

Model based methodology development for energy recovery in flash heat exchange systems

Report Contributors:

Andrei Korobeinikov^{1,2}, John McCarthy³, Andrey Melnik¹,
Emma Mooney⁴, Jaime Rojas⁴, Krum Semkov⁴,
James Varghese⁴, Toshko Zhelev⁴

Industry Representative:

Brendan Thorne⁵

¹MACSI, Department of Mathematics and Statistics, University of Limerick, Ireland

²Report coordinator, Andrei.Korobeinikov@ul.ie

³School of Mathematics, Trinity College Dublin, Ireland / Mathematics Department, Washington University, USA

⁴Department of Chemical and Environmental Science, University of Limerick, University of Limerick, Ireland

⁵RUSAL Aughinish Ltd, Aughinish Island, Asketon, Co Limerick, Ireland

Abstract

Flash tank evaporation combined with a condensing heat exchanger can be used when heat exchange is required between two streams and where at least one of these streams is difficult to handle (in terms of solid particles content, viscosity, pH, consistency etc.). To increase the efficiency of heat exchange, a cascade of these units in series can be used. Heat transfer relationships in such a cascade are very complex due to their interconnectivity, thus the impact of any changes proposed is difficult to predict. In this report, a mathematical model of a single unit flash tank evaporator combined with a condensing heat exchanger unit is proposed. This model is then developed for a chain of the units. The purpose of this model is to allow an accurate evaluation of the effect and result of an alteration to the system. The resulting model is applied to the RUSAL Aughinish Alumina digester area.

List of notations

A	Heat exchanger area, m ² ;
T	Slurry (hot stream) temperature, °C;
t	Spent liquor temperature, °C;
t _C	Condensation temperature, °C;
F	Hot stream mass flow rate, kg/s;
S	Cold stream (spent liquor) flow rate, kg/s;
G	Steam flow rate, kg/s;
L	Output hot stream mass flow rate, kg/s;
N	Number of flash tank stages;
h	Specific enthalpy, J/kg;
h _{fg}	Latent heat, J/kg;
U	Heat transfer coefficient, W/(m ² K);
Q	Power, W;
P	Pressure, Pa;
c _p	Specific heat capacity, J/(kgK);
ϕ	Temperature change due to non condensable gases, K;
ψ	Temperature change due to superheating, K;
S	Cold stream;
F	Input hot stream;
G	Steam flow;
L	Output hot stream;
W	Water;
in	Input;
out	Output;
*	Equilibrium value.

1 Introduction

Effective heat exchange between hot and cold streams is an important part of energy efficiency in process operations. Where heat exchange is required between two streams and where at least one of these streams is difficult to handle (in terms of solid particles content, viscosity, pH, consistency, etc.) in a standard counter-flow type heat exchanger, flash tank evaporation combined with a condensing heat exchanger can be used. Often to ensure maximum heat exchange, a series of these units in a chain is used, with the hot stream flowing through the flash tanks and the cold stream flowing in a counter current direction through the heat exchanger. Steam is generated through pressure drop in the flash tank from the hot stream and then condenses in the heat exchanger, transferring the latent heat to the cold stream. The interlinked nature of the flash tanks and the condensing heat exchangers in series makes analysis of the heat exchange process rather difficult, as any change in the operating parameters at any point in the chain impacts the other components of the chain, and therefore once a system is operating, any alteration of the system parameters could have an unknown impact on the process and the overall balance of the heat exchange throughout the chain. While flash tank heat exchange, as described above, is in use in many applications, the particular case study for this project is the system that is currently in use in the digestion area of RUSAL Aughinish Alumina, Co. Limerick, Ireland.

RUSAL Aughinish Alumina produces alumina from bauxite through the so-called Bayer process [1, 2, 3, 4], which consists of four main steps (see Fig. 1):

1. Dissolution of bauxite in an aqueous Sodium hydroxide (NaOH) solution (Digestion);
2. Removal of the insoluble materials (Clarification);
3. Precipitation of gibbsite from the liquor (Precipitation);
4. Removal of chemically bound water (Calcination).

