r) = T_0 \quad - \quad (T_0 - T_w)\frac{r^2}{R_1^2}.$$

The volume integral above then becomes

$$\iiint_{cheese} (T_0 - T_w)\frac{r^2}{R_1^2} c_e \, dV$$

$$\approx C_0 \frac{(T_0 - T_w)}{2} c_e$$

and

$$C = C_0 \left[1 - \frac{(T_0 - T_w)}{2}\right] c_e.$$

That is, the gap volume is increased by

$$C_0 \frac{(T_0 - T_w)}{2} c_e.$$

5.2 Cooling of the air in the gap

Assuming that the gas in the gap obeys the Perfect Gas Law, the pressure in the gap will be given by

$$\frac{P}{P_0} = \frac{T_w/T_0}{G/G_0}.$$

Here we are assuming that the air in the gap quickly comes to the temperature of the cooling water bathing the lid, G_0 and G are the gap volumes initially and after contraction respectively, and P_0 and P are the pressures before and after contraction. We can then estimate the change in pressure in the gap as follows:

$$\frac{P}{P_0} = \frac{T_w}{T_0}\frac{G_0}{G} \tag{16}$$

$$= \frac{T_w}{T_0}\frac{1-\alpha}{\left[1-\alpha(1-\frac{(T_0-T_w)}{2}c_e)\right]}, \tag{17}$$

where α is the proportion of the jar volume initially filled. Rearranging this expression gives

$$T_w = \frac{1-\alpha(1-\frac{T_0}{2}c_e)}{\frac{P_0}{PT_0}(1-\alpha)+\frac{\alpha}{2}c_e}.$$

Cheese (water) boils at about 83°C when the pressure is about 0.53 atm (Perry, 1984), so we want to keep the water temperature above the estimate given by the formula above for a given value of the jar-fill proportion, α. It is unclear what proportion of the gap is initially filled with steam. It also needs further investigation to determine how the steam/water behaves under the changing temperature and pressure conditions in the gap. We have taken $P_0 = 0.74$ atm to be the air pressure in the gap before further cooling takes place. The core axial temperature has been shown, both by experimental evidence and numerical simulation, to remain at or very near to the initial value, T_0, for some time — approximately 15–20 minutes. Thus the water temperature estimated for the prevention of cheese boiling remains relevant throughout at least the first two, and possibly the first three, zones.

To keep the centre of the cheese from boiling, we need to keep the spray temperature of the first three zones sufficiently high. Figure 5 shows that this is highly dependent upon the level to which the jar is filled. For the 500 g jar, the fill proportion lies between 0.92 and 0.966 (for clearly there is a problem with the size of the gap in this case). Figure 5 shows that for $\alpha = 0.966$ (worst case scenario for the 500 g jar) the critical value of T_w is around 50°C. We recommend therefore that the first few zones are sprayed with water at a temperature around 50°C, before cold water is introduced.

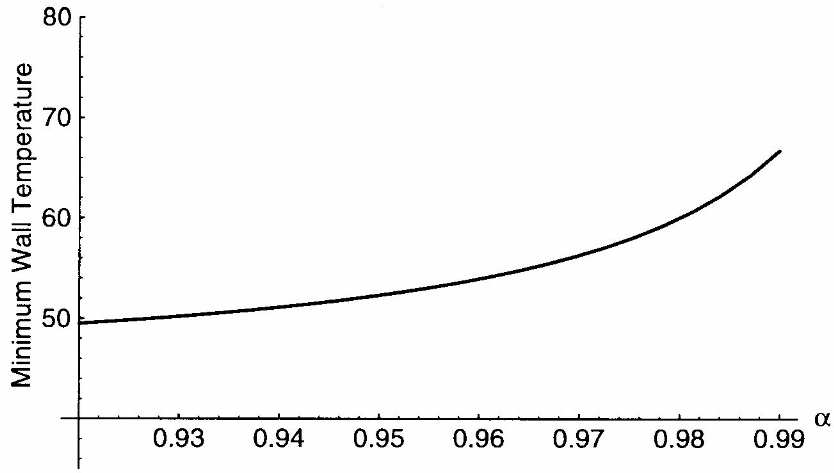

Figure 5: Estimated water temperature at which boiling will occur as a function of the proportion of the jar filled.

6. Discussion

The work at the MISG resulted in several new ideas about different aspects of the cooling of jarred cheese. An argument based on overall heat balance gives a way of estimating the size of the refrigeration unit required for a given number of jars of cheese per hour, water flow rate, etc.

An analysis of the cooling of a single jar is useful in a number of ways. It shows, for example, that a 1–D model of the heat conduction is sufficient to describe the cooling process. This 1–D model gives predictions for the core temperature of the cheese which are in very good agreement with the temperatures measured by Kraft as the jar travels through the cooler. This model also provides the basis for a zonal heat balance model that can be used for the development of new coolers.

The boiling of cheese which sometimes occurs is almost certainly the result of the lowering of the pressure in the jar associated with the cooling and shrinking of the cheese. A strategy to prevent boiling is to ensure that the cooling process begins slowly and that the jars are not filled too much.

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