# TRANSPORTATION OF A WATER BASED SLURRY IN AN OPEN FURROW, LAUNDER OR STREAM

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**Abstract**

The transport of large boulders in a furrow from a mining area to a nearby pond was considered. The furrow is filled with a mixture of water and soil particles flowing down to the pond at a very high velocity. Due to operating constraints, the slope of the furrow is reduced progressively. A formula is derived, relating the slope of the furrow and the composition of the fluid to the maximum size and shape of the transported boulders. The characteristics of the boulders carried all the way down to the pond may then be determined.

## 1 Introduction

A mineral processing plant produces vast amounts of waste material. This is made up of rocks of all sizes, ranging from very fine pebbles, with a diameter less than a millimetre, to rocks, with a diameter up to twenty centimetres. In the set-up presented at the study group, the waste is washed away with high pressure water and flows downhill to a pond three kilometres and approximately three hundred metres lower. As it flows the slurry carves a channel, typically one metre by one metre. Since the mining area, where the material

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is produced, is slowly moved away from the pond, the average angle of the furrow to the horizontal decreases progressively. As the angle decreases the possibility of larger rocks becoming stuck in the channel increases.

Every month, about one million tons of material is produced, most of it consisting of particles with a diameter less than a millimetre. They are washed away by water at the ratio 1000kg of water for 200kg to 400kg of material. Considering the size of the channel created by the flow, the flux of material leads to an average mixture velocity in the channel between $v = 1\text{m}\cdot\text{s}^{-1}$ and $v = 2\text{m}\cdot\text{s}^{-1}$. In these conditions, the flow is clearly turbulent, the Reynolds number is well above $Re = 10^6$.

Due to the high velocity, small particles should be carried away easily by the flow. This study therefore focuses on the largest boulders, with a diameter that may reach up to 20cm. In the following section, we determine a criterion for boulders to be washed away. The results depend on the Shields number, the ratio of shear stress and gravity, the two main forces applied on the boulder and the shape of the boulder. Numerical results are presented in Section 3.

## 2 Moving large boulders

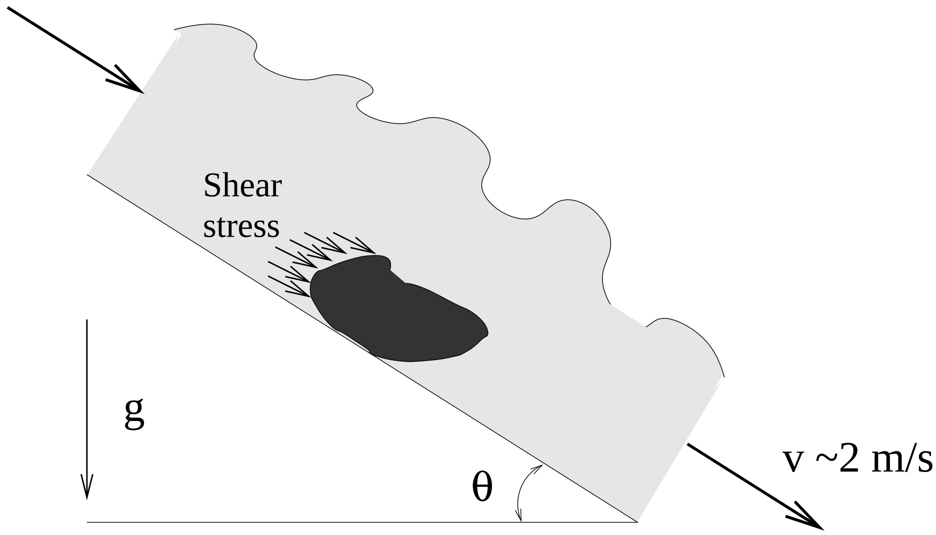


Figure 1: Typical configuration for large boulders.

Figure 1 shows a typical configuration for the transport of large boulders. The channel is inclined at an angle $\theta$ to the horizontal and the boulder lies at the bottom of it. The potential movement of the boulder is governed by two main forces:

1. Shear stress is applied on the upwards facing surfaces and pushes the boulder in the downstream direction.
2. Effective gravity, the difference between gravity and the Archimedes force.

