EFFICIENT DESIGN OF TALL TAPERED FEEDERS

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Toowoomba Foundry is seeking help with the design of tall tapered feeders, which supply extra molten iron to poured castings as they cool. There is a problem with the reliability of the feeders, particularly for thicker castings of the newer spheroidal graphite irons. An effective feeder remains molten until the casting has set. The setting of the melt in the feeder is delayed by making it large enough to retain its heat for longer than the casting, and by placing it close to the thermal centre of the casting. So a larger feeder is more reliable. But any metal remaining in the feeder is recycled, and has an associated energy cost. If a feeder is too small, it will set too soon, and the casting will have unwanted holes in it that may require the entire casting to be recycled. Thus there is a tension between making the feeder smaller so as to minimise recycled metal, and having the feeder large enough that the casting is good.

Existing design methods use purely conductive models of heat transport. We investigate the relevance of convection in the cooling feeder, and set up a boundary-layer model of flow driven by density differences. We find that convection is a significant factor in the design of a feeder, effectively maintaining constant temperature across it. The height of the feeder is important mainly in providing the driving force for this flow.

1. Introduction

Toowoomba Foundry uses both grey iron (GI) and spheroidal graphite (SG) iron when making castings. Both shrink when cooling and setting after being poured into moulds. Molten iron is provided to the cooling casting by feeders, which are designed to remain hot long enough to provide extra molten iron as required by the shrinking casting. Problems arise primarily in the moulding of SG iron (becoming increasingly popular as it is stronger by weight), when

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the casting section to be fed exceeds approximately 25 mm. Then the feeders become unreliable, resulting in shrinkage porosities in the castings of around 3%, compared with 1% with GI. In some cases, the feeder is observed not to have remained hot for long enough to provide molten iron to the casting. This is apparent by observing the feeder, and seeing that it has not *piped*, that is, it has none of the holes or hollow tube effects associated with having provided molten metal to the casting. Then the casting will have holes (porosity), and may have to be rejected (and recycled).

A popular feeder design is the tall tapered feeder (TTF), placed near the thermal centre of the casting, so that the only cavities to form should then be in the feeder itself. Experience with GI has led foundries to believe the taller thinner design of the TTF aids the supply of molten iron by providing extra head (pressure due to the height of the feeder). Figure 1 shows sketches of typical TTF and traditional feeder sections.

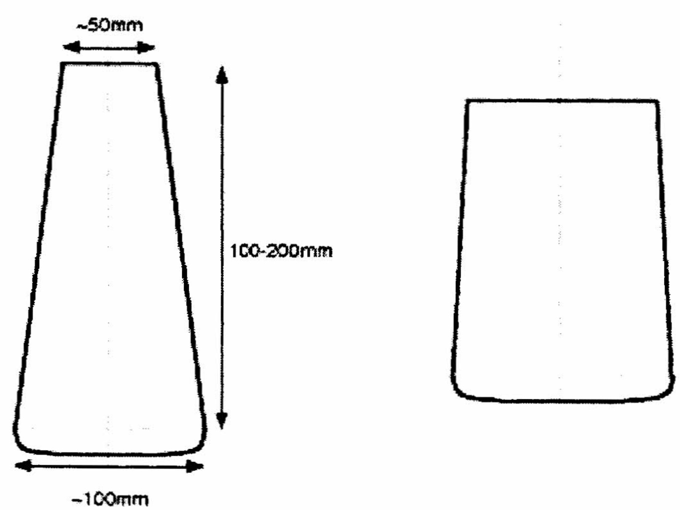

Figure 1: Sketches of feeder sections, showing a typical tall tapered feeder and a traditional feeder.

Figure 2 shows a typical TTF and the upper box (cope) of the casting it is intended to feed. The TTF is the upside-down funnel-shaped object at the centre of the cope. Typical dimensions of a feeder reservoir are 4–8 cm diameter and 10–20 cm tall. Metal that remains in the feeders and also in the channels through which the melt is poured (the whole mould needs to be poured in less than 25 seconds) is waste metal. Waste metal is removed from the casting and recycled but this has associated energy costs. Another source of waste metal is rejected castings from failed feeders. The aim is to reduce the amount of waste metal, from 40–50% to 20% if possible, in order to reduce recycling costs. One way to reduce the amount of waste metal is to make the feeder reservoir smaller. Another way is to improve the reliability of the TTF. Since a simple way to improve feeder reliability is to increase its size, there is some tension between these two requirements. This is particularly the case for SG castings, which have a failure rate of 4% to 10%, high compared with GI failure rates.

