Oxford Study Group with Industry 1990, Birmingham University

STUDY GROUP REPORT - DOWTY PROBLEM

1. Information available at the Study Group

On behalf of Dowty Defence and Air Systems Ltd, Dr. Mark Butcher presented a problem to the study group concerning an instability which had been detected in a steam driven high speed turbine. The instability had led in some cases to destruction and threatened to greatly reduce the power and efficiency of the turbine. Dr. Butcher's main concern was to be able to predict the instability theoretically, identify the main controlling mechanisms for the instability and project these predictions to other, hypothetical turbine conditions. Detailed drawings of the turbine were provided, but the most important pieces of information seemed to be as follows :

(i) The turbine was roughly 20cm in length and 3.5cm in diameter. The turbine disc was about 10cm in diameter. Under the envisaged operating conditions the turbine was expected to run at speeds in excess of 100,000 RPM.

(ii) The turbine shaft was mounted on a pair of angular contact bearings which were axially pre-loaded by springs of a given stiffness. The bearings were of standard ball race type (but see comments below).

(iii) At a shaft speed of about 45,000 RPM the first 'whirling' mode of the shaft was observed. Bending of the shaft between the bearings was discerned, but there was no sign of axial motion and the turbine could be driven through this instability. However, at approximately 90,000 RPM large amplitude axial vibrations ('chattering') of the shaft were noted. This is the potentially destructive instabilty, and all attempts to drive the turbine through the regime had proved unsuccessful.

(iv) A considerable number of experimental measurements had been performed using an instrumented rig. One of the key elements of the problem appeared to be the fact that, at the onset of the chattering instability, the circular motion of the centre of mass of the shaft due to whirling took on a pronounced elliptical character. It appeared from conversations with Dr. Butcher that the appearance of this elliptic shaft centre motion served to differentiate the chattering instability from all the other possible instabilities in the shaft motion.

(v) From the experiments which had been carried out, it seemed that there was only one practical way of influencing the onset of the chattering instability. This consisted of changing the axial preload which was applied to the springs at the opposite end of the shaft to the disc. An increase in effective spring stiffness was seen to lead to an increase in the rotational speed at which the onset of chattering occured, but the relationship appeared to be a nonlinear one.

2. Summary of models proposed and work carried out at the Study Group

Many different models were proposed at the Study Group, and most sought simply to find a physical mechanism for the chattering instability. In fact it proved hard to find such a mechanism and personally I am still not happy that the basic cause of the instability is understood.

One of the first models to be put forward was proposed by Professor P. Parks from Shrivenham. His idea was based on an examination of the out-of-plane tilting of the heavy turbine disc and consequent bending of the turbine shaft at its centre. The out-of-plane disc tilting was governed by the standard Euler equations for rigid body rotations and included couples produced by an assumed imbalance of the disc. When linearized for small disc tilt angles θ the result was a set of ordinary differential equations of the form

$$A_1\ddot{\theta} + A_2\ddot{\phi} + A_3\dot{\theta} = -m\ddot{x}r\sin\omega t$$

$$B_1\ddot{\phi} + B_2\dot{\phi} + B_3\dot{\theta} = -m\ddot{x}r\cos\omega t$$

where ϕ denotes the angle of rotation of the disc, x the axial displacement of the disc centre, and r the offset of the true centre of gravity of the disc from the centre of the disc. Clearly these equations provide a mechanism for producing axial vibrations of the sort observed, and much discussion took place concerning the addition of extra terms representing other effects. However it was noted that the experiments which Dowty had carried out had been extremely accurate and showed conclusively that the disc was not subject to any measurable tilt. Indeed, Dr. Butcher's opinion was that its weight tended to stabilize it. This experimental evidence should therefore be regarded as fatal to the above model.

The fact that chattering took place at roughly twice the rotational speed of the first whirling frequency then became a topic of discussion. It was clear at this point that some analysis of the basic whirling frequencies of the shaft was required, and David Parker and Graham Mc.Cauley produced a model (they have the details) of the whirling and bending shaft which depended on writing down a rather complicated Lagrangian for the motion. Some mistakes were made in the non-dimensionalization and insertion of the correct numerical parameters, but a reconsideration of this model predicted that (as far as I know - there may have been further reconsiderations !) the first rotational speed of whirling would be at about 45,000 RPM and the next at 180,000 RPM. The suggestion from this model is therefore that the second whirling frequency is not connected to the chattering.