The ratio between these two forces is summarised in a non-dimensional parameter called the Shields number:

$$S = \frac{\tau}{\Delta\rho g D} \ , \tag{2.1}$$

where:

- $\tau$ is the shear stress applied by the flow on the boulder,
- $\Delta\rho = \rho_b - \rho_f$ is the difference between the density of the boulder, $\rho_b$, and the density of the fluid, $\rho_f$,
- $g$ denotes gravity,
- $D$ is the diameter of the boulder.

Critical values of the Shields number may be found in tables. Below the critical value, gravity dominates and the boulder will remain motionless on the ground. When the Shields number exceeds the critical value, shear stress is the dominant force and movement is triggered. To calculate the minimum angle to the horizontal required to wash away boulders, the shear stress in Equation 2.1 will be expressed as a function of the flow characteristics, geometry of the channel, slope, particle concentration, density and dynamic viscosity, and the resulting expression will be combined with the critical value of the shear stress.

## 2.1 Expression for the Shields number

The average velocity of the particle and water mixture is such that the flow is highly turbulent. In this case, an appropriate expression for the shear stress applied to the boulder, $\tau$, is, see Patel (1999):

$$\tau = \frac{f}{8}\rho_f u^2 \ , \tag{2.2}$$

where

- $f$ is the friction factor. For the type of turbulent flow considered in this study, according to Patel (1999), the friction factor may be expressed as:

$$f = \frac{0.292}{Re^{1/4}} \quad \text{where} \quad Re = \frac{\rho_f u L}{\mu_f} \tag{2.3}$$

  and $Re$ is the Reynolds Number, $L$ is a typical length of the flow, $\rho_f$ and $\mu_f$ are the density and dynamic (or shear) viscosity of the mixture respectively. According to experimental observations, $L = 1$m is appropriate here. The density and dynamic viscosity vary significantly with the concentration of soil particles in the fluid mixture. The density of the mixture increases with particle volumetric concentration, $c$:

$$\rho_f = \rho_w + c(\rho_b - \rho_w) \ , \tag{2.4}$$

  where $\rho_w$ and $\rho_b$ denote the density of pure water and soil respectively, see Abraham *et al* (2001) and Guy *et al* (1990). The dynamic viscosity may be approximated by:

$$\mu_f = \mu_w \left(1 + \frac{0.75c}{0.605 - c}\right)^2 \ , \tag{2.5}$$

  where $\mu_w$ is the dynamic viscosity of pure water, see Coussot (1997). Equation (2.5) is valid for very concentrated solutions, up to $c = 0.45$. The average soil volumetric concentration in the fluid may be estimated around $c \approx 0.1$, therefore formula (2.5) is entirely appropriate here.

- $u$ is the velocity of the fluid. According to experimental observations, this may be estimated around 2 m$\cdot$s$^{-1}$. The velocity may be related to the slope of the channel using for example the Glaucker-Manning formula, see Chanson (1999), valid for fully rough turbulent flows:

$$u = \frac{1}{n} R_H^{2/3} \sqrt{\sin\theta} \ , \tag{2.6}$$

  where

  ◇ $n$ is the Manning coefficient. A typical value for earth surface roughness is $n = 0.025$. For gravels, the value is $n = 0.029$. The value for this parameter will therefore be taken as $n = 0.027$.

  ◇ $R_H$ denotes the hydraulic radius,

- ⋄ $\theta$ is the slope of the channel.

Combining Equations (2.2-2.6), the right hand side of Equation (2.1) may be rewritten:

$$S = \frac{0.292}{8gD}\frac{(\rho_w + c(\rho_b - \rho_w))^{3/4}}{(\rho_b - \rho_w)(1-c)}\left(\frac{\mu_w}{L}\right)^{1/4}\sqrt{1 + \frac{0.75c}{0.605 - c}}R_H^{7/3}\frac{(\sin\theta)^{7/8}}{n^{7/4}}. \quad (2.7)$$

The slope of the channel may then be expressed as:

$$\begin{aligned}\theta &= \arcsin\left[\frac{n^2}{R_H^{4/3}}\left(\frac{8gDS}{0.292}(\rho_b - \rho_w)(1-c)\right)^{8/7}\right. \\ &\quad \left.\times\left(\frac{1}{(\rho_w + c(\rho_b - \rho_w))^3}\frac{L}{\mu_w\sqrt{1 + 0.75c/(0.605 - c)}}\right)^{2/7}\right]. \quad (2.8)\end{aligned}$$

The Shields number, $S$, remains the only unknown quantity on the right hand side of Equation (2.8): the critical value of this parameter is required to determine when large boulders will move. It depends on the boulder, flow and fluid characteristics. This will now be detailed.