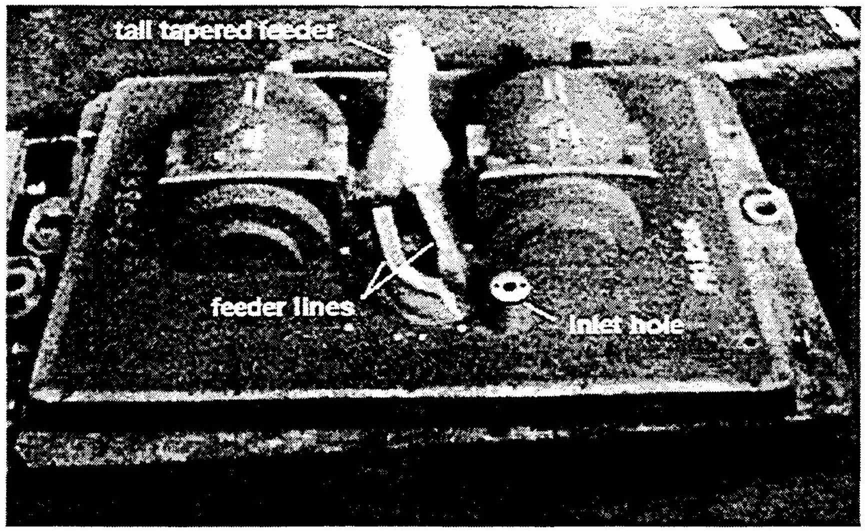

Figure 2: The cope, or upper box, of a casting, showing a typical tall tapered feeder in the centre.

Current design techniques used by Toowoomba Foundry involve a package called MAGMAsoft, which uses a purely conductive model for the solidification process, and models heat conduction in both the iron and the surrounding sand mould in some detail. Another design approach used in conjunction with this package is the concept of thermal modulus, or volume divided by area, of different parts of the casting. The larger the modulus, the longer that part of the casting is expected to take to solidify. The thermal centre of the casting is the last part to solidify, and the feeder is usually placed very near the thermal centre. All of this design is based on conduction of heat, and cannot include the possible effects of the pressure head provided by a TTF filled with molten iron, or the possibly important effects of convection currents within the liquid metal in a cooling casting.

The following factors and possible outcomes were outlined in the preliminary information provided to the Study Group.

1.1 Possible factors involved

The design developed in one foundry need not work effectively in another, since there are several factors which are peculiar to each foundry. For example, the melt composition and mould variables such as mould rigidity may vary from foundry to foundry.

The following are some of the factors that could influence the performance of TTFs:

- Melt composition
  - if there is less carbon percentage in the melt, the consequence will be increased shrinkage as there will be less graphite expansion;
  - melt composition also depends on the solidification mechanism (GI vs. SG).
- Melt temperature
  - There will be more feeder melt required if the temperature is higher because the casting melt will contract more as it cools down, and since the hotter melt will have expanded the mould more due to the excess heat it transfers, the casting cavity will be larger.
- Casting height and size of section to be fed
  - The feeder height should take into account the height of the casting, and the feeder diameter should be greater than the effective thickness of the section to be fed.
- Connection size
  - The size of the connection between the feeder and the casting should be large enough to keep the metal there in liquid form until after the casting has set.
- The size of the sprue
  - The feeder and sprue compete to feed the casting. Ingates to the feeders and the casting from the sprue are generally made thinner so that they freeze first, allowing the feeder to provide melt as required by the casting.

- Mould variables
  - If the mould is less rigid (e.g. due to increased sand moisture levels), then the casting cavity will expand more during the graphite expansion phase of solidification and will necessitate additional feed metal for soundness.

1.2 Possible investigations

- Investigate the mechanisms of the cooling of a mould and a TTF so they can be better understood and parameterised.
- With some model of the cooling, investigate minimal volume (axi-symmetric) shapes of a feeder that can maintain hydrostatic pressure into the mould for a set time.
- Toowoomba Foundry are at present using a software package called MAGMAsoft which seems to resolve the feeder at a relatively coarse level. Some higher resolution numerical solutions of the reservoir and cooling air/sand matrix may be useful.

