Prediction of sanding in subsurface hydrocarbon reservoirs

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1. Introduction

Sand production in oil and gas wells can occur if the fluid velocity exceeds a certain value. Due to drilling operations, the mechanical stresses can exceed the load bearing capacity of the rock. As the local stresses exceed certain level, a certain amount of rock is fractured into sand. Then, the sand is carried by the fluid through the wellbore depending on the flow rate. The amount of the solids can be less than a few grams per cubic meter of reservoir fluid or an essential amount. In the later case erosion of the rock and removing sufficient quantities of rock can occur. This can produce subsurface cavities which collapse and destroy the well.

When sanding is unavoidable it is necessary to estimate the characteristics of the process. Our aim was to generate a simple one-dimensional local model, which predicts the volume of sanding, the radius and the porosity of the yielded zone. Such model will help the company in the development of complex 3D models.

2. Mathematical model

The model we have generated is based on the continuum model presented in the paper [1], assuming radial propagation of the yielded zone around the wellbore (see Fig. 1). First we quote only the main results of the modelling in [1], concerning the quantities we are interested in.

By assuming small variations of the porosity, the relationship between the radius $R(t)$ of the yielded zone and the cumulative solids production $S(t)$ is derived:

$$S(t) = \frac{(1+\alpha)(\phi_y - \phi_n) r_w^{\frac{2\alpha}{1+\alpha}}}{2(1-\phi_y)} \left(R(t)^{\frac{2}{1+\alpha}} - r_w^{\frac{2}{1+\alpha}} \right), \tag{1}$$

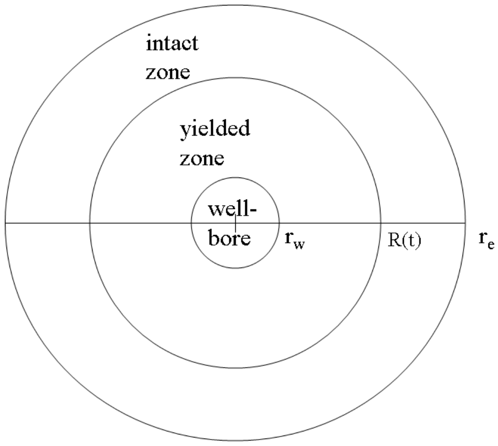


Figure 1: Wellbore, yielded and intact zones

where r_w is the radius of the wellbore, ϕ_n is the porosity of the intact zone, ϕ_y is the porosity of the yielded zone at $r = R(t)$, α is the coefficient of dilation.

From this equation we find:

$$\frac{dS(t)}{dt} = \frac{dR(t)}{dt} \frac{(\phi_y - \phi_n) r_w^{\frac{2\alpha}{1+\alpha}} R(t)^{\frac{1-\alpha}{1+\alpha}}}{(1-\phi_y)}. \tag{2}$$

Further, assuming continuity of the radial stress at the boundary between yielded and non-yielded zones, the following equation is obtained in [1]:

$$\frac{\beta\mu}{k} \frac{dS(t)}{dt} \left(1 - \frac{\ln\rho}{\ln\rho_e}\right) = \frac{p_c - p_w(t)}{\ln\rho} - 2\kappa + \beta \frac{p_e - p_w(t)}{\ln\rho_e} - \frac{2\alpha|\kappa'|}{r_w^2} \frac{(1-\rho^{-2})}{\ln\rho} S(t), \tag{3}$$

where $\rho = R(t)/r_w,\ \rho_e = r_e/r_w$, r_e is the radius of the reservoir (see Fig. 1), p_c is the critical wellbore fluid pressure, p_e is the reservoir pressure, β is the Biot's constant, μ is the viscosity of the fluid, k is the permeability, κ is the strength function.

The strength κ and the porosity are related by (see [1]):

$$\kappa(\phi) = \kappa(\phi_y) + |\kappa'| \frac{2\alpha(1-\phi_y)S(t)}{(1+\alpha) r_w^{\frac{2\alpha}{1+\alpha}} r^{\frac{2}{1+\alpha}}}. \tag{4}$$

For the evolution in time of the wellbore pressure $p_w(t)$ we have used

$$p_w(t) = \frac{p_e(1+\exp(-10t/T))}{2}, \quad T = 500 \times 24 \times 3600 \text{ sec.} \tag{5}$$

Equation (3) (together with the relations (1), (2), (4) and (5)), is an ordinary differential equation for the radius $R(t)$ of the yielded zone. The initial condition is

$$R(0) = r_w.$$

This initial value problem was solved in Maple and Mathematica by the fourth-fifth order Runge-Kutta method.

