Adaptive Light Control System

Problem Presenter

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Problem Statement

An adaptive traffic light system for crossroads is to be developed with the control being the data obtained through fixed cameras attached to the light system. The control itself is to be adaptive as there is no need for collecting data during the time when there is no traffic at all. Thus the problem is to collect data adaptively and control the light system accordingly. The idea, of course, is not to have people wait for unnecessary amount of time along the way, while there is no traffic across roads. Though looks rather reasonable, a very good adaptive strategy and an accompanying algorithm need to be developed. The study group is asked for such an algorithm. The problem requires collaborative work of mathematicians, computer scientists and electrical engineers.

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Content

The study groupe has identified two essential components of the problem:

maximize the traffic flow through the intersection and

minimize total waiting time at the intersection.

Both are formulated and combined into a single linear objective function with linear constraints by using appropriate relation between the density and speed.

1. Model

Given the lengths of the queues in four directions (obtained from camera images)

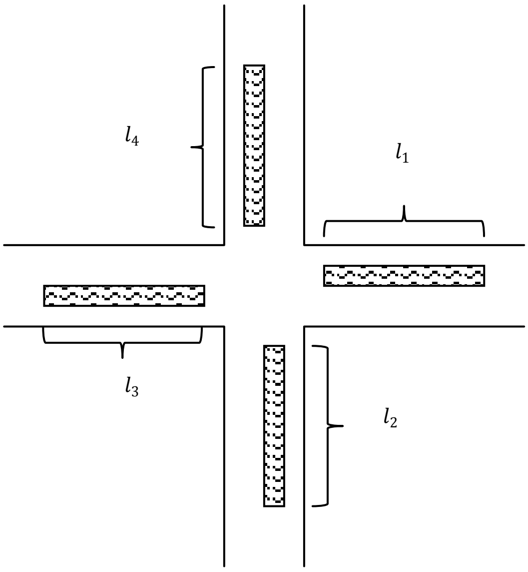

Figure 1. Final distribution of red and green lights for $E-W$, $N-S$ directions.

Aim-1

- Maximize flow through intersection

$$a) \sum_i \int_0^{t_i^g\,(green)} \rho_i v_i(t)dt \rightarrow maximize \tag{1}$$

or

$$\max\left(t_1^g v_1\rho_1 + t_2^g v_2\rho_2 + t_3^g v_3\rho_3 + t_4^g v_4\rho_4\right) \tag{2}$$

where ρ_i, v_i are the traffic density and velocity, respectively, in the $i-th$ direction as shown in Figure 1 and $t_i^{g\,(green)}$ is the duration of green light. The symbols E, W,N,S stand for the directions East, West, North and South.

We assume that

$$v_1 = v_2 = v_3 = v_4 = \bar{v} \tag{3}$$

so the maximization problem reduces to

$$t_1^g(\rho_1 + \rho_3) + t_2^g(\rho_2 + \rho_4) \to max \tag{4}$$

Aim-2

- Minimize the total waiting time

$$b)\ t_1^r \max(l_1, l_3) + t_2^r \max(l_2, l_4) \to minimize \tag{5}$$

If we define $l_{13} = \max(l_1, l_3)$and $l_{24} = \max(l_2, l_4)$ and the problem reduces to

$$(t_1^r l_{13} + t_2^r l_{24}) \to minimize \tag{6}$$

On the other hand, it is known that

$$t_1^r + t_1^g = T \tag{7a}$$
$$t_2^r + t_2^g = T \tag{7b}$$

So, the optimization problem becomes

$$t_1^g l_{13} + t_2^g l_{24} \to maximize \tag{8}$$

Thus, we have introduced two separate cost functions which are the minimum waiting time and maximum flow. These cost functions, Eq. (4) and (8), can be combined into

$$\alpha\left[\left(t_1^g l_{13} + t_2^g l_{24}\right)\right] + (1-\alpha)\left[t_1^g \rho_{13} + t_2^g \rho_{24}\right] \to maximize \tag{9}$$

Where $\alpha \in [0,1]$ ve $\rho_{13} \equiv \rho_1 + \rho_3$; $\rho_{24} \equiv \rho_2 + \rho_4$. It should be noted here that t_1^g and $t_2^g$are the unknowns to be determined throughout this optimization problem.