A further mechanism was proposed for the chattering. This relied on the fact that whilst the shaft was whirling, its centre of mass described a circle so that the effective length of the shaft was constant. Assuming that this whirling was subject to unstable amplification at certain rotational speeds, so that elliptic motion would be produced, it was noted that this would lead to a change in the effective length of the shaft which may provide the mechanism for producing a resonance when coupled with the motion of the springs attached to the bearings. It is important to note however that resonance must be achieved for this to explain the chattering, as the amplitude of axial vibrations which could be set up due only to the bending of the shaft from elliptical motion is an order of magnitude too small to account for the chattering. A very simple model of the form

$$M\ddot{x} + kx = \epsilon \cos \Omega t \tag{1}$$

was proposed by considering the whole system simply to be attached to the bearing springs. Here k is the stiffness of the bearing spring, ϵ corresponds roughly to the eccentricity of the ellipse, Ω is the frequency of effective shaft shortening, which is related in an obvious way to the transit time around the ellipse, and M is the mass of the shaft (or the mass of the shaft and disc assembly, depending on ones personal interpretation of this very simple model). As usual, resonance takes place when

$$k = M\Omega^2 \tag{2}$$

but although it is possible to argue about the correct values for M and Ω one fact is quite clear - an increase in k from 261 to 2600 (the two values used in the Dowty experiments) increased the chattering instability onset from approx. 72,000 RPM to 98,000 RPM - nothing like the factor of $\sqrt{10}$ which is predicted by (2).

Some discussion also took place concerning a modification of the above model which took the form of an impact oscillator. The physical basis for this was the fact that under some circumstances it seemed possible that the shaft was not constrained by the springs. Apparently some numerical calculations were performed which gave encouraging numbers. (P. Wilmott has details ?)

My conclusion from the work carried out at the Study Group was therefore that although many mechanisms and models had been considered, further work was still required and no concensus had been reached.

3. Information received after the Study Group

After the Study Group at Birmingham, a meeting was held at Oxford on Friday 11th May to

further consider the problem. At this meeting, some further information came to light which may materially affect the theoretical approach which must be taken. The important new facts which were revealed were;
(i) The ball races in the angular contact bearings are themselves rotating with a significant angular velocity. Typically the order of magnitude of this velocity is half the angular speed of the shaft.
(ii) The whole turbine assembly actually includes an alternator mechanism which rotates with the speed of the shaft. This mechanism is not in direct lateral contact with either the shaft or the outer housing, but is connected to the shaft via a series of gears. This may provide a whole new mechanism for the production of a 'chattering' instability.
(iii) The first whirling mode described in (iii) above did not suddenly appear at 45,000 RPM as we had thought, but was in fact observed very soon after start-up of the turbine and gradually increased in amplitude until a peak was reached at about 45,000 RPM. P. Wilmott and J. Ockendon were rumoured to be considering a slowly varying frequency driving the turbine through this instability in order to determine orders of magnitude for the amplitudes which might be expected.

4. Potential for Further Work

There is no doubt in my mind that the problem still requires a good deal of work before the important mechanisms and models are for the observed behaviour are properly understood. This work would have to begin with a collection of all the models produced at the Study Group (Prof. Tryfan Rogers has all the relevant bits of paper) and especially a re-examination of all the information which is relevant to the Parker and Mc.Cauley model. I would envisage that the next step would be to make a more realistic model of the situation modelled in (1) and see if any realistic numbers could be produced. My opinion at this stage is that the ball race and the alternator are not crucial to the instability (with no alternator the instability was still observed - it is not possible to coduct experiments with no ball race !) and that the mechanism modelled by (1), perhaps with allowances made for impact oscillator type behaviour is the most likely candidate.

