# PIPED WATER COOLING OF CONCRETE DAMS

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**Abstract**

Piped water is used to remove hydration heat from concrete dams during construction. By examining simple models we obtain an estimate for the temperature rise along the pipe network and within the concrete. To leading order, for practically useful networks, the temperature distribution is quasi-steady, so that exact analytic solutions are obtained. The temperature in the water increases linearly with distance along the pipe and varies logarithmically with radial distance from the pipe in the concrete. Using these results we obtained estimates for the optimal spacing of pipes and pipe length. Some preliminary work on optimal network design has been done. This is work in progress.

## 1 Introduction

During dam construction large slabs of concrete are poured (typically 3m $\times$ 10m $\times$ 10m). The concrete is made of a mixture of cement (powder) and water which react to generate heat. The chemical reaction can lead to temperature rises in excess of 50K and continues for months, indeed years.

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Obviously this can lead to problems due to thermal stress, such as cracking and resultant structural weakening.

To alleviate the problem pipe networks are included in the block and chilled water is pumped through the pipes. Later the pipes are filled with concrete. The aim of this study was to estimate the temperature profile in the concrete and the effect of pumping water, with the ultimate goal being to provide a strategy for improved heat removal.

## 2 Simple energy balance model

We start by considering a simple global energy balance model. This will give a crude estimate for the effect of water cooling on the temperature rise in the concrete.

Under steady-state conditions the heat removal rate per unit concrete volume by circulating water will be $Q\rho_w c_w(T_{\text{out}} - T_0)$ where $T_0$ is the entry temperature of the water, $T_{\text{out}}$ is the exit temperature and $Q$ is the volume flux of water through the concrete, of volume $V$ say. If $q$ represents the rate at which heat is generated within the slab per unit volume then this must balance with the heat removed requiring

$$Vq = Q\rho_w c_w(T_{\text{out}} - T_0) \ . \tag{2.1}$$

A crude estimate for the concrete temperature (and water exit temperature) is thus given by

$$T_{\text{out}} = T_0 + \frac{qV}{Q\rho_w c_w} \tag{2.2}$$

Of course this is a crude estimate for a number of reasons. The concrete temperature is unlikely to be uniform and may not approximately match the water temperature at exit. The efficiency of heat removal from the concrete slab decreases as the water temperature increases and no heat transfer from the slab to the circulating water will occur once the water temperature reaches that of the slab. Optimistically one might hope that a good pumping system could, however, perform close to this design limit. Of course efficient heat removal networks are likely to be expensive (obviously an extensive network of fine pipes with many individual elements would be efficient) so that an appropriate balance between thermal efficiency and expense will need to be struck. Nevertheless in a real sense the above calculation provides us with an objective measure for the efficiency of practical water network designs.

The above discussion assumes the hydration heat release remains fixed and steady state conditions are realised and appropriate. In fact, the hydration heat release varies with time, so that the aim of the network may be to simply extract sufficient heat during the significant hydration period to keep temperature levels within acceptable limits. We now move on to a more complex, but hopefully more accurate, model.

## 3 A simple cylindrical model

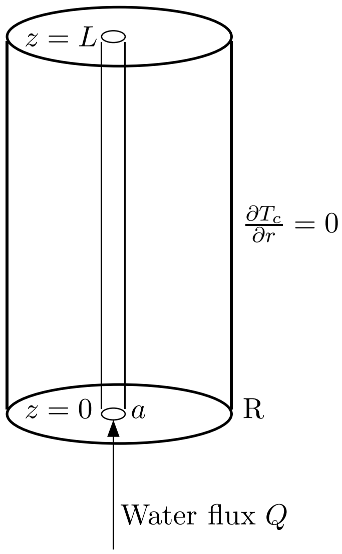


Figure 1: Cylindrical model: water flux $Q$ at temperature $T_0$ through a pipe cools an insulated concrete cylinder.

We consider a pipe radius $a$(m) embedded in a long insulated cylindrical slab of concrete of radius $R$(m), length $L$, see Figure 1. The concrete produces hydration heat at a prescribed rate $q$ per unit volume. Cool water, at temperature $T_0$, enters the pipe at $z = 0$ and removes hydration heat from the slab. The volume flux of water flowing through the pipe is $Q$. The above configuration models a section of pipe within a concrete sleeve within the slab. An insulated boundary condition, $\frac{\partial T_c}{\partial r} = 0$, is chosen at $r = R$ to take into account the local periodicity of the pipe network; the situation thus approximately models a periodic array of pipes with spacing $2R$. We will modify the solutions obtained here to treat the periodic pipe array problem later. Information from this model, and its extensions, will enable us to estimate the optimal spacing $R$ between elements of the pipe network and the flux levels $Q$ required to maintain concrete temperature levels within prescribed bounds. Also we will use the model to estimate the appropriate length of piping.