1.3 Outcomes for Toowoomba Foundry

To expand the boundaries and understanding of feeder design so that TTFs can be used more reliably on the thicker SG castings generally made in a commercial foundry. In particular,

- Arrive at an effective thermal design for the top section of the TTF such that the melt in that top region remains molten for as long as possible, thereby keeping a path open for the atmospheric pressure to aid the feeding process in the making of large SG castings.
- To effectively calculate (or simulate) the heat convected from all sides of the TTF by the air that flows between sand granules, and the radiated heat, and heat conducted by sand granules.

2. Orders of magnitude

This section contains some order of magnitude calculations, as a first step towards discovering the important mechanisms in the cooling process.

2.1 Static pressure head in the TTF

There is some indication in the literature on feeders that pressure head might be of importance, and that a taller feeder provides more driving pressure to more readily supply molten metal to a shrinking casting. However, with a height of about 0.1 m, the extra pressure provided is only 0.1 atmospheres, roughly 10% more than, say, using a horizontal feeder. So we do not consider that the static pressure advantage of a TTF over a regular feeder is significant. However, it is clear that the provision of melt to the casting does require some head in the feeder, or otherwise the casting may draw away from the top of the sand mould.

It is clearly important that the metal in the feeder does not set completely on the outside surfaces, as this will remove the atmospheric pressure driving the feeding of liquid metal. In other words, if the metal at the outside of the feeder sets, then the liquid metal inside will not be available to flow into a shrinking casting — atmospheric contact must be maintained. This is the first clue that a purely conductive model is inadequate to explain how a feeder works, since such a model predicts that the metal at the surfaces will set first. It is clear from pictures of feeders that have piped correctly (see Figure 3), that there is molten metal very near the top of the feeder when it pipes (provides liquid metal to the casting). With this in mind, foundries have tried changing the detailed shapes of the tops of feeders, for example, by inserting slots or dimples, in an attempt to keep them hotter for longer. Such changes increase surface area, which may actually increase cooling rates overall, but they do introduce a path for air to penetrate deeper into the feeder from the top, possibly facilitating access of molten metal to atmospheric pressure.

2.2 Heat conduction in the metal

We find that the time scale of cooling of the TTF by conduction to an assumed constant temperature sand bed is of the order of minutes. In practice, the cooling process takes hours. Thus the heat transport through the sand bed is the limiting factor in the cooling of the TTF. That is, based on a conductive model of heat flow, the temperature in the casting will be almost constant spatially, compared to the temperature in the sand mould.

We use a length scale of the TTF radius $r = 0.03\,\mathrm{m}$, density of melt $\rho = 7000\,\mathrm{kg/m^3}$, heat capacity of melt $C = 700\,\mathrm{J/kg/^\circ K}$ and conductivity of $K = 22\,\mathrm{W/m/^\circ K}$. Then the time scale of cooling is

$$t = \frac{\rho C r^2}{K} \approx 200\,\mathrm{s}\,.$$

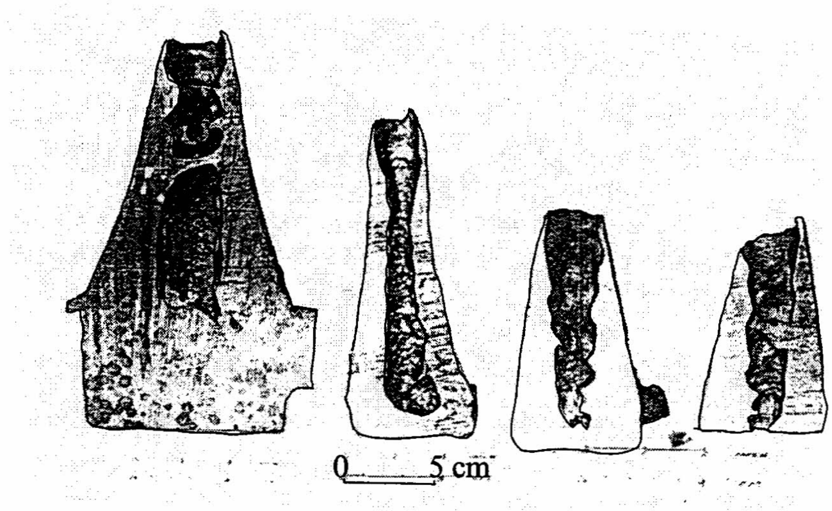

Figure 3: Photographs showing feeders that have piped successfully.

This time scale of 2–3 minutes is significantly smaller than the observed cooling time scale of hours, for a typical casting and TTF. Thin castings (≈ 1 cm) are believed to solidify in about 20 minutes, while thicker ones (≈ 3 cm) solidify in about 1 or 2 hours.