ADF, Southampton University 26/5/90

Dynamic Instabilities in Turbines - the Dowty problem

Introduction

An up-to-date diagrammatic description of the shaft and bearings relevant to the Dowty turbine problem has already been given by Mark (2.4.90), together with a "definitive" version of the instability observations and a chronological account of the Study Group's search for an explanation of the axial "chatter" at 79,000-95,000 rpm.

Apart from a general consensus that the problem needed substantial further work, it appeared that the Parker et al model merited further investigation, as also did the Wilmott et al. This report concentrates on the former (Parker) analysis, from which some conclusions (speculations?) may be drawn.

Notation (also refer figure)

s = arc length along the central axis of the shaft with $s = 0$ at the 'fixed' bearing (the 'disc' end) and $s = \ell$ at the 'sprung' bearing.

ℓ = length of shaft ~ 0.114 m.

t = time (one revol^n ~ 7.5×10^{-4}s)

A = cross-sectional area ~ 0.00045 m^2

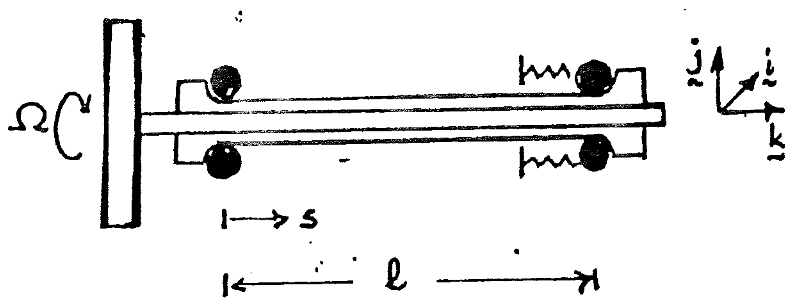
I = (polar) second moment of area of shaft ~ $3.3 \times 10^{-8} m^4$

C = polar moment of inertia of disc ~ 0.85×10^{-3} kg.m^2

Ω = angular speed of shaft ~ 8×10^4 rpm ≡ 8400 s^{-1}

ρ = density ~ 8×10^3 kg m^{-3}

E = Young's modulus of shaft material ~ 1.86×10^{11} Nm^{-2}



Governing equations

We denote the position of the central axis at s and at time t by

$$\underline{r} = x\underline{i} + y\underline{j} + (s+w)\underline{k}$$

so that x, y and w are its displacement components. The unit tangent vector $\underline{e}$ is thus parallel to $\underline{r}'$ ($\equiv \partial \underline{r}/\partial s$) with

$$\underline{r}' = x'\underline{i} + y'\underline{j} + (1+w')\underline{k}$$

The angular velocity of the cross-section at s is

$$\underline{\omega} = \Omega\underline{e} + \dot{\theta}\underline{i} + \dot{\varphi}\underline{j} \qquad (\dot{\theta} \equiv \partial\theta/\partial t, \text{ etc.})$$

where

$$\dot{\theta} \sim -\dot{y}' \ , \ \dot{\varphi} \sim \dot{x}' \ .$$

Hence the kinetic energy T per unit length is

$$T = \tfrac{1}{2}\rho A(\dot{x}^2+\dot{y}^2+\dot{w}^2) + \tfrac{1}{4}\rho I\{(\dot{x}')^2+\dot{y}')^2 + 2(\Omega+\dot{x}'y'-x'\dot{y}')^2\} \qquad (1)$$

For the potential energy V per unit length we first note that the shaft is effectively inextensional, since any extensional wave will traverse the shaft and back in $2\ell\rho^{\frac{1}{2}}/E^{\frac{1}{2}} \sim 0.45 \times 10^{-4}$s whilst the shaft performs one revolution in approx. 7.5×10^{-4}s. This inextensional condition gives

$$(x')^2 + (y'^2) + (1+w')^2 = 1 \qquad (2)$$

and introduces a Lagrange multiplier S ($\equiv$ tensile load in the shaft) into the potential energy:

$$V = \tfrac{1}{4}EI \ \{(x'')^2 + (y'')^2\} + \tfrac{1}{2}S\{(x')^2 + (y')^2 + (w')^2 + 2w'\} \qquad (3)$$