Once a solution for $R(t)$ is obtained in the time domain, the solids production $S(t)$ can be calculated from equation (1).

To find the porosity of the yielded zone, the second continuity equation from [1]

$$-\frac{\partial \phi}{\partial t} + \operatorname{div}[(1-\phi)v_s] = 0 \tag{6}$$

has been simplified there by letting $\phi = \phi_y$. By using this simplification the following dependence of the porosity on the radial coordinate r and time t has been proposed in [1]:

$$\phi(r,t) = \phi_y - \frac{2\alpha(1-\phi_y)}{1+\alpha} \frac{S(t)}{r^{\frac{2}{1+\alpha}} r_w^{\frac{2\alpha}{1+\alpha}}}. \tag{7}$$

However, as we have found in the computational experiments, this formula gives non-physical behavior of the porosity, namely, it becomes negative for r close to r_w.

In the next section we derive another dependence for the porosity without making the above mentioned simplification. Besides we verify the expression (1) for $S(t)$.

3. Improvement of the expression for porosity

Let $\lambda = \dfrac{1-\alpha}{1+\alpha}$. From the equation for the solid velocity in [1]:

$$v_s(r,t) = -\frac{1}{r^{\frac{1-\alpha}{1+\alpha}}} \frac{q(t)}{r_w^{\frac{2\alpha}{1+\alpha}}}, \tag{8}$$

where $q(t)$ is the volumetric rate of solid production, we get

$$v_s = \frac{-q(t)}{r^\lambda r_w^{1-\lambda}}.$$

Now we rewrite the continuity equation (6) in polar coordinates and obtain

$$-\frac{\partial \phi}{\partial t} - \frac{q(t)}{r_w^{1-\lambda} r} \frac{\partial}{\partial r} \left(r^{1-\lambda}(1-\phi) \right) = 0.$$

After the substitution $u = r^{1-\lambda}(1-\phi)$ this equation takes the form

$$r^\lambda u_t - \frac{q(t)}{r_w^{1-\lambda}} u_r = 0.$$

This is a first order hyperbolic equation for u. Its characteristic curves are given by

$$\frac{dt}{r^\lambda} = -\frac{dr}{q(t) r_w^{\lambda-1}}.$$

Since $\int_0^t q(u)du = S(t)$ we get

$$S(t) + \frac{r^{\lambda+1}}{r_w^{\lambda-1}} \frac{1}{\lambda+1} = \text{const.}$$

The solution u is constant along the characteristics, thus:

$$S(t) + \frac{r^{\lambda+1}}{(\lambda+1) r_w^{\lambda-1}} = F\left((1-\phi) r^{1-\lambda} \right), \tag{9}$$

where F is an arbitrary sufficiently smooth function. Taking into account the condition

$$\phi = \phi_y \quad \text{on} \quad r = R(t)$$

we arrive at

$$S(t) + \frac{R(t)^{\lambda+1}}{(\lambda+1) r_w^{\lambda-1}} = F\left((1-\phi_y) R(t)^{1-\lambda} \right). \tag{10}$$

To relate $S(t)$ and $R(t)$ we use the equation for the mass balance at the boundary (from [1]):

$$(1-\phi_n)\frac{dR}{dt} = (1-\phi_y)\left(\frac{dR}{dt} - v_s \right)$$

and (8):

$$(\phi_y - \phi_n)R'(t) = \frac{(1-\phi_y)q(t)}{R^\lambda r_w^{1-\lambda}}.$$

Then we integrate with respect to t and use $R = r_w$ at $t = 0$ to get

$$S(t) = \frac{(\phi_y - \phi_n)(1+\alpha)r_w^{\frac{2\alpha}{1+\alpha}}\left(R^{\frac{2}{1+\alpha}} - r_w^{\frac{2}{1+\alpha}}\right)}{2(1-\phi_y)},$$

which verifies (1).

To find ϕ we substitute $S(t)$ in (10):

$$A\left(R^{\frac{2}{1+\alpha}} - r_w^{\frac{2}{1+\alpha}}\right) + BR^{\frac{2}{1+\alpha}} = F\left((1-\phi_y)R^{\frac{2\alpha}{1+\alpha}}\right),$$

where

$$A = \frac{(\phi_y - \phi_n)(1+\alpha)r_w^{\frac{2\alpha}{1+\alpha}}}{2(1-\phi_y)}, \quad B = \frac{(1+\alpha)r_w^{\frac{2\alpha}{1+\alpha}}}{2}.$$

We set

$$z = (1-\phi_y)R^{\frac{2\alpha}{1+\alpha}}$$

and obtain

$$F(z) = (A+B)\left(\frac{z}{1-\phi_y}\right)^{\frac{1}{\alpha}} - Ar_w^{\frac{2}{1+\alpha}} \quad (\text{for } z > 0).$$