In the digestion area the bauxite ore is mixed with caustic soda and the reheated spent liquor. High pressure steam is added to heat the mixture to a required temperature of 250 °C. The resulting hot slurry then passes through a chain of flash tanks, where the mix is cooled through indirect heat exchange with a cold liquid stream of the spent liquor. Cooling the slurry to a temperature of 40 °C is essential for the next stages in the alumina production process. By increasing the amount of heat transferred in the flash tank heat exchange chain from the hot bauxite slurry to the cold spent liquor, the amount of high pressure steam required in the digester can be reduced, resulting in an increase in the efficiency of the process.

The purpose of this report is to create a model of a cascade of heat exchange units composed of a flash tank and a condenser that would allow accurate evaluation of the effect and result of alterations to the system. The resulting model is applied to the RUSAL Aughinish Alumina digester area.

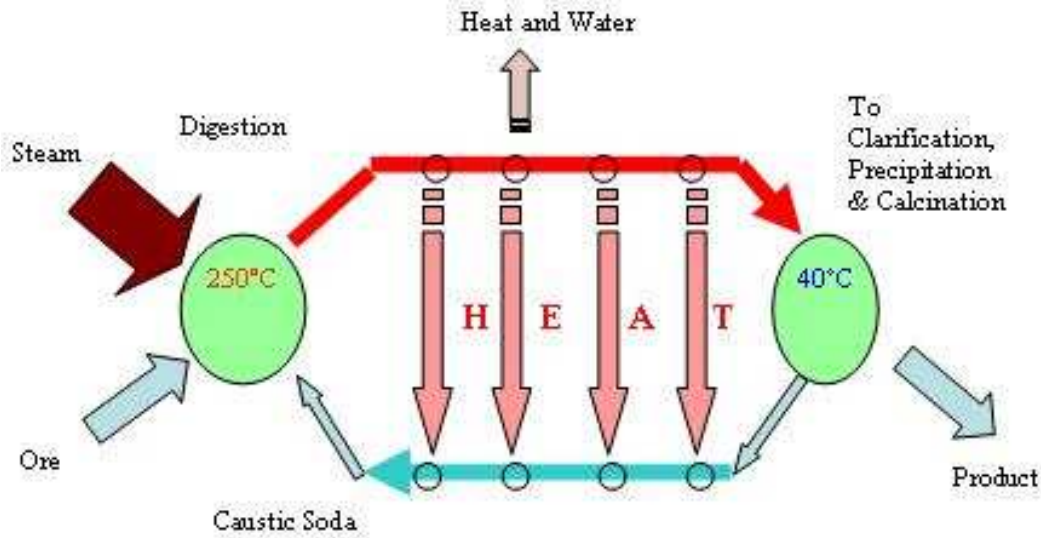


Figure 1: Basic thermal flow chart of the digester area

2 A model of flash tank - condenser system

In a unite that is composed of a flash tank and a condenser, as shown in Fig. 2, the liquid slurry is coming in with flow rate F , specific heat c_{pF} , pressure P_0 , temperature T_F , and enthalpy h_F into the flash tank. Steam is evaporating in the flash tank and flows to the heat exchanger with flow rate G , specific heat c_{pG} , pressure P_1 , and temperature t_G . The liquid is leaving the flash tank at flow rate L , pressure P_1 , temperature T_L , and enthalpy h_L . In the condenser the steam condenses at temperature t_c and pressure P_1 , and exchanges heat Q with the liquid (the spent liquor) entering the exchanger at temperature t_{in} , flow rate S and specific heat c_{pS} . The liquid leaves the heat exchanger at temperature t_{out} . The mass and enthalpy balances are expressed by the following equations:

$$F = L + G, \quad (2.1)$$

$$Fh_F = Lh_L + Gh_G, \quad (2.2)$$

$$h_G = h_L + h_{fg}, \quad (2.3)$$

$$Q = Gh_{fg} = F(h_F - h_L) = Fc_{pF}(T_F - T_L). \quad (2.4)$$

The description of heat exchange includes the equation for generated and consumed heat accounting for the change of temperature and pressures (see Fig. 3). The heat that is generated