The heat equations in the concrete and water are, respectively,

$$\rho_c c_c \frac{\partial T_c}{\partial t} = \kappa_c \nabla^2 T_c + q \, , \tag{3.1}$$

$$\rho_w c_w \left( \frac{\partial T_w}{\partial t} + \mathbf{u} \cdot \nabla T_w \right) = \kappa_w \nabla^2 T_w \, , \tag{3.2}$$

where $T_c, T_w$ are the temperature of the concrete and the water in the pipe, $q$ is the rate of heat production per unit volume in the concrete, and $(\rho_c, c_c, \kappa_c)$, $(\rho_w, c_w, \kappa_w)$ are the density, specific heat and conductivity of concrete and water respectively. Due to the low kinematic viscosity of water ($\nu \sim 10^{-6}$), the Reynolds number is high even for low volume fluxes and so the flow in the pipe will be turbulent (providing the fluid velocity is greater than 1cm/s). Under such circumstances the average radial velocity is zero and the mean flow is in the $z$-direction, $\mathbf{u} = (0, w)$. Since the fluid is incompressible we can state $w = Q/\pi a^2$ is constant, where $Q$ is the water volume flux in the pipe. We may simplify the water heat equation further by considering the average temperature, since the flow is axisymmetric we may write

$$\overline{T_w} = \frac{2\pi \int_0^a T_w \; r dr}{\pi a^2} \, .$$

Further, since the flow is turbulent and the fluid well-mixed we expect the $r$ variation to be small, except perhaps for in a boundary layer near the pipe wall. Hence, integrating the whole equation, gives

$$\begin{aligned} 2\pi \rho_w c_w \int_0^a \left( \frac{\partial T_w}{\partial t} + \frac{Q}{\pi a^2} \frac{\partial T_w}{\partial z} \right) \; r dr &= 2\pi \kappa_w \int_0^a \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_w}{\partial r} \right) + \frac{\partial^2 T_w}{\partial z^2} \right) \; r dr, \\ 2\pi \rho_w c_w \frac{a^2}{2} \left( \frac{\partial \overline{T_w}}{\partial t} + \frac{Q}{\pi a^2} \frac{\partial \overline{T_w}}{\partial z} \right) &= 2\pi \kappa_w \left( a \frac{\partial T_w}{\partial r} \bigg|_{r=a} + \frac{a^2}{2} \frac{\partial^2 \overline{T_w}}{\partial z^2} \right) \, . \end{aligned} \tag{3.3}$$

At the boundary between the water and concrete, $r = a$, a standard cooling condition gives

$$\kappa_w \frac{\partial T_w}{\partial r} = H(T_c - T_w) \, . \tag{3.4}$$

Note, the heat transfer coefficient can include the effect of the pipe wall through the relation

$$H = 2\pi\kappa_p \frac{s}{a} + H' \ , \tag{3.5}$$

where $\kappa_p$, $s$ and $H'$ represent the conductivity of the pipe, the pipe thickness and the heat transfer coefficient of water on the pipe, see Carslaw and Jaeger (1959). Substituting the boundary condition into equation (3.3) gives the final form for the dimensional governing equation for heat flow in the water

$$\pi\rho_w c_w a^2 \left(\frac{\partial \overline{T_w}}{\partial t} + \frac{Q}{\pi a^2}\frac{\partial \overline{T_w}}{\partial z}\right) \quad = \quad 2\pi a H \left(T_c|_{r=a} - \overline{T_w}\right) + \pi\kappa_w a^2 \frac{\partial^2 \overline{T_w}}{\partial z^2} \ . \tag{3.6}$$

Note, since the temperature is constant for a given $z$, *i.e.* $T_w \sim \overline{T_w}$ is a function of $z$ alone we have replaced the value of $T_w$ at $r = a$ with the average value across the flow $T_w(a, z, t) \approx \overline{T_w}(z, t)$.

