Numerical Predictions for Submarine Sound

INTRODUCTION

Basic idea: to decrease the noise made by submarines so that they are less easy for the enemy to detect.

Another key idea: in general "normal" submarine hull coating leads to quite a lot of noise. However, there exists an alternative "smooth" coating which reduces the noise. This smooth coating is very expensive, so cannot be used everywhere.

Keynote question: if it is only possible to coat *some* of the hull, which parts should be coated?

Clearly there is no chance of doing closed form calculations for real submarine shapes and numerical analysis will be necessary. So the general plan is:

(1) Use numerical method to solve flow problem for given hull smoothness

(2) Use Lighthill noise model to argue that noise pressure p is given by

$$p_{tt} - c_0^2 \nabla^2 p = K \nabla . (\omega \wedge \mathbf{u})$$

where $\omega = \nabla \wedge \mathbf{u} = \omega \hat{\mathbf{e}}_z$ is the vorticity, $\mathbf{u} = $ velocity and $p = $ pressure.

(3) Using numerical flow solution, calculate p numerically

(4) Hence we know the submarine noise as a function of the hull smoothness distribution.

Of course, we wish to *minimize* the noise

NUMERICAL METHOD
Professor Zhou Liandi and his student are using a very interesting numerical method to solve the basic flow problem. (Proceed entirely in 2/D, but do unsteady flow).

They write the Navier-Stokes equations as

$$\omega_t + \mathbf{u}.\nabla\omega = \nu\nabla^2\omega$$

where $\nu = $ kinematic viscosity.

ADVANTAGE: the pressure p is completely removed from the problem. (Normally p enters the equations only as a Lagrange multiplier and is extremely hard to deal with numerically.)

DISADVANTAGE: normally we know boundary conditions for $\mathbf{u}$ and not for ω: so some work is required to transform the boundary conditions to the right form.

Details of numerical method: not important, but essentially the idea is to solve

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x}, \mathbf{t})$$
$$\frac{d\omega}{dt} = \nu\nabla^2\omega$$

(Lagrangian form) by replacing the right hand sides of both of these equations by integral operators. Then assume that the fluid is made up of "particles" (quadrature points).

The vorticity field is now a discrete sum of the individual vorticity fields of "all the particles".

At solid boundaries we use the boundary condition

$$\delta t \nu \frac{\partial \omega}{\partial n}(s) = -\gamma(s)$$

where n is the normal, δt is the time interval and $\gamma(s)$ is the vortex sheet strength of the body surface.

- This condition was much discussed

SMOOTHNESS CONDITION

The real modelling in the problem concerns the alteration of the surface vorticity boundary condition to model the effects of smoothness. Currently the idea is to write

$$\delta t \nu \frac{\partial \omega}{\partial n}(s) = -\alpha(s)\gamma(s)$$

where $\alpha(s)$ accounts for the smoothness.

Interpretation: we certainly know that $\alpha = 1$ gives no-slip and $\alpha = 0$ gives inviscid ("smooth") flow. But can we extrapolate this idea to assert that this boundary condition is still valid for other values of α? For example:

- $0 < \alpha < 1$: "smoother" wall?

- $1 < \alpha$: "rougher than normal" wall?

- $\alpha < 0$: "super-smooth" wall?

Justification of inclusion of α:

If this is to be anything other than a complete guess, then we need to look at some KNOWN exact solutions to see whether or not a boundary condition of this form is at all valid.

Possible known solutions that may be examined:

(1) Injection through a porous wall (Cole & Aroesty, 1968, Int. J. Heat Mass Transfer 11, 1167-1184)

(2) Boundary layer similarity solution (sucking or blowing) - asymptotic injection/suction profile (Rosenhead, Laminar Boundary Layers Oxford University Press 1960)

(3) Flow over small bumps and depressions in boundary layer (F.T. Smith - various publications including J. Fluid Mech. IMA J. Appl. Math.)

(4) There is a great deal of literature on compliant boundaries.

NOISE CALCULATION

We note also that the noise is calculated from

$$p_{tt} - c_0^2 \nabla^2 p = K \nabla . (\omega \wedge \mathbf{u}).$$

Much care is required here as it is well know that using a vortex method calculate ω and then numerically differentiating can lead to errors.

- Need for careful error results and checking for calculation of noise. Again scope for using exact solutions to check that noise calculation is accurate.

SEPARATION PREDICTIONS

The numerical/experimental comparisons carried out so far show that the code is working very well in predicting velocity profiles *as long as* there is no separation.

For cases where the flow separates the results are not so good.
Since wakes from separations seem to be the main causes of noise,
it is ESSENTIAL that the code handles separation well. Scope for
much interesting numerical work.
Need to consider effects of transition/relaminarization on boundary
layer separation and consequent noise production.

STUDY GROUP RECOMMENDATIONS:
(Note: some of these may already have been implemented.......)

- To justify adopting boundary condition involving α, interpret
  known exact solutions to see if they can be cast in this form.

- A very careful programme of comparisons between experiments
  and numerical calculations needs to be carried out in order to
  see exactly where the present numerical code is giving good
  agreement and where it is failing. This should also be done in
  parallel with existing literature.

- Compare results of other vortex codes with experiments where
  boundary layer separation occurs. Do they give good predic-
  tions of separation? If not, why do vortex codes struggle to
  predict separation?

- Eventually turbulence effects should be included in the equa-
  tions and thus the numerical model. Turbulence will occur in
  boundary layer first.