2.3 Air flow in the sand mould

We also find that air flow generated by natural convection in the sand mould is not a significant cooling mechanism. Hence MAGMAsoft calculations (which ignore convection) are expected to be accurate, for sand mould temperatures.

If there is air flow through the porous sand, it has to be driven by the atmospheric pressure drop across the height, 0.6 m, of the sand mould. When the air heats in the sand mould it becomes less dense, and the density difference could drive a flow of air. The most the pressure drop could be is one atmosphere, and then the pressure gradient is simply $\rho g = 10\,\mathrm{Pa}$. Darcy's law then says that the air velocity through the porous sand is

$$w = \frac{k^*}{\mu}\rho g \approx 2 \times 10^{-6} \times 10 = 2 \times 10^{-5}\,\mathrm{m/s}\,,$$

where we have used an estimate $\frac{k^*}{\mu} \approx 2 \times 10^{-6}$ m^2/Pa/s. Thus in a period of an hour the air moves 7 cm at most — air flow cannot convect significant heat away from the casting.

Note that as the sand heats up, the interstitial air heats and expands and so is rapidly forced out by the higher generated pressure gradients. This occurs similarly for water vapour and burning Bentonite and wood. This is not the mechanism we are modelling here, as it is a once-only effect, over in seconds.

2.4 Approximate rate of cooling through the sand mould

The volume of a TTF is taken to be $V = 4 \times 10^{-4}\,\mathrm{m}^3$. To cool the melt to 1134°C from the pouring temperature of 1400°C we need to extract energy which is $\int \rho C\, dT \approx 3 \times 10^9\,\mathrm{J/kg}$. For this size TTF the total energy to be extracted is thus about 10^6 J. To extract this in about 1.5 hours we need to do it at an average rate of 200 W.

We have not tried to calculate how much of the heat lost by the TTF is lost through the casting. The casting is designed to cool faster than the TTF, so that eventually heat loss to the casting could become significant. In the next section on similarity solutions, we do calculate the rate of heat loss from the TTF through the surrounding sand bed.

2.5 Mushy zone formation

The transition from liquid to solid does not occur at a single temperature, but over a temperature range, within which the metal forms a mush that is a mixture of solid and liquid, modelled successfully as a porous medium with a temperature-dependent permeability (see, for example, [7, 2, 3]). SG iron and grey iron solidify over different temperature ranges. A mush begins to develop in grey iron at 1150°C and it is completely solid at 1142°C, a range of 8°C. However, in SG iron a mush begins to develop at 1170°C and it is completely solid at 1134°C, a range of 36°C. To get a feel for the effects of these different ranges of solidification, we have numerically simulated the conductive cooling at a fixed heat flux of an iron block.

In this simulation we assume a fixed heat flux into the sand mould. The heat capacity is modelled by an expression of the form $C\rho\{1 + 226/T_r/\sqrt{\pi}\exp[-(T - T_f)^4/T_r^4]\}$. The numerical grid is made of 50 points spread over 3 cm from the sand at $x = 0$ to the middle of the TTF at $x = 3\,\mathrm{cm}$. The temperature gradient at $x = 0$ is set to -1300°C/m in order to get the time scale of the cooling about right. The results are illustrated in Figure 4.

Note in Figure 4 that the resulting mush for SG iron progresses very quickly across the block. In about a minute the entire block has transformed to mush.

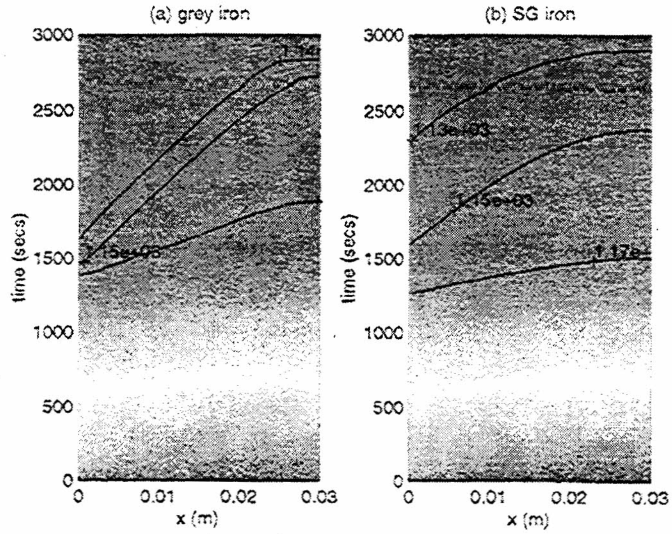

Figure 4: Progress of the mushy zone in a 1D conductive cooling model. The black lines show the leading edge, the trailing edge, and the centre of the mushy zone as it progresses across a cooling block of metal, for GI and SG iron.