Now we go back to (9)

$$S(t) + Br^{\frac{2}{1+\alpha}} = (A+B)\frac{(1-\phi)^{\frac{1}{\alpha}} r^{\frac{2}{1+\alpha}}}{(1-\phi_y)^{\frac{1}{\alpha}}} - Ar_w^{\frac{2}{1+\alpha}}.$$

Further we express $\phi(r,t)$ in terms of $R(t)$:

$$AR^{\frac{2}{1+\alpha}} + Br^{\frac{2}{1+\alpha}} = (A+B)\left(\frac{1-\phi}{1-\phi_y}\right)^{\frac{1}{\alpha}} r^{\frac{2}{1+\alpha}}.$$

Thus, solving this equation for $\phi(r,t)$, we get

$$\phi = 1 - \frac{1-\phi_y}{(A+B)^\alpha}\left[A\left(\frac{R(t)}{r}\right)^{\frac{2}{1+\alpha}} + B\right]^\alpha.$$

We substitute A and B and finally obtain the new formula for ϕ:

$$\phi(r,t) = 1 - \frac{1-\phi_y}{(1-\phi_n)^\alpha}\left[(\phi_y - \phi_n)\left(\frac{R(t)}{r}\right)^{\frac{2}{1+\alpha}} + (1-\phi_y)\right]^\alpha. \tag{11}$$

4. Numerical results

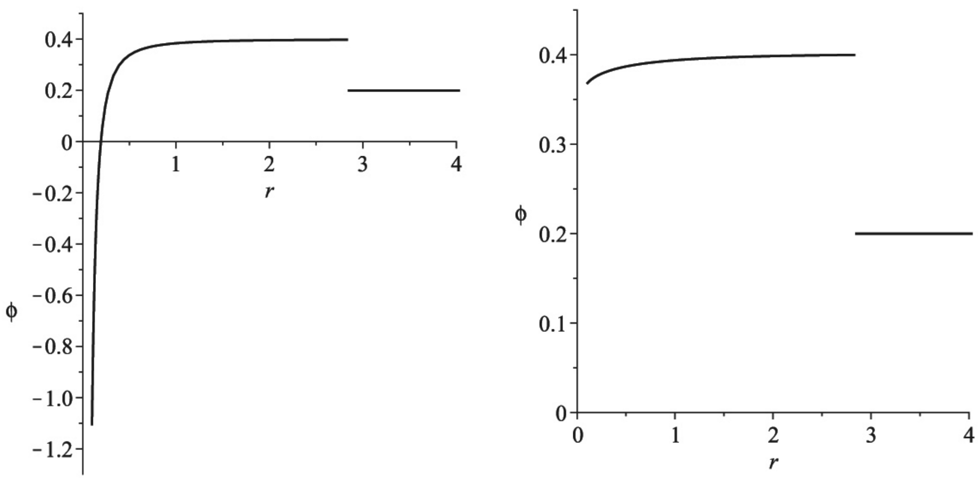
By using the described model we have investigated

- the evolution of the radius of the yielded zone $R(t)$,
- the cumulative solids production $S(t)$ and
- the porosity $\phi(r,t)$

for the following values of the model parameters:

$p_c = 4\times 10^7$, $p_e = 4\times 10^7$, $r_w = 0.1$, $r_e = 500$, $k = 10^{-14}$, $\mu = 10^{-3}$,
$\phi_y = 0.4$, $\phi_n = 0.2$, $\beta = 0.9$, $\alpha = 0.01;\ 0.1$, $\kappa(\phi_y) = 10^5$, $|\kappa'| = 10^7;\ 10^8$.

First we show the porosity ϕ as a function of r at time $t = 500$ days, computed by formula (7) from paper [1] (Fig. 2, left), and by the corrected formula (11)



by formula (7) by the corrected formula (11)

Figure 2: Porosity $\phi(r,t)$ for $t = 500$ days and $\alpha = 0.01$, $|\kappa'| = 10^7$.

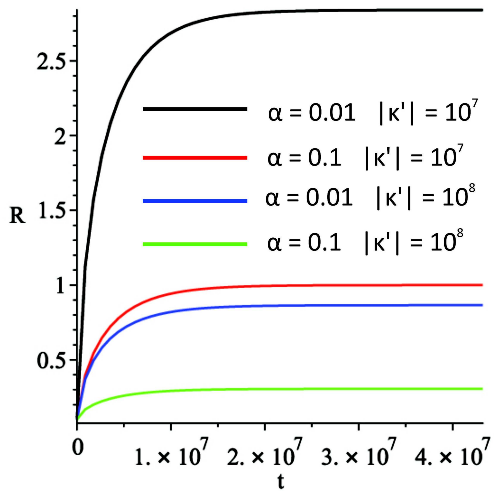


Figure 3: Evolution of the radius of the yielded zone for several values of α and $|\kappa'|$.