## 3.1 Dimensional analysis

In the following dimensional analysis we will determine which terms may be neglected in the governing equations. However, a simple statement of conservation of energy can provide us with the leading order balance in the water. Assuming all the energy enters the water through the pipe wall and is then convected away gives

$$\pi a^2 \rho_w c_w w \frac{\partial T_w}{\partial z} = 2\pi a \kappa \frac{\partial T_c}{\partial r} = 2\pi a H (T_c - T_w) \ .$$

We expect this same balance to result from the non-dimensionalisation of the defining equations (2.3, 2.4).

We non-dimensionalize the equations using the scales

$$r = Rr' \ , \quad z = z_0 z' \ , \quad t = \tau t' \ , \quad T_c = T_0 + \Delta T \ T' \ , \quad \overline{T_w} = T_0 + \Delta T_c \ T'_w ,$$

where $T_0$ is the water temperature on entering the pipe, $\Delta T$ is a typical increase in temperature within the concrete and $z_0, \tau$ are the length-scale and time scales for significant temperature variations in the pipe; $\Delta T, \tau, z_0$ are yet to be determined. For ease of notation, from now on we drop the primes. The heat equation in the concrete becomes

$$\frac{\rho_c c_c \Delta T}{\tau}\frac{\partial T_c}{\partial t} \quad = \quad \kappa_c \Delta T \left(\frac{1}{R^2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_c}{\partial r}\right) + \frac{1}{z_0^2}\frac{\partial^2 T_c}{\partial z^2}\right) + q \ . \tag{3.7}$$

Anticipating the fact that radial diffusion is the dominant method for heat transferral in the concrete we rearrange this to

$$\frac{\rho_c c_c R^2}{\tau \kappa_c} \frac{\partial T_c}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_c}{\partial r} \right) + \frac{R^2}{z_0^2} \frac{\partial^2 T_c}{\partial z^2} + \frac{q R^2}{\kappa_c \Delta T} \ . \tag{3.8}$$

In the water we expect energy to be carried along with the fluid and so rearrange the water heat equation accordingly to give

$$\frac{\pi a^2 z_0}{Q\tau} \frac{\partial T_w}{\partial t} + \frac{\partial T_w}{\partial z} = \frac{2\pi a H z_0}{\rho_w c_w Q} (T_c|_{r=\epsilon} - T_w) + \frac{\pi a^2 \kappa_w}{\rho_w c_w z_0 Q} \frac{\partial^2 T_w}{\partial z^2} \ . \tag{3.9}$$

Here $\epsilon = a/R$. The final term on the right hand side of (3.9) is $\mathcal{O}(10^{-9})$, indicating diffusion does not play a significant role in heat transfer in the water. This term will be neglected from now on.

There are three unknown scales in equations (3.8, 3.9), the length-scale $z_0$, the time-scale $\tau$ and the temperature scale $\Delta T$. Clearly the temperature rise is driven by heat production in the concrete, so we choose

$$\Delta T = q R^2 / \kappa_c. \tag{3.10}$$

In the water the temperature rise is due to forced convection at the boundary, so we choose

$$z_0 = \rho_w c_w Q / 2\pi a H. \tag{3.11}$$

The time derivatives indicate two distinct time scales. In the concrete

$$\tau = \tau_c = \rho_c c_c R^2 / \kappa_c, \tag{3.12}$$

in the water

$$\tau = \tau_w = \pi a^2 z_0 / Q \ . \tag{3.13}$$

Substituting typical values, as given in Table 1, indicates $\Delta T \sim 54.7$K, $z_0 \sim 5.35$m, $\tau_c \sim 3.8 \times 10^5$s $\sim 4.4$ days, $\tau_w \approx 52.5$s$\approx$ 1min. These scales indicate a typical temperature rise of 50°K in the concrete, and also that the appropriate pipe length should be around 60m, see later. These values do fit in with engineering practice, which suggests the above scaling and numbers are correct.

*Comment on the two time scales*: When the tap is turned on it takes a time of order 3 minutes ($\tau_w$) for the cool water to adjust to its immediate concrete temperature environment. It then takes about 4 days ($\tau_c$) for the concrete to adjust to the presence of the cooling water. Over this period of time the heat flux into the water increases, leading eventually to a steady state situation in which the hydration heat supply rate is balanced by the heat removal rate by the water. The two time-scales allow us to observe the cooling process from two different viewpoints. Over the water flow time-scale, $\tau_w$, the concrete temperature does not vary, over the much longer time-scale, $\tau_c$, the concrete temperature will change. We will now briefly examine the thermal problem over the water time-scale and then move on to the problem over the longer time-scale, $\tau_c$, which is our main concern.