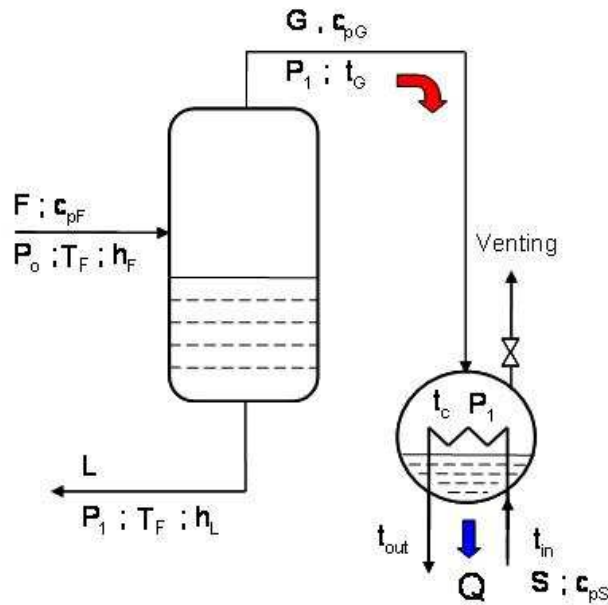


Figure 2: A schematic representation of a flash tank - condenser unit

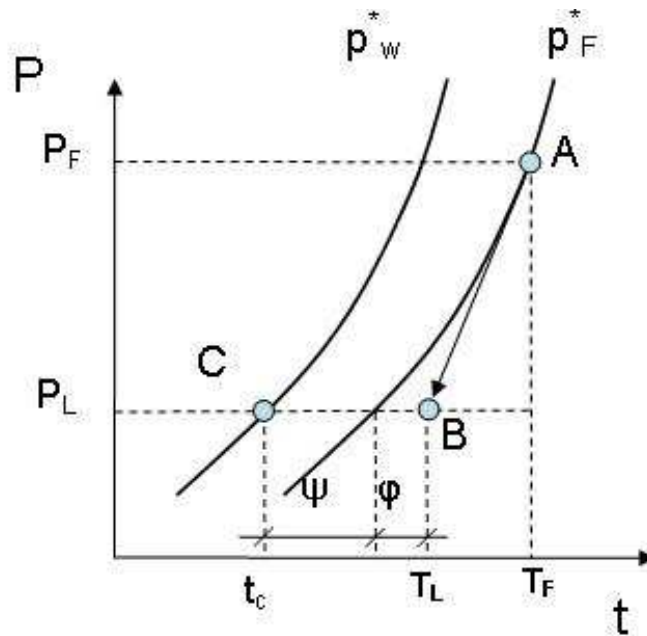


Figure 3: Heat exchange in the unit

at the source (in the flush tank) is

$$Q = Fc_{pF}(T_F - T_L), \quad (2.5)$$

where $t_G = T_L = t_c + \phi + \psi$. Here ψ expresses the superheated state of the vapours, since the evaporated liquid in the flash tank is not pure water and contains dissolved non-organic components; ϕ is the temperature correction that is due to the presence of non-condensable gases in the vapour phase. Hence, the generated heat is

$$Q = Fc_{pF}(T_F - t_c - \phi - \psi). \quad (2.6)$$

The consumed heat at the sink is

$$Q = UA \Delta t_{\log}, \quad (2.7)$$

where U is overall heat transfer coefficient, A is the heat exchanger area and

$$\Delta t_{\log} = \frac{(t_c - t_{in}) - (t_c - t_{out})}{\ln \frac{t_c - t_{in}}{t_c - t_{out}}} = \frac{t_{out} - t_{in}}{\ln \frac{t_c - t_{in}}{t_c - t_{out}}}. \quad (2.8)$$

Besides,

$$Q = Sc_{pS}(t_{out} - t_{in}). \quad (2.9)$$

Now, excluding t_{out} from eqs.(2.7–2.9), we receive

$$Q = K_S \frac{\exp(UA/K_S) - 1}{\exp(UA/K_S)} (t_c - t_{in}), \quad (2.10)$$

where $K_S = Sc_{pS}$. Excluding t_c from eqs. (2.6) and (2.10), we receive

$$Q = \frac{K_S \frac{C-1}{C}}{1 + \frac{K_S(C-1)}{K_F C}} (T_F - t_{in} - \psi - \phi), \quad (2.11)$$

where $K_F = Fc_{pF}$ and $C = \exp(UA/K_S)$.