## 2.2 Critical Shields number

The value of the critical Shields number corresponding to the present setting may be found in Julien (1994):

$$S_c = 0.06\tan\phi_s. \quad (2.9)$$

This holds when the nondimensional particle parameter, $d_*$, satisfies:

$$d_* = D\sqrt[3]{\frac{\rho_f(\rho_b - \rho_f)g}{\mu_f^2}} > 50\ .$$

The angle $\phi_s$ is called the angle of repose. This is the critical angle of the channel floor at which a particle would move of its own accord, *i.e.* with no fluid motion, see Julien (1994). The angle of repose is zero for a sphere on its own and increases with the angularity of the boulder. Equation (2.9) is valid if the bottom of the channel is entirely covered with large boulders of diameter $D$. In the present situation the channel is likely composed of smaller particles, in which case Equation (2.9) will over estimate the critical Shields number.

The minimum slope of the channel required to transport large boulders may then be expressed as a function of the angle of repose and Equation (2.8) becomes:

$$\theta = \arcsin\left[\frac{n^2}{R_H^{4/3}}\left(1.64gD\tan\phi_s(\rho_b-\rho_w)(1-c)\right)^{8/7} \times\left(\frac{1}{(\rho_w+c(\rho_b-\rho_w))^3}\frac{L}{\mu_w\sqrt{1+0.75c/(0.605-c)}}\right)^{2/7}\right]. \quad (2.10)$$

This shows that the angle of the slope, $\theta$, required to move a large boulder increases with the angle of repose, $\phi$ and the boulder diameter, $D$. This is confirmed by the observations at the end of the channel (and common sense). A straightforward analysis of the formula shows that the angle $\theta$ decreases when the volumetric particle concentration in the flow increases. Note the volumetric concentration is the only parameter that may be adjusted in practice. A more quantitative analysis of the formula will now be performed.

# 3 Numerical results

Figure 2 shows the angle of the slope necessary to move boulders with a diameter varying up to $d$ = 20cm and angle of repose up to $\phi = 50^o$. As could be expected, the required angle of the slope for small particles is extremely low, whatever their angularity. When the diameter of the boulder increases, the effect of the angle of repose appears more clearly: low values of this parameter (corresponding to almost spherical boulders) do not require a high channel inclination. The more angular the boulder gets, *i.e.* the larger the angle of repose, the steeper the slope of the channel required to move it.

In the configuration presented at the study group, the channel is 3 kilometres long and the decrease in altitude is 300metres. The corresponding average angle of the slope is therefore $\theta_0 = 5.7^o$. In this configuration with a volumetric concentration $c \approx 0.1$, boulders with a diameter $d = 20$cm may only be transported if their angle of repose is lower than $\phi \approx 11^o$ *i.e.* they must be close to spherical but again, the critical value of the shear stress is over estimated and real values could be significantly higher. If the diameter is $d = 10$ cm, the allowed angle of repose increases to $\phi \approx 18^o$. However, due to the operating conditions, the volumetric concentration of particles in the fluid may vary significantly: the waste is piled up at the top of the hill and a