### 3.1.1 Thermal variation over time-scale $\tau_w$

If we work on the time-scale of the water flow, $\tau = \tau_w$, we can obtain detailed information about the water temperature profile. This may be used for example when the tap is initially turned on or in fact at any given time if we require a more accurate description of the water temperature than can be obtained from using the time-scale $\tau_c$.

Setting $\tau = \tau_w$ the time derivative in (3.8) is $\mathcal{O}(10^4)$ and the leading order concrete temperature is simply $T_c = T_c(r, z)$. If we consider the start-up period then the initial concrete temperature must be approximately constant, $T_c = T_i$. Since $T_c$ is independent of time (to leading order) it will remain at this temperature for the duration of any calculation on this time-scale, *i.e.* for sufficiently short time periods the concrete temperature will not change due to heat production or loss. The heat equation in the water, (3.9), reduces to

$$\frac{\partial T_w}{\partial t} + \frac{\partial T_w}{\partial z} = \frac{2\pi a H z_0}{\rho_w c_w Q}(T_i - T_w) = (T_i - T_w) \ . \tag{3.14}$$

For simplicity we consider the steady-state, which will be achieved after a few minutes. If the water enters at temperature $T_0 (< T_i)$ then we obtain the solution

$$T_w = T_i - (T_i - T_0)\exp(-z) \ .$$

In dimensional form this is

$$\begin{aligned} T_w &= T_i - (T_i - T_0)\exp(-z/z_0) \ , \\ &= T_i - (T_i - T_0)\exp(-2\pi a H z/\rho_w c_w Q) \ . \end{aligned} \tag{3.15}$$

So, the water temperature increases exponentially along the pipe. As it nears the end the exponential decay means that the temperature increases

| | | | | | |
|---|---|---|---|---|---|
| $\rho_c$ | 2350 | $kg/m^3$ | $c_c$ | 880 | J/kg °C |
| $\rho_w$ | 1000 | $kg/m^3$ | $c_w$ | 4200 | J/kg °C |
| $\kappa_c$ | 1.37 | W/m °C | $R$ | 0.5 | m |
| $q$ | 300 | $W/m^3$ | $H$ | 1000 | $W/m^2$ °C |
| $Q$ | $2\times 10^{-4}$ | $m^3/s$ | $a$ | 0.025 | m |
| $\kappa_w$ | 0.59 | $kg/m^3$ | | | |

Table 1: Parameter values

more slowly, *i.e.* the water is less efficient at removing heat from the concrete as it becomes hotter itself. The factors affecting the water temperature can be seen from equation (3.15). The water temperature increases more slowly with a decrease in heat transfer coefficient or pipe radius or an increase in the flux. Obviously colder water at the inlet results in colder water at the outlet.

In general our interest lies with the concrete temperature and this is best examined by working on the time-scale $\tau_c$.

### 3.1.2 Thermal variation over time-scale $\tau_c$

Note that when we work with this time scale, the time derivative term in (3.9) is $\mathcal{O}(10^{-4})$ and may be neglected throughout the calculation.

We assume (sensibly) that the pipe-spacing is considerably smaller than the pipe-length $R \ll L$ and so denote $\delta = R/L \ll 1$. The leading order heat equations in the concrete and water may now be written

$$\begin{aligned} \frac{\partial T_c}{\partial t} &= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_c}{\partial r}\right) + 1, & (3.16)\\ \frac{\partial T_w}{\partial z} &= T_c|_{r=\epsilon} - T_w \ . & (3.17) \end{aligned}$$

The neglect of the term involving $T_{c,zz}$ indicates that diffusion in the $z$-direction is small. However, this is the largest term so far neglected, so a more accurate solution could be determined by including this term in an asymptotic expansion in powers of $\delta^2$ (in fact the solution obtained below is accurate to order $\delta^2$).