3 Cascade of the units

In order to increase the efficiency of heat exchange, the flush tank - condenser units are often assembled in chain. The schematic representation of a series of flash tanks and heat exchangers are shown in Fig. 4. In Fig. 4, the top line represents the flow of hot slurry from the digester D. It starts at a temperature T_1 , and goes through a series of N flush tanks. At the i th flush tank,

the slurry's temperature decreases from T_i at the input to T_{i+1} at output (which is the input temperature for the $(i+1)$ th flash tank), and the slurry finally exits the cascade at temperature $T_{N+1} = T_{\text{out}}$. The bottom line represents the flow of spent liquor into the digester. The flow starts at temperature $t_{N+1} = t_{\text{in}}$, and as it passes through the i^{th} condenser, its temperature increases from t_{i+1} to t_i . The spent liquor enters the digester at temperature t_1 . The closer this temperature is to the temperature at which the digester operates (255°C) the less energy is needed to heat it up.

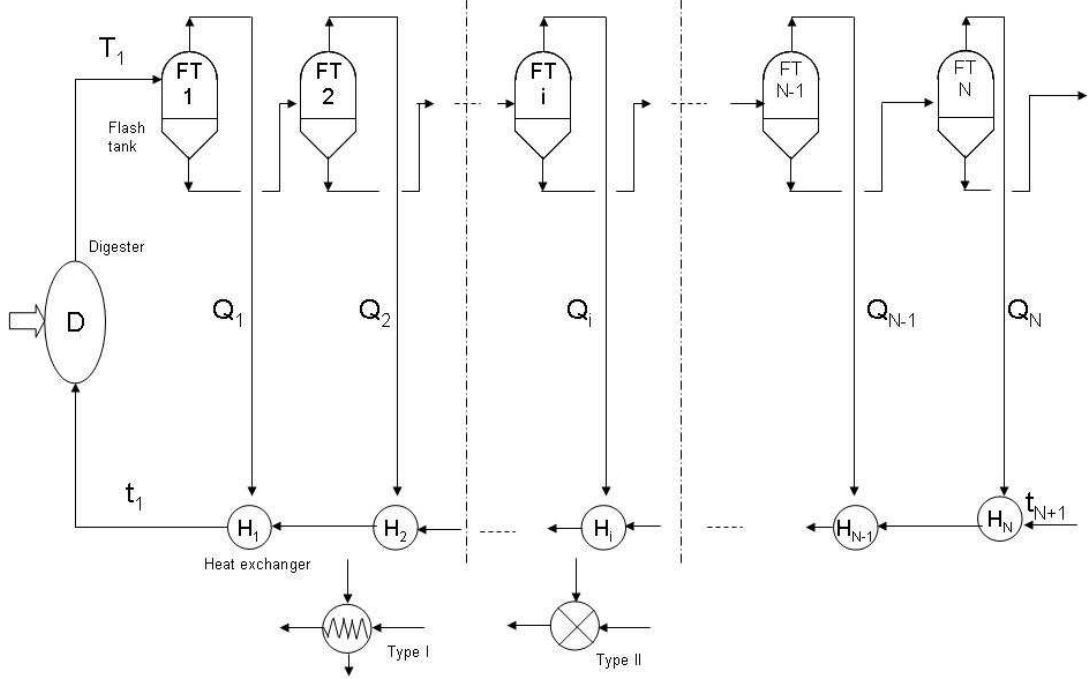


Figure 4: A cascade of the flash tank - condenser units

The analysis of a single condenser in Section 2 shows that the four quantities, T_i , T_{i+1} , t_i and t_{i+1} , are related by two (approximately) linear equations. In this paper, we assume that these equations are precisely linear. Then for the $2N + 2$ variables T_1, \dots, T_{N+1} and t_1, \dots, t_{N+1} we have $2N$ linear equations. In addition, we have a fixed boundary values, namely $T_1 = 255^\circ\text{C}$, which is the temperature at which the digester operates. This leaves one free variable, which we shall choose to be $t_{N+1} = t_{\text{in}}$. It follows from the general theory of linear systems that all the other variables are then linearly functions of t_{in} and T_1 . Thus, we have

$$t_1 = \mathbf{a} t_{\text{in}} + \mathbf{b} \quad (3.1)$$

where \mathbf{a} is a real number, which from physical principles must be between 0 and 1.