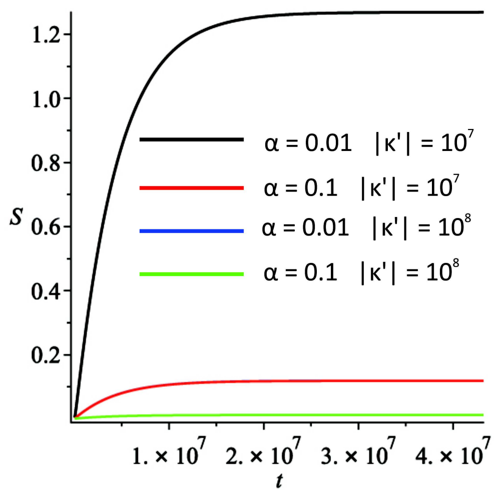


Figure 4: Cumulative solids production $S(t)$ for several values of α and $|\kappa'|$.

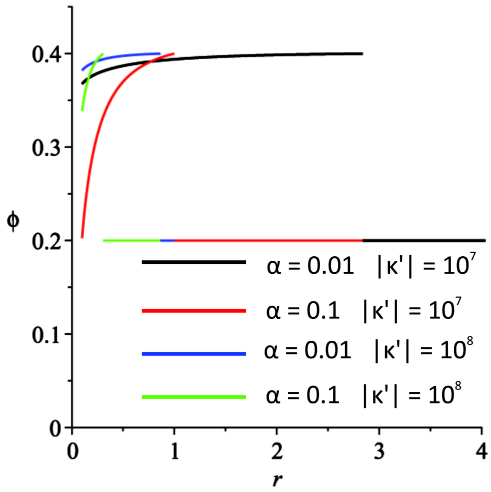


Figure 5: Porosity $\phi(r,t)$ for $t = 500$ days, for several values of α and $|\kappa'|$.

(Fig. 2, right). It can be seen that for small r, close to r_w, the porosity on Fig. 2, left, is negative.

The radius $R(t)$ of the yielded zone as a function of time is presented on Fig. 3, for four sets of parameters $\alpha > 0$ and $|\kappa'|$. It can be seen that the radius of the yielded zone increases at the initial time interval and then it reaches a steady-state value, thus the yielded zone stops growing. The steady-state value of the radius of the yielded zone depends on the model parameters.

The sand production $S(t)$ is obtained by using the computed radius $R(t)$ of the yielded zone and equation (1). It is presented on Fig. 4 for the same values of the parameters $\alpha > 0$ and $|\kappa'|$. The graphs of $S(t)$ for parameters $\alpha = 0.1$, $|\kappa'| = 10^7$ and $\alpha = 0.01$, $|\kappa'| = 10^8$ coincide within the plotting resolution.

Finally, the porosity is obtained from equation (11). It is shown on Fig. 5 for the same values of α and $|\kappa'|$. It can be seen that by using the new approximation (11) the porosity of the yielded zone is positive for $r \geq r_w$, moreover $\phi(r,t) \geq \phi_n$, which is physically realistic.

5. Conclusion

The model presented in [1] gives reasonable approximations for the radius $R(t)$ of the yielded zone and for the cumulative solids production $S(t)$, but it does not give physically reasonable results for the porosity $\phi(r,t)$. With the new approximation of the porosity, the numerical results are physically reasonable.

The described model and its computer implementation can be used to predict the radius of the yielded zone $R(t)$, the cumulative solids production $S(t)$ and the porosity $\phi(r,t)$ for various scenarios.

The model can be improved taking into account the sand erosion due to abrasion. This could be done by including extra terms in the equations of continuity:

$$-\frac{\partial \phi}{\partial t} + \operatorname{div}[(1-\phi)v_s] = -c(v_f - v_s)\frac{\partial \phi}{\partial t},$$

$$\frac{\partial}{\partial t}(\phi\, c) + \operatorname{div}[\phi\, c\, v_f] = c(v_f - v_s)\frac{\partial \phi}{\partial t},$$

where c is the sand concentration in the fluid.

Alternative possibility is to construct elastoplastic model for the rock behavior.

References

[1] M. B. Gelikman, M. B. Dusseault, F. A. Dullien, Sand production as a viscoplastic granular flow, Society of Petroleum Engineers 27343 (1994), 41–50.