The problem has now reduced to solving equations (3.16, 3.17). The water enters the pipe at a dimensional temperature $T_0$ which means (3.17) must be solved subject to $T_w = 0$ at $z = 0$. At the pipe boundary, $r = \epsilon =$

$a/R$, a cooling condition applies to the concrete

$$\frac{\partial T_c}{\partial r} = \xi(T_c - T_w), \tag{3.18}$$

where

$$\xi = HR/\kappa_c \ . \tag{3.19}$$

Note the heat transfer $H$ enters the equations through the dimensionless parameter $\xi$. Halfway between the current pipe and the next, $r = 1$, symmetry requires that the temperature gradient must be zero:

$$\frac{\partial T_c}{\partial r} = 0 \ . \tag{3.20}$$

To simplify the calculations we will now consider the steady-state solution. Since changes in the concrete temperature occur over a time-scale of $\tau_c = 4$ days which is much less than the time span (weeks) of interest this makes sense (in practice the hydration heat input varies with time; the solutions obtained below really represent a quasi-steady state approximation). Under steady-state conditions equation (3.16) may be integrated to give

$$T_c = -\frac{r^2}{4} + A\log r + B \ , \tag{3.21}$$

where $A$ and $B$ may be functions of $z$. Applying the condition (3.20) gives $A = 1/2$. Applying (3.18) gives

$$B = T_w + \frac{1}{\xi}\left(-\frac{\epsilon}{2} + \frac{1}{2\epsilon}\right) + \frac{\epsilon^2}{4} - \frac{1}{2}\log\epsilon \ , \tag{3.22}$$

that is $B$ depends on the water temperature. Substituting for $T_c$ in equation (3.17) leads to

$$\frac{\partial T_w}{\partial z} = \frac{1}{\xi}\left(-\frac{\epsilon}{2} + \frac{1}{2\epsilon}\right) \ . \tag{3.23}$$

The temperature in the water and concrete is therefore

$$T_w = \frac{1}{\xi}\left(-\frac{\epsilon}{2} + \frac{1}{2\epsilon}\right) z, \tag{3.24}$$

$$T_c = -\frac{r^2}{4} + \frac{1}{2}\log r + \frac{1}{\xi}\left(-\frac{\epsilon}{2} + \frac{1}{2\epsilon}\right)(z+1) + \frac{\epsilon^2}{4} - \frac{1}{2}\log\epsilon, \tag{3.25}$$

$$= T_w(z) + \left[-\frac{r^2}{4} + \frac{1}{2}\log\frac{r}{\epsilon}\right] + \left[\frac{1}{\xi}\left(-\frac{\epsilon}{2} + \frac{1}{2\epsilon}\right) + \frac{\epsilon^2}{4}\right] \ . \tag{3.26}$$

The maximum temperature in the water is of course reached at the exit $z = L/z_0$ and in the concrete is reached at the edge of the domain at the exit, that is at $(r, z) = (1, L/z_0)$. These are given to order $\epsilon$ by

$$T_{w\max} = \frac{1}{2\epsilon\xi}(\frac{L}{z_0}), \; T_{c\max} = \frac{1}{2\epsilon\xi}(\frac{L}{z_0} + 1) - \frac{1}{4} - \frac{1}{2}\log\epsilon, \tag{3.27}$$

which in dimensional form are

$$T_{w\max} = T_0 + \frac{\pi R^2 L q}{Q\rho_w c_w} \equiv T_0 + \frac{Vq}{Q\rho_w c_w}, \tag{3.28}$$

where $V$ is the volume of the cylinder, and

$$T_{c\max} = T_0 + \frac{qR^2}{\kappa_c}\left[-\frac{1}{4} + \frac{\kappa_c}{HR}\left(\frac{R}{a}\right) - \frac{1}{2}\log(a/R)\right]. \tag{3.29}$$