From Section 2, we get that the heat exchanged in the i^{th} unit is given by

$$Q_i = D_i(T_i - t_{i+1} - \pi_i), \quad (3.2)$$

where D_i is a constant depending on the physical properties of the unit,

$$D_i = \left(\frac{K_S \frac{C-1}{C}}{1 + \frac{K_S(C-1)}{K_F C}} \right)_i, \quad (3.3)$$

and $\pi_i = \psi_i + \phi_i$ is a loss in potential caused by superheating (the ψ_i term) and the presence of non-condensable gases (the ϕ_i term). Let the flow rate of the spent liquor be r_i and its specific heat be s_i (these may change from stage to stage as the amount of water in the spent liquor changes). Let the flow rate of the slurry be R_i and its specific heat S_i (again, changing quantities of water in the stream can lead these quantities to vary from stage to stage). Then we have

$$\begin{aligned} (T_i - T_{i+1})R_i S_i &= Q_i, \\ (t_i - t_{i+1})r_i s_i &= Q_i. \end{aligned}$$

Let

$$\begin{aligned} K_i &= D_i/R_i S_i, \\ k_i &= D_i/r_i s_i. \end{aligned}$$

Using 3.2, we get the equations

$$T_i - T_{i+1} = K_i(T_i - t_{i+1} - \pi_i), \quad (3.4)$$

$$t_i - t_{i+1} = k_i(T_i - t_{i+1} - \pi_i). \quad (3.5)$$

As i ranges from 1 to N , equations (3.4) and (3.5) give $2N$ linear equations. If we augment them with the two equations

$$T_1 = 255, \quad (3.6)$$

$$t_{N+1} = t_{in}, \quad (3.7)$$

we get a system of $2N+2$ linear equations in the $2N+2$ variables T_1, \dots, T_{N+1} and t_1, \dots, t_{N+1} . They can be written in matrix form as $A\mathbf{t} = \mathbf{b}$, where, using a superscript \mathbf{t} to denote the transpose,

$$\mathbf{t} = (t_1, T_1, t_2, T_2, \dots, t_{N+1}, T_{N+1})^t,$$

$$\mathbf{b} = (-k_1\pi_1, -K_1\pi_1, -k_2\pi_2, -K_2\pi_2, \dots, -k_N\pi_N, -K_N\pi_N, 255, t_{in})^t$$

and A is the coefficient matrix

$$\begin{pmatrix} 1 & -k_1 & k_1 - 1 & 0 & 0 & 0 & \dots \\ 0 & 1 - K_1 & K_1 & -1 & 0 & 0 & \dots \\ 0 & 0 & 1 & -k_2 & k_2 - 1 & 0 & \dots \\ 0 & 0 & 0 & 1 - K_2 & K_2 & -1 & \dots \\ \vdots & & & & & & \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & 1 \\ 0 & 0 & \dots & & & 0 & 0 & 1 \end{pmatrix}.$$

The current plant has an assembly of 10 heat exchangers in a row, so there are 30 physical parameters that we need to know: K_1, \dots, K_{10} , k_1, \dots, k_{10} and π_1, \dots, π_{10} . If π_i is known, we could use equations (3.4) and (3.5) to calculate K_i and k_i .

The following Theorem holds for this model:

Theorem 1. *An assembly of N heat conversion units, where heat exchange in every unit described by the system*

$$\begin{aligned} T_{i+1} &= T_i - Q_i/R_i S_i, \\ t_i &= t_{i+1} + Q_i/r_i s_i, \\ Q_i &= D_i(T_i - t_{i+1} - \pi_i), \end{aligned}$$

can be described by a model

$$\begin{aligned} T_{N+1} &= (1 - \bar{K})T_1 - \bar{K}t_{N+1} + \bar{K} \bar{\delta}, \\ t_1 &= (1 - \bar{k})t_{N+1} + \bar{k}T_1 - \bar{k} \bar{\Delta}. \end{aligned}$$

Proof. At every stage,

$$\begin{aligned} T_{i+1} &= (1 - K_i)T_i + K_i t_i + K_i \pi_i, \\ t_i &= (1 - k_i)t_{i+1} + k_i T_i - k_i \pi_i, \end{aligned}$$

where $k_i = D_i/r_i s_i$ and $K_i = D_i/R_i S_i$, hold. The boundary conditions T_1 and t_{N+1} are assumed to be given. Hence we have $N + 1$ linear algebraic equations for $N + 1$ unknown variables, and the system is well defined.