Substituting for L = T − V in Lagrange's equations of motion

$$\left[\frac{\partial L}{\partial x''}\right]'' - \left[\frac{\partial L}{\partial x'}\right]' = \frac{\partial}{\partial t}\left\{\frac{\partial L}{\partial \dot{x}} - \left[\frac{\partial L}{\partial \dot{x}'}\right]'\right\}$$

gives

$$\tfrac{1}{2}EIx'''' + \rho A\ddot{x} - 2\rho I\Omega\dot{y}'' - \tfrac{1}{2}\rho I\ddot{x}''$$

$$= \left[Sx'\right]' - \rho I \ \left[\dot{y}'(x'\dot{y}'-\dot{x}'y') + \frac{\partial}{\partial t}\left\{y'(x'\dot{y}'-\dot{x}'y')\right\}\right]' \qquad (4)$$

$$\tfrac{1}{2}EIy'''' + \rho A\ddot{y} + 2\rho I\Omega\dot{x}'' - \tfrac{1}{2}\rho I\ddot{y}''$$
$$= (Sy')' - \rho I\left[\dot{x}'(y'\dot{x}'-\dot{y}'x') + \frac{\partial}{\partial t}\left\{x'(y'\dot{x}'-\dot{y}'x')\right\}\right]' \qquad (5)$$

$$\left\{S(1+w')\right\}' = \rho A\ddot{w} \qquad (6)$$

Equations (2), (4), (5) and (6) are the four differential equations governing the four unknown functions, x, y, w and S.

Along the shaft ($0 < s < l$), $x(s,t)$ and $y(s,t)$ are governed approximately by the linearised equations obtained from (4) and (5); in each the two leading terms are the dominant ones for the present problem. However, the Coriolis terms ($(\pm 2\rho I\Omega)$) introduce coupling between x and y, and show that modes rotating with, and those rotating counter to, the shaft rotation have (slightly) different natural frequencies.

The axial load $S(s,t)$ introduces restoring forces into (4) and (5). Its time dependence is coupled to any axial vibrations through (6). A resonance phenomenon will arise if these axial vibrations are themselves related to the lateral displacements (x, y). From (2) we see that to leading order w' is zero, i.e.

$$w \sim W(t) \qquad (W(0) = \dot{W}(0) = 0) \qquad (7)$$

so that (6) gives

$$S = S_0(t) - \rho A(s-\ell)\ddot{W}(t) \;, \quad 0 < s \leqslant \ell \;. \qquad (8)$$

Here $S_0(t)$ is the load at the (sprung) bearing at $s = \ell$, and is related to the preload P and the instantaneous length of the springs:

$$S_0(t) = P + f(W(t)) \qquad (9)$$

where f describes the non-linear dependence of the spring stiffnesses on the applied load.

If there is a mechanism whereby oscillations in x and y produce oscillations in W(t), then (8) and (9) show that S(s,t) becomes oscillatory, so implying possible quadratic forcing in equations (4) and (5). Once such oscillations produce $W < 0$ at any stage, $t = t_0$ say, then the bearing at $s = 0$ has some lateral freedom. Assuming, for simplicity, that the bearings are conical with angle $\bar{\alpha}$ then

$$S(0,t) = 0 \quad \text{for } x^2(0) + y^2(0) < (\tan^2\bar{\alpha})W^2(0)$$

$$\text{for } t > t_0 ,$$

$$S(0,t) > 0 \quad \text{for } x^2(0) + y^2(0) = (\tan^2\bar{\alpha})W^2(0)$$

until W becomes positive again. This highly non-linear behaviour could lead to the "chattering" phenomenon, and is the subject of the Willmott, Ockenden et al. investigation.

It has also been observed that the geometry of the bearings can relate the lateral displacements (x,y) of the shaft to its deflections (x',y'), though this is a linear effect and probably gives a negligible adjustment to the whirling modes. More important is the possibility (DFP) that the angular contact bearings induce a coupling between the lateral reaction at s = 0 and the axial load $S_0 - \rho A \ell \ddot{W}$ there. Finally, there is the coupling between the shaft ($0 < s < \ell$) and the rotating disc at $s = -\bar{\ell}$, a short shaft-length $\bar{\ell}$ the other side of the fixed bearing at s = 0; Parker and McCauley deduced that this implied a linear relation between the slope and the curvature of the shaft at s = 0.