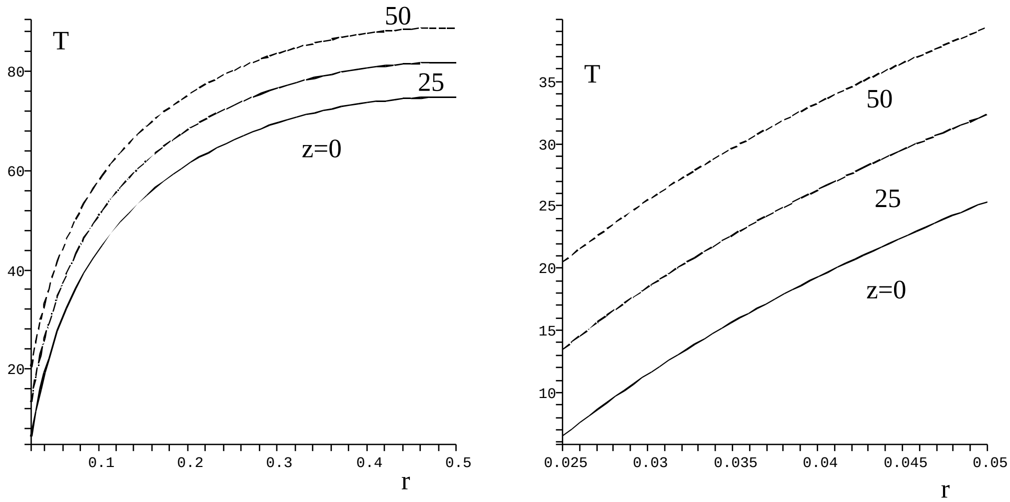


Figure 2: Temperature profile in concrete

Figures (2 a,b) show the temperature profile in the concrete at three different $z$ locations, $z = 0, 25, 50$, where the length-scale $L = 50$m. Water is pumped in at a temperature of 5°C. Figure (2a) shows the profile for $r \in [a, R]$ and clearly the temperature increases away from the pipe in each case. The lowest profile represents the temperature at $z = 0$, the highest at $z = 50$. Figure (2b) shows a close-up of the region near the pipe wall. From this it is clear that as the water flows through the pipe it is heating up and consequently the concrete temperature next to the pipe must also increase.

### 3.1.3 Observations

It is interesting to note that the water temperature increases linearly with distance along the pipe. The explanation is straightforward. Since we ignore axial diffusion then, under steady conditions, all the locally (i.e. at the z location) released hydration heat must be locally absorbed by the flowing water; the water temperature thus increases linearly along the pipe's length as in (3.24), emerging at a temperature $T_w(L/z_0)$. More surprising perhaps is that the temperature of the water at each position $z$ along the pipe does not depend on the efficiency of the heat transfer from the concrete into the water as reflected in $H$, as can best be seen from (3.28); again this is a consequence of the steady state. Of course if the heat transfer process is inefficient ($H$ small) then the temperature in the concrete will need to be high to drive the required heat flux into the water, see later (also if $H$ is small then the time required to reach steady state will be large). Note that the result (3.28) coincides with that obtained using the simple energy balance model of section 2, see (2.1). The simple model however fails to determine the concrete temperature; the assumption that the concrete temperature is the same as that of the exit water temperature is not good. Further, this result is not inconsistent with that obtained over the time-scale $\tau_w$. With the current scaling the factor which appears in the exponential term of equation (3.15) is significantly smaller and so the exponential is approximately linear.

The concrete temperature form is quite special; it consists of separated and added $r$ and $z$ pieces, see (3.26). The $r$ portion of the solution exhibits the typical logarithmic behaviour close to the pipe associated with a line sink, with a quadratic modification further away. The $z$ portion consists of a *constant* temperature jump,

$$T_c(\epsilon, z) - T_w(z) = \frac{1}{2\epsilon\xi}(1 - \epsilon^2) \ ,$$

across the pipe skin which simply adds onto the water temperature $T_w(z)$. Note that the size of the jump depends on the transfer coefficient as reflected in $\xi$ and the surface area of the pipe as reflected in $\epsilon$. We can think of this term simply as the *pipe resistance* to heat flow.

### 3.1.4 Design implications

Recall again that the primary issue for the engineer is to reduce the maximum temperature in the concrete to an acceptable level using water cooling, *and* using a minimal (least costly) network[1]. To a very limited extent one may improve the efficiency of the cooling system by reducing the pipe resistance to heat flow as defined above, for example by increasing $\xi$ (changing the conductivity of the pipe, or pipe diameter) or $\epsilon$ (increasing the pipe diameter). As we have seen above the effect of such a change will be to change the temperature throughout the slab by $\frac{1}{2\epsilon\xi}(1-\epsilon^2)\Delta T$ in dimensional terms. The effect of reducing the input water temperature $T_0$ will also be to uniformly reduce the temperature of the concrete slab. However, it is in the design of the network *as a whole* (that is the choice $R, L$ and $Q$ for our simple model) that most gains can be made, and the scales obtained above provide the qualitative answers. When supplemented with the solutions above quantitative answers result.