Constraints of the optimization problem:

1. $t_i^g \leq T$, $i = 1,2,3,4$ (10a)

2. $t_1^g = t_3^g \; and \; t_2^g = t_4^g$ (10b)

3. $t_i^g \geq t_{min}$, $if \;\; l_i > 0$ (10c)

4. $\int_0^{t_i^g} \rho_i(t,x) v_i(t,x) dt \leq l_i$ (10d)

The constraints, (10a-10d), are set to maximize the objective function (9) in a realistic scenario shown in Fig.1. A linear programming method is suggested to solve (9),(10a-d) however constraint (10d) is not linear, as it stands. So, in order to reduece (9),(10a-d) to a linear programming problem, a relation between density and speed ($\rho \; and \; v$) is needed. For this reason the following continuous traffic flow model is considered.

2. Continuous traffic model

Let $\rho = \rho(x,t)$ and $v = v(x,t)$ represent density and velocity of traffic at the the position x and time t. Then the conservation law reads

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0, \qquad (11)$$

where $v = F(\rho, \rho_x, \rho_t, \ldots)$.

(11) is a nonlinear hyperbolic PDE problem. We only need a linear relation between speed and density.

The following is a linear approximation for the required relation

$$v(t,x) = V_m \frac{\rho_m - \rho(t,x)}{\rho_m}, \qquad (12)$$

where V_m and ρ_m are the maximal values of the speed and density.

Assume that, the light tuns into green at $t = 0$ and

$$\left.\begin{matrix} \rho = \rho_m \,; x \leq 0 \\ \rho = 0 \,; x > 0 \end{matrix}\right\} at \; t = 0$$

For $t > 0$ solution is called an expansion fan. In this case the density can be defined as $\rho(t,x) = F\left(\frac{x}{t}\right)$. In order to determine $F(z)$, $\rho(t,x) = F\left(\frac{x}{t}\right)$ and $v(t,x) = V_m \frac{\rho_m - \rho(t,x)}{\rho_m}$ are substituted into (11). Then the following relations are obtained:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0\Rightarrow\frac{\partial\rho}{\partial t}+v\frac{\partial\rho}{\partial x}+\rho\frac{\partial v}{\partial x}=0\Rightarrow \tag{13}$$

$$-\frac{x}{t^2}F^{\prime}+V_m\left(\frac{\rho_m-F}{\rho_m}\right)\frac{1}{t}F^{\prime}+FV_m\left(-\frac{1}{\rho_m}\right)\frac{1}{t}F^{\prime}=0 \tag{14}$$

For $F^{\prime}\neq 0$, multiplying both sides of (14) with $\frac{\rho_m}{V_m}$ gives

$$-\frac{\rho_m}{V}\frac{x}{t}+\rho_m-F-F=0. \tag{15}$$

Then,

$$F\left(\frac{x}{t}\right)=\frac{\rho_m}{2}\left(1-\frac{x}{V_m t}\right) \tag{16}$$

or

$$\rho(t,x)=F\left(\frac{x}{t}\right)=\frac{\rho_m}{2}\left(1-\frac{x}{V_m t}\right) \tag{17}$$

is obtained as a *Fan Solution*.

Using (17), we obtain the following result:

$$\begin{aligned}v(t,x)&=V_m\left(\frac{1}{\rho_m}\frac{\rho_m}{2}\left(1-\frac{x}{V_m t}\right)\right)\\&=\frac{V_m}{2}+\frac{x}{2t}=\frac{V_m}{2}\left(1+\frac{x}{V_m t}\right).\end{aligned} \tag{18}$$

Then the flow is obtained as

$$\rho(t,x)v(t,x)=\frac{\rho_m V_m}{4}\left(1-\frac{x^2}{V_m^2 t^2}\right). \tag{19}$$

The maximum flow is attained at the light position, $x=0$, and becomes

$$\rho(t,0)v(t,0)=\frac{\rho_m V_m}{4}. \tag{20}$$

If we substitute (20) into (10d), we have

$$\int_0^{t^g}\rho(t,0)v(t,0)dt=\frac{\rho_m V_m}{4}t^g \tag{21}$$

Using (21), the constraint (10d) becomes

$$\frac{\rho_{mi} V_{mi}}{4} t_i^g \leq l_i, \quad i = 1,2,3,4 \tag{22}$$

Thus the new constraint equation (22) should be used instead of the constraint (10d). Then all the constraints would be linear in this optimization process. Therefore a linear programming method can be used to solve the optimization problem defined in (9) to determine the optimal values of t_1^g and t_2^g .

Since the method described above has been considered for very idealised form of the traffic light problem, it would be appropriate to express the view of the group for the general form of the traffic ligh optimization problem.

For example, the density function $\rho(t, x)$ can be discussed in the following figures for a real application.