The hypothesis of Theorem definitely holds for $N = 1$. Assume that it holds for an assembly of $i - 1$ heat exchangers, from the first to the $(i - 1)$ st. That is, we assume that the following equations hold:

$$T_i = (1 - K_{1,i-1})T_1 + K_{1,i-1}t_i + K_{1,i-1}\delta_{1,i-1}, \quad (3.8)$$

$$t_1 = (1 - k_{1,i-1})t_i + k_{1,i-1}T_1 - k_{1,i-1}\Delta_{1,i-1}. \quad (3.9)$$

We now have to prove that then it also holds for an assembly of i heat exchangers.

For the i th unit the following equations hold:

$$T_{i+1} = (1 - K_i)T_i + K_i t_{i+1} + K_i \pi_i, \quad (3.10)$$

$$t_i = (1 - k_i)t_{i+1} + k_i T_i - k_i \pi_i. \quad (3.11)$$

Substituting (3.11) into (3.8), and (3.8) into (3.11), we obtain

$$\begin{aligned} T_i &= (1 - K_{1,i-1})T_1 + K_{1,i-1}[(1 - k_i)t_{i+1} + k_i T_i - k_i \pi_i] + K_{1,i-1}\delta_{1,i-1}, \\ t_i &= (1 - k_i)t_{i+1} + k_i[(1 - K_{1,i-1})T_1 + K_{1,i-1}t_i + K_{1,i-1}\delta_{1,i-1}] - k_i \pi_i, \end{aligned}$$

and hence

$$\begin{aligned}(1 - k_i K_{1,i-1}) T_i &= (1 - K_{1,i-1}) T_1 + K_{1,i-1} (1 - k_i) t_{i+1} - K_{1,i-1} k_i \pi_i + K_{1,i-1} \delta_{1,i-1}, \\ (1 - k_i K_{1,i-1}) t_i &= (1 - k_i) t_{i+1} + k_i (1 - K_{1,i-1}) T_1 + k_i K_{1,i-1} \delta_{1,i-1} - k_i \pi_i.\end{aligned}$$

Substituting these equalities into (3.10) and (3.9), we finally get

$$\begin{aligned}T_{i+1} &= \frac{(1 - K_i)}{(1 - k_i K_{1,i-1})} ((1 - K_{1,i-1}) T_1 + K_{1,i-1} (1 - k_i) t_{i+1} - K_{1,i-1} k_i \pi_i + K_{1,i-1} \delta_{1,i-1}) \\ &\quad + K_i t_{i+1} + K_i \pi_i, \\ t_i &= \frac{(1 - k_{1,i-1})}{(1 - k_i K_{1,i-1})} ((1 - k_i) t_{i+1} + k_i (1 - K_{1,i-1}) T_1 + k_i K_{1,i-1} \delta_{1,i-1} - k_i \pi_i) \\ &\quad + k_{1,i} T_1 - k_{1,i-1} \Delta_{1,i-1}.\end{aligned}$$

That is,

$$T_{i+1} = (1 - K_{1,i}) T_1 + K_{1,i} t_{i+1} + K_{1,i} \delta_{1,i}, \quad (3.12)$$

$$t_i = (1 - k_{1,i}) t_{i+1} + k_{1,i} T_1 - k_{1,i} \Delta_{1,i}, \quad (3.13)$$

where the coefficients $K_{1,i}$, $k_{1,i}$, $\Delta_{1,i}$, $\delta_{1,i}$ are defined by recurrent formulae

$$k_{1,i} = \frac{k_{1,i-1} + k_i - k_i k_{1,i-1} - k_i K_{1,i-1}}{1 - k_i K_{1,i-1}} = 1 - \frac{(1 - k_i)(1 - k_{1,i-1})}{1 - k_i K_{1,i-1}}, \quad (3.14)$$