Again it is absolutely essential to recognise that we are designing a heat absorption system, *not* a water transport system. If the aim was to transport water, then large diameter (high volume flux) pipes are the answer, whereas to absorb heat it makes sense to use many small pipes; because the surface area (for absorption) of such small pipes is much larger than that of the larger pipe for the same total volume flux. Indeed from a purely heat absorption point of view very fine pipes are the answer, but the resulting system would be expensive. This is, however, simplistic; a good design is one in which the total temperature variation along the pipe is about the same size as the variation between adjacent pipes (i.e. at $r = R$), and of course the maximum concrete temperature must be acceptable; this determines the spacing $R$. Our present concern is with the very simple cylindrical network of Figure 1 with a single pipe running through it, and our aim in this context is to determine an appropriate flux and pipe radius $(Q, a)$ using all other parameters as in Table 1.

Thus we have a 250m length of pipe surrounded by a cylindrical sleeve of concrete radius 0.5m.[2] We will parallel observations with calculations to display the connections. Note:

- Scaling arguments indicated a time scale of order $\tau_c = \rho_c c_c R^2/\kappa_c$ for significant temperature changes to be brought about by the circulating water. We found that, at least for the parameters used earlier, this gave 4.3 days, which suggests that tuning this time scale is not likely to be important in practice.

[1]More correctly the engineer wants to minimise the thermal stress, but one would expect the maximum temperature to be a good indicator

[2]This would correspond to a 4m by 10m by 1m slab of concrete with the above pipe winding back and forward through its centre.

- We found that radial temperature variations in the concrete are of order $\Delta T = \frac{qR^2}{k_c}$, (see 3.10); note especially that this scales up with the square of the network pipes separation distance $R$ (equivalently the sectional area). Using the values in Table 1 we obtain $55°K$.

- The temperature change at the end of the water pipe, see (3.28) is given by $Vq/(Q\rho_w c_w)$, and if this is to balance the radial variation then the required volume flux of water is given by

$$\frac{qR^2}{k_c} = Vq/(Q\rho_w c_w) \; ,$$

  which for the specific case described gives $Q = 2.56 \times 10^{-3}\mathrm{m}^3/\mathrm{sec}$, slightly more than that quoted in Table 1. This flux level could be realised using a slightly more powerful pump with the 1 inch radius pipe. Alternatively a larger pipe could be used with a less powerful pump.

The ultimate aim is to ensure that temperature levels do not exceed a specified value. To ensure this (3.29) can be used to determine the appropriate spacing $R$ between pipes and then the arguments above can be employed to determine the required $(Q, a)$. In general terms this is the procedure to be used, but a better network model is required and an optimisation model is required. The optimal network will balance costs associated with network construction with expenses (or risks) associated with thermally induced structural problems. We now go on to determine a better network model.

## 4 Modelling periodic arrays or networks

The above cylindrical model is inadequate in two ways, both of which are easily overcome. Firstly the geometry in practice is not cylindrical; a periodic array of pipes is more realistic. The solution obtained for the cylindrical model, see (3.25, 3.24) can also be thought of as the first term of the steady state solution corresponding to a uniform hydration heat input together with a periodic array (size 2) of matching sinks, see Figure 3. The complete solution is obtained by adding image sinks in each of the neighbouring cells:

$$T_c(x, y, z) = \frac{1}{4}(1 - r^2) + \frac{1}{2}(\ln r + \sum_{i,j} \ln r_{ij}) + T_R(z),$$

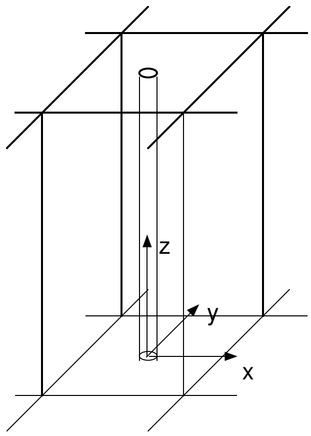


Figure 3: A periodic array model.

where

$$r_{ij} = \sqrt{(x-i)^2 + (y-j)^2}, \ i, j \neq 0 \ .$$

(More usefully a Fourier expansion could be used). This solution of course also corresponds to the solution within an insulated cell of width 2 due to a uniform hydration heat input and with a matching sink located at $r = 0$. Note that because heat flux levels are unaffected by a uniform temperature shift the solution is determined up to an arbitrary 'constant' $T_R$ which in our case needs to be adjusted to fit in with the known temperature along the pipe network. Thus the temperature distribution in the concrete slab is given locally by

$$T_c(s,r) = T_w(s) + \frac{1}{4}(1-r^2) + \frac{1}{2}\left[\ln r + \sum_{i,j} \ln r_{ij}\right] \ ,$$

where $T_w(s) = T_0 + \frac{s}{2\epsilon\xi}s$ is the temperature of the pipe water at a distance $s$ measured along the pipe from the inlet.