In contrast, grey iron takes nearly ten minutes. The reason is that although the same total latent heat is released upon solidifying, because SG iron needs much less cooling to initiate the formation of mush, the mush front moves more quickly. Indeed, in Carslaw and Jaeger [1] in section 11.2 part VI, one method for treating mushy zones is to give the liquid a thermal capacity inversely proportional to the temperature range of solidification. Hence a larger temperature range of solidification gives a smaller thermal capacity in the mushy zone, and a faster freezing front speed. The freezing front speed, from the work of Carslaw and Jaeger [1], is roughly proportional to the temperature range. Hence the onset of the mushy zone is expected to travel eight times faster for SG iron than for GI, in a conductive model.

Hence SG iron forms a wider mushy zone than GI, and the leading edge of this mushy zone moves much faster than it does in GI. Once the mush fills the feeder, we expect the melt flow will slow dramatically, as it would need to travel through what has essentially become a porous medium, with large increases in effective viscosity in mushy regions. Then flow will effectively cease, and conductive heat loss will dominate.

3. Similarity analyses and convective flow

In this section we consider heat flow through the sand mould in more detail, and use a semi-infinite model to generate a standard similarity solution. This solution then provides a heat flow boundary condition on the walls of the TTF. We then consider the question of convective flow in the TTF, as this is not modelled by MAGMAsoft.

3.1 Heating of the sand mould

We assume the mould is made of dry sand at an initial temperature of 30°C, the casting is at a constant temperature of 1300°C, and heat flow is one dimensional so that the standard similarity solution for the temperature in the sand a distance x away from the casting after time t is [1]

$$T(x,t) \approx 1300 \text{ erfc}\left(\frac{x}{\sqrt{t}}\sqrt{\frac{\rho C}{4K}}\right) . \tag{1}$$

Inserting the approximate values $\rho = 2 \times 10^3\,\text{kg/m}^3$, $C = 10^3\,\text{J/kg/°K}$, and $K = 0.7\,\text{W/m/°K}$ for sand gives

$$T(x,t) \approx 1300 \text{ erfc}\left(0.8 \times 10^3 \frac{x}{\sqrt{t}}\right) . \tag{2}$$

Then this model predicts that the temperature gradient in the sand at the wall of the TTF is given by

$$\frac{\partial T}{\partial x} \approx \frac{1.2 \times 10^6}{\sqrt{t}} . \tag{3}$$

The time for the temperature in the sand to change appreciably at a distance 0.3 m away from the TTF is then

$$t = \frac{\rho C}{4K}x^2 \approx 0.6 \times 10^5 \text{ s} \approx 18 \text{ hrs} . \tag{4}$$

This means the sand takes a long time to heat up, and transient solutions are needed when considering the sand temperature.

The heat loss rate from a TTF of surface area A is $AK\frac{\partial T}{\partial x} \approx 0.8 \times 10^6 \frac{A}{\sqrt{t}}$ W, which integrates in time to give the total heat lost from the TTF after time t as $1.6 \times 10^6 A\sqrt{t}$ J. This may be equated to the sensible heat required to lower a volume V of liquid metal by a temperature ΔT, which is $V\rho C\Delta T \approx 6.3 \times 10^6 V\Delta T$. Then the time to cool the liquid metal in the TTF through a temperature range ΔT is given by

$$t^* \approx 16\left(\frac{V}{A}\Delta T\right)^2 .$$

Volume to area ratios for cylindrical TTF's are about 0.02 m, giving a time to cool from 1400 to 1200°C for the TTF of about 4–5 minutes. This is too fast for the TTF to be effective. However, our model ignores the significant thermal contact between TTF and casting, and also has rather small V/A values since the TTF has a partly spherical shape on the lower half.

Water is observed in practice to drip out of the mould for the first few minutes after pouring. We have not considered the possible effects of moisture transport on the temperature field about the TTF.

3.2 Liquid flow in the TTF

The techniques used by Toowoomba Foundry for modelling heat flow are thorough in all aspects except that of convective flow in the TTF as the liquid metal cools to the liquidus temperature where it begins to turn to mush. We model this flow, driven by density differences in the cooling fluid, and assume that once the mushy zone has formed any convective flow is rapidly choked off, and thereafter the metal in the TTF effectively sets conductively from the outside surfaces. An interesting and challenging direction for further study would be to consider flow in the mushy zone.