All the above possibilities (and no doubt others) imply that the conditions at the two bearings s = 0 and s = ℓ merit further careful treatment.

Simplified theory

Linearisation of the governing equations (2), (4), (5), (6) leads to the equations

$$x_{ssss} + x_{tt} - \bar{\Omega} y_{tss} - \alpha x_{ttss} - \bar{S} x_{ss} = 0$$

$$0 < s < 1 \qquad (10)$$

$$y_{ssss} + y_{tt} + \bar{\Omega} x_{tss} - \alpha y_{ttss} - \bar{S} y_{ss} = 0$$

where all the lengths x, y, s have been non-dimensionalised with respect to the shaft length ℓ, and time t with respect to $t^* = (2\rho A \ell^4/EI)^{\frac{1}{2}}$; the boundary conditions are

$$x = y = 0,\ x' = \gamma x'' ,\ y' = \gamma y'' \quad \text{at } s = 0 \qquad (11)$$

$$x = y = x'' = y'' = 0 \quad \text{at } s = 1 \qquad (12)$$

Here α, $\bar{\Omega}$ and $\bar{S}$ are dimensionless parameters defined by

$$\alpha = \frac{I}{2A\ell^2}, \quad \bar{\Omega} = 2\left[\frac{2\rho I}{EA}\right]^{\frac{1}{2}} \Omega, \quad \bar{S} = \frac{2S_0\ell^2}{EI} \qquad (13)$$

whilst γ is the coupling constant derived by DFP/GMcC assuming a single simple whirling mode of angular frequency ω as

$$\gamma = \frac{EI}{C\Omega \ell \omega} \tag{14}$$

It is important to note the dependence on the whirling frequency ω, which is obtained by looking for non-trivial solutions of (10)–(12) in the form

$$x = X(s)\cos\omega t \ , \quad y = Y(s)\sin\omega t \ .$$

Then

$$\begin{bmatrix} X \\ Y \end{bmatrix}'''' + (\alpha\omega^2 - \bar{S}) \begin{bmatrix} X \\ Y \end{bmatrix}'' - \omega^2 \begin{bmatrix} X \\ Y \end{bmatrix} = \omega\bar{\Omega} \begin{bmatrix} Y \\ X \end{bmatrix} \tag{15}$$

subject to

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \underset{\sim}{0} \ , \quad \begin{bmatrix} X \\ Y \end{bmatrix}' = \gamma \begin{bmatrix} X \\ Y \end{bmatrix}'' \text{ at } s = 0 \ , \quad \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}'' = \underset{\sim}{0} \text{ at } s = 1 \ .$$

Clearly $X = Y$ and $X = -Y$ satisfy these equations, which therefore uncouple as

$$z_i'''' + 2\delta_i z_i'' - \omega^2 z_i = 0 \ , \qquad (i = 1, 2) \tag{16}$$

with

$$z_i(0) = z_i(1) = z_i''(1) = 0 \ , \quad z_i'(0) = \gamma z_i''(0) \tag{17}$$

where

$$z_1 = X + Y \ , \quad z_2 = X - Y$$

and

$$2\delta_i = \alpha\omega^2 - \bar{S} + (-1)^i \omega\bar{\Omega} \ . \tag{18}$$

The solution is

$$z_i = A_i \sin(s-1)\beta_i - B_i \sinh(s-1)\, k_i \tag{19}$$

where

$$\beta_i^2 = (\omega^2+\delta_i^2)^{\frac{1}{2}} + \delta_i \ , \quad k_i^2 = (\omega^2+\delta_i^2)^{\frac{1}{2}} - \delta_i \tag{20}$$

and

$$\frac{A_i}{B_i} = \frac{\sinh k_i}{\sin \beta_i} = \frac{k_i}{\beta_i} \frac{(\cosh k_i + \gamma k_i \sinh k_i)}{(\cos\beta_i - \gamma\beta_i \sin\beta_i)} \tag{21}$$

(21) can be rearranged to show that ω^2 must be such that

$$\beta_i \cot\beta_i = k_i \coth k_i + \gamma\,(k_i^2+\beta_i^2) \qquad (i=1,2) \tag{22}$$