$$K_{1,i} = \frac{K_{1,i-1} + K_i - K_i K_{1,i-1} - k_i K_{1,i-1}}{1 - k_i K_{1,i-1}} = 1 - \frac{(1 - K_i)(1 - K_{1,i-1})}{1 - k_i K_{1,i-1}}, \quad (3.15)$$

$$k_{1,i} \cdot \Delta_{1,i} = k_{1,i-1} \Delta_{1,i-1} - (1 - k_{1,i-1}) \frac{k_i K_{1,i-1} \delta_{1,i-1} - k_i \pi_i}{1 - k_i K_{1,i-1}}, \quad (3.16)$$

$$K_{1,i} \cdot \delta_{1,i} = K_i \pi_i + (1 - K_i) \frac{K_{1,i-1} \delta_{1,i-1} - k_i K_{1,i-1} \pi_i}{1 - k_i K_{1,i-1}}. \quad (3.17)$$

□

Equations (3.12–3.17) enable us to find explicitly the temperatures at the ends of a given cascade of heat converters. We have to stress that these equations also hold for any ordered sub-chain composed of any number of units. Furthermore, for many practically relevant heat exchanges assemblies equations (3.12, 3.13) can be further simplified. To show this we have to recollect firstly that parameter π_i is depends exclusively on the chemical properties of the hot flow, and in many practically relevant cases these properties can be assumed constant throughout all units of an assembly. Indeed, by definition, term π_i is the sum of two parts, ψ_i and ϕ_i , and it reflects a loss in potential caused by superheating (the ψ_i term) and the presence of non-condensable gases (the ϕ_i term) in the hot flow. Superheating occurs because the liquid is a mixture of chemicals, and hence vaporization starts at a temperature higher than this for

pure water. We estimate that ψ_i is approximately 6.5 K. The term ϕ_i is trickier to estimate, but should be less than 1 K. We assume that

$$\pi_i \approx 7.5. \quad (3.18)$$

It is apparent, however, that the value of π does not change significantly from unit to unit and is near constant throughout the chain. For such a case, that is if $\pi_i = \pi$ for all i ,

$$\Delta_{1,i} = \delta_{1,i} = \pi \quad (3.19)$$

hold, and the equations (3.12, 3.13) take the form

$$T_{i+1} = (1 - K_{1,i})T_1 + K_{1,i}t_{i+1} + K_{1,i}\pi, \quad (3.20)$$

$$t_1 = (1 - k_{1,i})t_{i+1} + k_{1,i}T_1 - k_{1,i}\pi. \quad (3.21)$$

It is obvious that (3.19) hold for a single unit. For two units such that $\pi_1 = \pi_2 = \pi$ hold, equations (3.14–3.17) give

$$k_{1,2} \cdot \Delta_{1,2} = \left(k_1 - (1 - k_1) \frac{k_2 K_1 - k_2}{1 - k_2 K_1} \right) \pi = k_{1,2} \pi,$$

$$K_{1,2} \cdot \delta_{1,2} = \left(K_2 + (1 - K_2) \frac{K_1 - k_2 K_1}{1 - k_2 K_1} \right) \pi = K_{1,2} \pi.$$

This recurrent process can be carried further on.

Acknowledgements

1. We thank Mathematics Applications Consortium for Science and Industry (MACSI) for the excellent organization of this Study Group and hospitality they provided.
2. Partially carried out in the frame work of Innovation of Ireland for Energy Efficiency (I2E2) project.

Bibliography

- [1] P. Smith, The processing of high silica bauxites — Review of existing and potential processes, *Hydrometallurgy*, **98** (2009) 162–176.
- [2] Shu-hua Ma, Zong-guo Wen, Ji-ning Chen, Shi-li Zheng. An environmentally friendly design for low-grade diasporic-bauxite processing. *Minerals Engineering*, **22** (2009) 793–798.
- [3] Huixin Li, Jonas Addai-Mensah, John C. Thomas, Andrea R. Gerson. A study of colloidal Al(III)-containing species in fresh/caustic aluminate solutions. *Colloids and Surfaces A: Physicochemical Engineering Aspects*, **223** (2003) 83–94.
- [4] Yousry L. Sidrak. Dynamic Simulation and Control of the Bayer Process. A Review. *Industrial & Engineering Chemistry Research*, **40** (2001) 1146–1156.