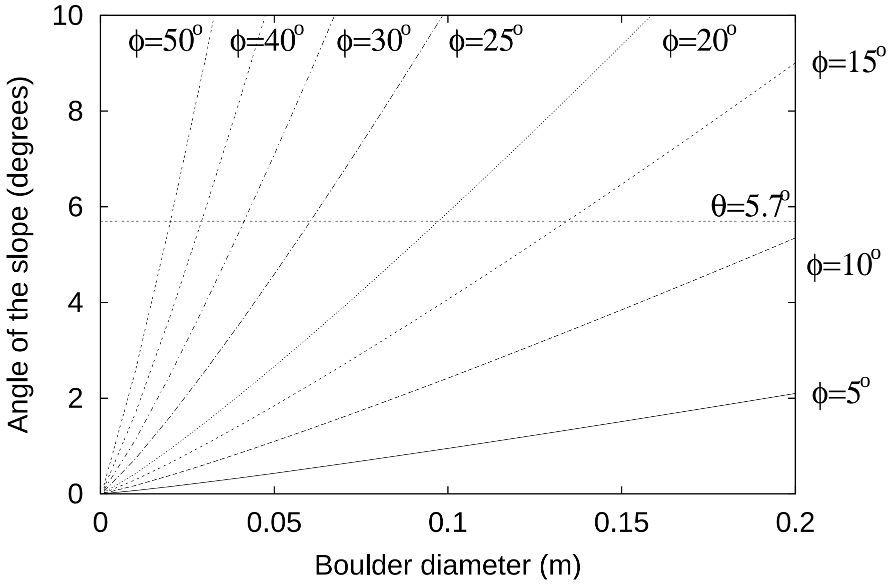


Figure 2: Slope angle, $\theta$, required to move a boulder as a function of its diameter and angle of repose, $\phi$, in a slurry with the particle volumetric concentration $c = 0.1$.

much larger quantity of material may suddenly be washed away, triggering movement for more angular boulders. For the channel slope $\theta_0 = 5.7^o$and volumetric concentration $c = 0.3$, a 20 centimetre diameter boulder may have an angle of repose up to $\phi \approx 16^o$. For a 10 centimetre diameter boulder, the angle increases to $\phi \approx 26^o$ and the boulder will still be washed away. These values may be read from Figure 3. Once movement has been triggered, the boulder's inertia allows it to keep on rolling down the hill, even if the starting conditions are no longer satisfied. The boulder may then either roll down to the pond or settle at the bottom of the channel for a while, until movement conditions are once again met, due to the varying volumetric concentration.

Practically, the model states that boulders with low angularity may reach the pond. This agrees with the experimental observations: the largest boulders reaching the pond are all rather round.

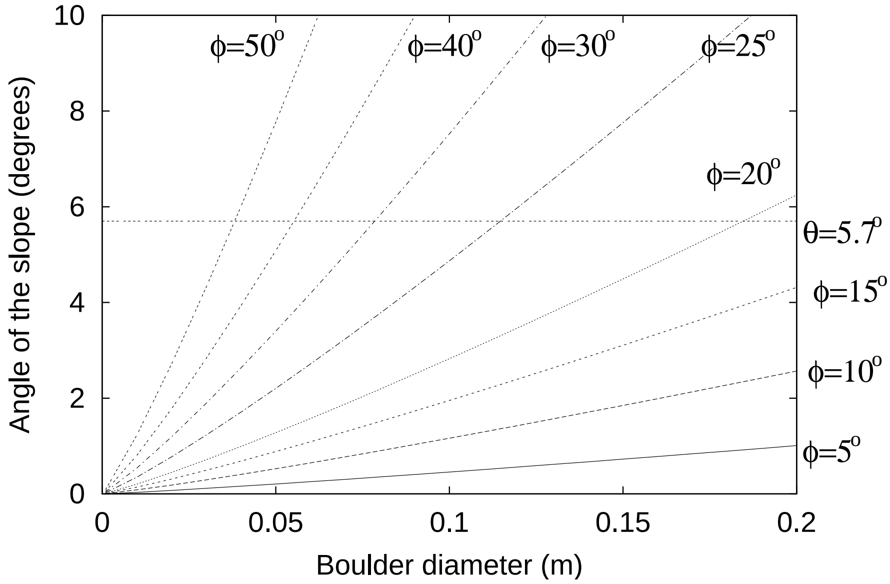


Figure 3: Slope angle, $\theta$, required to move a boulder as a function of its diameter and angle of repose, $\phi$, in a slurry with the particle volumetric concentration $c = 0.3$.

# 4 Conclusion

Turbulent flow in an open channel was studied to derive a formula relating the slope of a channel to the characteristics of the boulders the flow may wash away. This study is based on the critical value of the Shields number, which determines when the boulders to be washed away will start rolling down the channel. The study did not focus on the later course of the boulders: the condition derived here only indicates when movement is triggered. Inertia then allows boulders to continue moving even when the criterion is not verified. A geometrical study of the channel shape could complete the study and ensure that the boulders roll as far as possible

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