The interest is in what flows might develop inside the cooling (but still liquid) TTF, driven by the heat loss from the side walls. Cooler fluid near the wall is denser, and sinks, driving convection. This is similar to the problem considered by Turner [5], in which convection is driven by wall cooling, but Turner uses a constant temperature wall, whereas we wish to consider a specified heat flux at the wall, provided above from the semi-infinite similarity solution in the sand mould. The temperature gradient in the metal at the wall of the feeder is given by

$$\frac{\partial T}{\partial x} = \frac{K_{\text{sand}}}{K_{\text{metal}}} \left.\frac{\partial T}{\partial x}\right|_{\text{sand}} \approx \frac{8 \times 10^{11}}{\sqrt{t}} .$$

The modelling described here is almost identical to work done in the 1993 MISG on stratification in steelmaking ladles [6], and also to be found in [4]. That work also uses a constant flux boundary condition at the wall. We use a flux that varies with time, but this is equivalent to a constant flux condition since time only enters the equations as a parameter.

As in Turner [5] and in Sparrow and Greig [4], when the flow is driven by natural convection by wall cooling, a boundary layer is set up near the wall. The boundary layer equations are

$$u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z} = \nu\frac{\partial^2 w}{\partial x^2}+g\beta T \quad (5)$$

$$u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z} = \kappa\frac{\partial^2 T}{\partial x^2} \quad (6)$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z} = 0 \quad (7)$$

where u and x are velocity and distance normal to the wall, w and z are velocity and distance down the wall of the TTF, β is the coefficient of thermal expansion at constant pressure, ν is the kinematic viscosity, and $\kappa \equiv K/(\rho C)$ is the thermal diffusivity of the molten metal. A weakness of this model is that it approximates the wall of the TTF as a flat plate, but we proceed with this analysis for simplicity, with the intention of using it to inform about qualitative behaviour without expecting accurate quantitative results.

Boundary conditions are that $u = w = 0$ at the wall where $x = 0$, $\frac{\partial T}{\partial x} \equiv q = \alpha/\sqrt{t}$ at $x = 0$, where $\alpha \approx 8 \times 10^{11}$.

A similarity variable is evident in the nondimensionalisations

$$T = xF(z/x^5), \qquad u = \frac{G(z/x^5)}{x}, \qquad w = x^3W(z/x^5), \quad (8)$$

which reduce the three partial differential equations to ordinary differential equations in the similarity variable z/x^5. An alternative approach, as in [5, 4], is to introduce a stream function ψ so that the continuity equation is automatically satisfied, namely

$$w = \frac{\partial \psi}{\partial x}, \qquad u = -\frac{\partial \psi}{\partial z}. \quad (9)$$

Then we make the similarity transformations

$$\eta = axz^{-\frac{1}{5}} = (\mathrm{Gr})^{\frac{1}{5}}\left(\frac{x}{H}\right)\left(\frac{z}{H}\right)^{-\frac{1}{5}}, \ \psi = a\nu z^{\frac{4}{5}}f(\eta),\ g\beta T = a^4\nu^2 z^{\frac{1}{5}}h(\eta), \quad (10)$$

where

$$a^5 = \frac{\beta g q}{\nu^2},$$

and the Grashof number based on the height H of the TTF is

$$\mathrm{Gr} = \frac{\beta g q H^4}{\nu^2}.$$

Then f and h satisfy the ordinary differential equations [6]

$$f''' + \frac{4}{5}ff'' - \frac{3}{5}f'^2 + h = 0, \quad (11)$$

$$h'' + \frac{4}{5}\sigma f h' - \frac{1}{5}\sigma f' h = 0, \quad (12)$$

where $\sigma = \nu/\kappa \approx 0.25$ is the Prandtl number. These equations may be solved numerically for f' and h, as illustrated in [6], and this needs to be done only once, since the only parameter left in the equations is the (fixed) Prandtl number, and all other dependencies lie in the transformations. The solutions for f' and h are of order one, and vary appreciably when η is of order one.