The second deficiency of the cylindrical model is that in practice the pipes wind through the slab, and the flow in adjacent pipes will normally be in opposite directions. The temperature continually increases as one moves along a continuous pipe; this will tend to make the temperature distribution in the concrete more uniform. To model this it is useful to notice that our scaling results strongly suggest that to a high degree of accuracy the temperature field will be determined by the requirement that all the hydration heat will be locally absorbed. The temperature variation along the pipe will be necessarily linear and the local temperature field within the concrete must be such that this requirement is satisfied.

### 4.1 A simple 2D network example

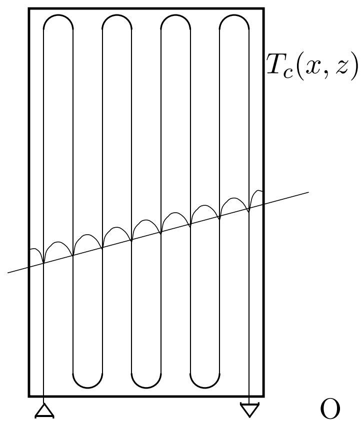


Figure 4: A Water Network: The steady state concrete temperature increases linearly in $x$ with superimposed periodic variations due to the piped water.

In order to clarify the above network issue we will examine the simple 2D network example illustrated in Figure 4. We have a network of 1D 'pipes' embedded in an infinite concrete slab of width $L$ and height $h$. The water flux through the network is $Q_0$ per unit depth (into the page). After a time scale of order $t_0 = h^2/\kappa_c$ a quasi-steady equilibrium will be reached, as described above, with the hydration heat being completely absorbed by the network pipes, resulting in a linear increase in temperature along the pipe from the entry point; thus the temperature along the pipe is determined to be:

$$
\begin{aligned}
T_w(1,z) &= T_0 + \alpha z \ , \\
T_w(2,z) &= (T_0 + \alpha h) + \alpha(h - z) \ , \\
T_w(3,z) &= (T_0 + 2\alpha h) + \alpha z \ , \\
\cdots &= \cdots \\
T_w(n,z) &= (T_0 + (n-1)\alpha h) + \alpha(h - z) \ , \ (n \ even) \\
T_w(n+1,z) &= (T_0 + n\alpha h) + \alpha z \ ,
\end{aligned}
$$

where $1 < n < L/h$ refers to the nth column, see Figure 3. The quasi-steady state (cell) equation is

$$T_{c,xx} = -1 + \delta(x),$$

with quadratic solutions in the n cells given by

$$T_c(x,z) = T_w(n,z) + (x - n) - \frac{1}{2}\,(x-n)^2$$

for

$$n - 1 < x < n + 1 \ .$$

Figure 4 indicates the form of the temperature profile. The base state is a linear increase across the block, superimposed on this is quadratic solution which exhibits peaks in the temperature in the region furthest from the pipes. Generally speaking this solution will not be in *global* equilibrium. Certainly the temperature levels near $x = 0$ will be less than at $x = L$. There will thus normally be a redistribution of heat driven by surface driving conditions. The time scale for global equilibrium to be reached will be of order $L^2/\kappa_c \gg t_0$. If the slab is insulated the above solution is compatible with this requirement, so the solution is correct. If the surface temperature around the block is for example required to be constant then the adjustments will occur to accommodate this requirement. A perturbation procedure can be used to determine the transient.

## Conclusions

The basic aim of water cooling is to decrease the maximum temperature reached in the concrete to an acceptable level. For the simple cylindrical model we have obtained explicit expressions for the maximum concrete temperature in the concrete as a function of the driving parameters, and have determined expressions for the pipe length and separation distance required to limit the temperature rise in the concrete to a prescribed level. Our calculations give a pipe length of 60m for a typical pipe size, flux level and slab. The general results obtained for this model will carry through for more realistic models. Preliminary investigations on more realistic networks have been carried out and surprisingly analytic results can be obtained. When combined with a financial model, optimum design parameters can be determined. This is ongoing work appropriate for a postgraduate student.

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## 5 References

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