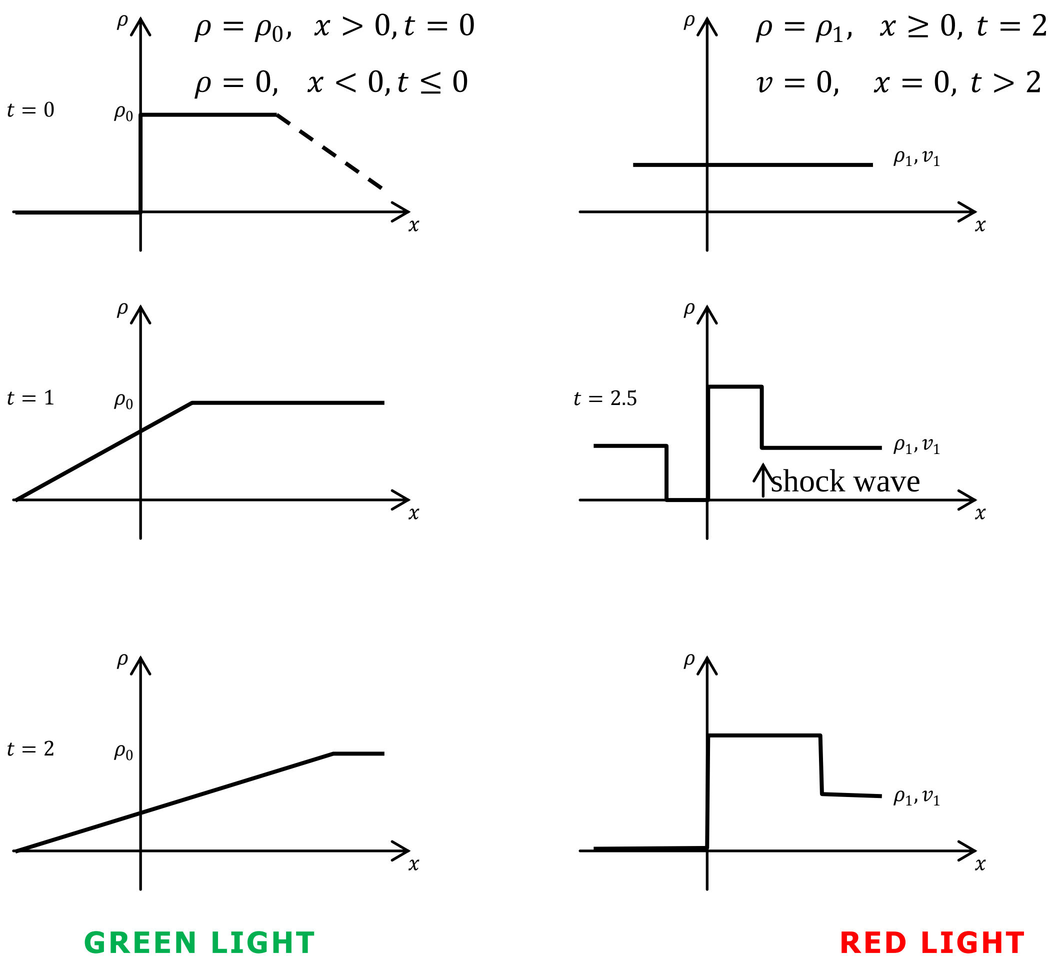

Figure 2 Various density distributions

On the other hand, general first-order conservation PDE is expressed as

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{23}$$

At a shock wave:

$$\left(\frac{dx}{dt}\right)_{shock} = \frac{[Q]}{[\rho]}, \qquad [F] = F^{+} - F^{-}, \ [\rho] = \rho^{+} - \rho^{-} \tag{24}$$

As an example, the cases subjected above can be discussed as following:

$$\rho^{+} = \rho_m \tag{25}$$

$$\rho^{-} = fan\ solution \tag{26}$$

$$Q = \rho v = V_m \rho \frac{\rho_m - \rho}{\rho_m}. \tag{27}$$

3. Conclusions

In this workshop, several aspects of traffic light optimization problem are discussed. The partial differential equation (11) is proposed to explain the relation between speed and density. (11) is based on the Haberman's method and helps to solve the traffic movements as it is in fluid dynamics. Finally, this report provides an accurate way to calculate how much time is required to finish a queeu when its length is known.

4. Recommendations

The problem of left turn is the next step to be solved.,

The relation between speed v and density ρ is assumed linear in this solution, however some nonlinear types of the relation could be included in this optimization, which is left for further studies.

The yellow light duration is fixed, so it may be excluded from the optimization process. Thanks for the yellow light duration which validates our solution and helps to make traffic light work.

5. Acknowledgements

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6. References

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