Then the thickness of the boundary layer is given by $\eta = 1$ with $z = H = 0.1$, that is,

$$x = \frac{z^{\frac{1}{5}}}{a} \approx 3 \times 10^{-3} \text{ m}. \tag{13}$$

The velocity of the descending molten metal near the wall is estimated to be

$$w = \frac{\partial \psi}{\partial x} = z^{\frac{3}{5}} a^2 \nu f' \approx 10^{-2} \text{ m/s}. \tag{14}$$

This descending melt flows through an annular region of area $\approx 10^{-3}\text{ m}^2$, hence the velocity of the rising melt in the central region of the feeder with area $\approx 10^{-2}\text{ m}^2$ is $w \times 10^{-3}/10^{-2} \approx 10^{-3}$ m/s. Hence the time taken to cycle the entire contents of the feeder once past the wall is ≈ 100 s. This is comparable to the time taken for conductive cooling of the feeder estimated above, so that convective and conductive cooling processes are taking place on similar time-scales in a typical TTF. In particular, fluid flow within the TTF will help to keep it at more uniform temperatures than if there was no flow. An estimate of the temperature difference that will exist over the boundary layer is given by setting

$$T = \frac{a^4 \nu^2 z^{1/5} h}{g\beta}, \tag{15}$$

and setting z to 0.1 m, h to 1, to get $T \approx 1.6°$C. That is, the slow flow in a feeder is expected to be rather effective in maintaining almost constant temperatures to within one or two °C, provided the temperatures are above liquidus. Once liquidus is reached, significant temperature gradients are to be expected across the feeder, and the outer regions will freeze first.

3.3 *Fastflo* simulations

Fastflo is a general purpose PDE solver using the Finite Element Method, and is applicable to fluid flows.

To support the above boundary layer analysis, a *Fastflo* model was set up. No latent heat was included (so the behaviour down to liquidus only is being modelled), viscosity was made temperature dependent to imitate the tabulated values available, and heat transfer from feeder to sand mould was treated approximately. Solutions after one minute are shown in Figure 5.

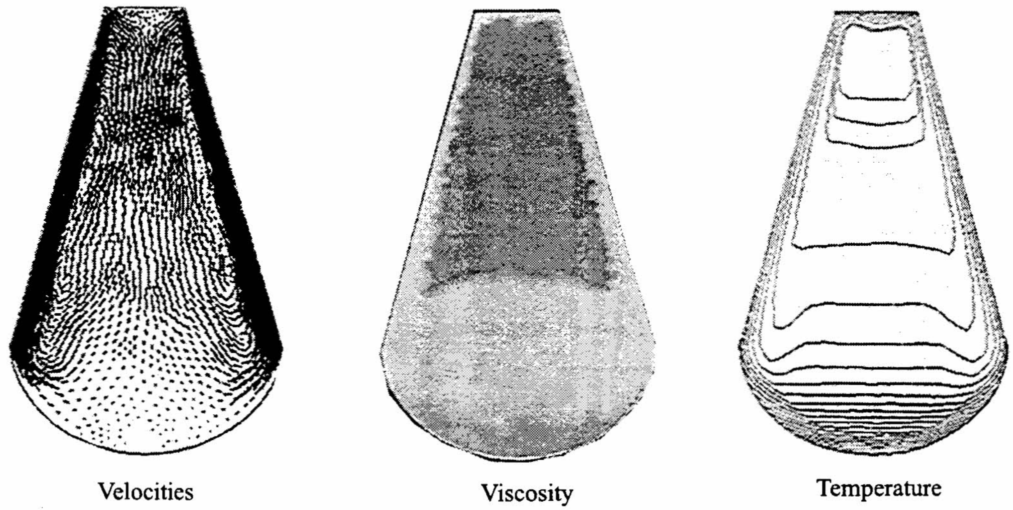

Figure 5: *Fastflo* solutions for flow in a feeder, after one minute. Velocities are represented as lines, viscosity by grey scale (paler means more viscous), and temperature by contours.

Key findings of this simulation are that there is downflow of melt close to the walls, with slower upflow in the centre, that the flow is laminar but not steady, the temperature in the bulk of the melt is almost constant, and there is significant cooling at the bottom of the feeder to which the cooled melt descends. Reynolds numbers for the flow are about 30.

4. Minimum surface area feeders are hemispherically capped cylinders

This is reasonably obvious from common knowledge: spheres minimise surface area for a given volume, but we here may require that the top of the feeder is at some height so we stretch the feeder vertically by 'inserting' a cylinder at the equator of the sphere.

The mathematical proof that spherical caps are needed follows from the Euler-Cauchy equations for the shape $z = h(r)$ over radii $0 < r < a$. For simplicity assume an axisymmetric shape and then in cylindrical coordinates the surface area to be minimised is

$$A = \int_0^a 2\pi r\sqrt{1 + h'^2}\, dr \quad \text{such that} \quad V = \int_0^a 2\pi r h\, dr$$

is constant. Consider the Euler-Lagrange equations for $A - \lambda V$ to deduce (though we should be more careful with signs)

$$\lambda r = \frac{\partial}{\partial r}\left[r\frac{h'}{\sqrt{1+h'^2}}\right].$$

Integrate, set the integration constant to zero as everything else goes to zero as $r \to 0$, and rearrange to

$$(4/\lambda^2 - r^2)h'^2 = r^2.$$

Rearranging and integrating gives

$$h = C \pm \sqrt{4/\lambda^2 - r^2}.$$

Clearly we take the minus case for the top of the feeder, then further work would show $4/\lambda^2 = a^2$ whence

$$h(r) = b + a - \sqrt{a^2 - r^2},$$

is the hemispherical cap to a cylinder of radius a and reaching up to height b.

5. Conclusions

1. MAGMAsoft's assumption of cooling via conduction through sand is realistic because air flow is insignificant.

2. MAGMAsoft's assumption of pure conduction in the melt is workable given that the diffusion time is comparable to the flow turnover time in the TTF. One way to modify MAGMAsoft to accommodate the convection results is to allocate a very large thermal conductivity to the metal in the feeder to ensure almost constant temperatures inside it.

3. Taller feeders have no static pressure drive advantage, but they will have a dynamic advantage — convection in the feeder is driven by wall cooling, and the convection velocity is proportional to the 3/5 power of the height of the feeder. This means that doubling the height of the feeder will increase the flow velocities by 50%. However, temperature differences scale as the 1/5 power of the feeder height, and hence are much less sensitive. Changing feeder height by a factor of two will only change the temperature difference across the feeder by 15%.

4. Because the convective flow mixes the melt, for as long as it remains molten, the shape of the feeder has no appreciable impact on the pattern of cooling inside the feeder.

(a) One design thrust is to minimize surface area to volume ratio simply to minimize the rate of heat loss and hence suggests the use of hemispherically capped cylinders.

(b) Another design suggestion is to place a deep dimple or pencil of sand at the top of the feeder, to give air access to molten metal deep inside even after the outside edges have frozen. The dimple would need to reach as deep as the thermal centre of the feeder, to have any appreciable effect. It should not have much effect on flow within the feeder, provided it is much smaller in diameter than the feeder top.

5. Cooler melt gathers in the bottom of the feeder, suggesting that it needs a deepish bottom and feeder lines somewhat high up on the feeder to access molten metal.

6. The possibility of an oxide skin could be checked by trying experiments in a nitrogen atmosphere to see if avoiding an oxide skin improves reliability.

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A Material properties

The environment is about 30°C and the melt is poured at about 1400°C. The void fraction of the sand is estimated to be $\epsilon = 0.42$ (from 1998 MISG) and the permeability of sand is such that $k^*/\mu = 2.4 \times 10^{-6}\,\mathrm{m}^2/\mathrm{s}/\mathrm{Pa}$, where k^* is effective permeability.

T (°C)	λ (W/m/°K)	C (J/kg/°K)
1	0.680	676
97.9		715
99.0	0.680	800
99.1		75000
101.0		75000
101.1		810
127.0		858
200.0	0.640	
280.0		961
327.0		993
400.0	0.586	
527.0		1074
550.0		3800
600.0	0.590	
727.0		1123
800.0	0.640	
927.0		1166
1000.0	0.710	
1127.0		1201
1327.0		1230
2000.0	0.730	1230

Table 1: **Sand properties:** λ is the calibrated coefficient of diffusivity in sand; C is the effective heat capacity accounting for some moisture and the Bentonite.

T (°C)	ρ (kg/m^3)	ν (m^2/s)	K (W/m/°K)	C (J/kg/°K)
1	7100.0	1000	36	540
100	7074.5		36	548
200	7049.1			561
300	7023.8		35	573
400	6998.6		34	586
500			32	
600			25	619
700				644
730				1000
800				703
900				720
1000				732
1100				745
1134	6880.0	1000	22	5833
1143		0.01		5833
1170	6930.0	1.6e-6	21	5833
1200	6900.0	1.478e-6		916
1300	6860.0	1.160e-6		
1400		9.42e-7		
1500		7.75e-7	30	
1600		6.56e-7		
1700		5.66e-7		
2000	6620.0	5.66e-7	30	916

Table 2: **Metal properties:** ρ density; ν kinematic viscosity; K conductivity; C heat capacity.