For the parameter values appropriate to the Dowty turbine,

$$\alpha \sim 0.003 \ , \quad \bar{\Omega} \sim 0.042 \ , \quad \bar{S} \sim 0.003$$

so that $\delta_1 \sim -\delta_2$ and both co- and counter-rotating modes have the same behaviours. Furthermore since δ_1 , δ_2 are small, then

$$\left|\delta_i\right| \ll \omega \Rightarrow \beta_i \sim k_i \tag{23}$$

and the roots of (22) are close to those of

$$\cot \beta = \coth \beta \pm 2\gamma\beta \ . \tag{24}$$

The coupling γ of slope to bending moment tends to zero as Ω increases; if it is neglected (equivalent to imposing a "built-in" condition at the "fixed" end) then the lowest resonances are close to the lowest non-zero root of $\cot \beta = \coth \beta$, i.e. $\beta \sim 1.25\pi$, which corresponds to a "physical" value of $f_0 \sim 45{,}000$ rpm.

If we denote the corresponding frequencies determined by (22) as ω^+ and ω^- , then the whirling solution is

$$\begin{bmatrix} x \\ y \end{bmatrix} = C^+ \begin{bmatrix} \cos\,(\omega^+ t+\varphi^+) \\ \sin\,(\omega^+ t+\varphi^+) \end{bmatrix} z_1(s) + C^- \begin{bmatrix} \cos(\omega^- t+\varphi^-) \\ \sin(\omega^- t+\varphi^-) \end{bmatrix} z_2(s)$$

Comments

The non-linear terms neglected for (10) will cause gradual change of the amplitudes C^+ and C^- and phases φ^+ and φ^- . Also if there is any forcing having a component close to the frequencies ω^+ or ω^- then a large response C^+ or C^- should be expected.

The forcing frequency has a harmonic midway between ω^+ and ω^- , and a dangerous regime could be its second harmonic $\Omega = \omega^+ + \omega^- \sim 90{,}000$ rpm ! However, to induce this resonance some quadratically non-linear effect must be important. This has been discussed in an earlier section, but it should also be observed that any irregularity in the ball-race could excite the amplitude of the ω^+ mode. This is because the ballrace rotates at frequency $\sim \frac{1}{2}\Omega$, and if $\Omega = 2\omega^+$ the modeshape rotates at $\omega^+ - \Omega = -\omega^+$ relative to the shaft, just the same as the ballrace! In the Dowty case, $2\omega^+ \sim 90{,}000$ rpm ....

DFP/TGR/GMcC

P.S.

In rechecking the calculations of parameters it was found that the value of $t^* = 4.46 \times 10^{-4}$ sec gives $f_0 \sim 330{,}000$ cycles/min. This appears quite unrelated to observations and design theory. No error in the calculations has yet been traced. Modification of the boundary conditions (11), (12) seems unlikely to provide sufficient change in β or ω. To resolve this discrepancy, comparison with the original design calculations for the whirling modes of the shaft may be called for.

DFP/TGR

Report on the Dowty Turbine Vibration Problem

Professor P.C. Parks, Applied and Computational Mathematics Group, RMCS, Shrivenham, Swindon, SN6 8LA.

1. INTRODUCTION

1.1 It was an unfortunate feature of this study that much important information did not become available until the second meeting held on 11 May 1990 in Oxford. This information, had it been available during the Study Group week in Birmingham (26-30 March), would have prevented a number of false trails being persued at that time.

1.2 A particularly difficult feature of this vibration problem is the presence of axial vibrations as opposed to lateral vibrations. The voluminous literature on shaft vibration and whirling problems is almost entirely concerned with lateral vibrations of rotating shafts and their bearings. However, in the present Dowty problem it appears that it is axial vibrations that are responsible for severe damage or destruction of the bearings and shaft of the turbine. A key problem is therefore to identify a mechanism which can couple lateral and axial motions of the shaft.

2. GEOMETRICAL CONSIDERATIONS

As it appeared that shaft bending was associated with the axial vibration problem, it seemed natural to consider possible changes in length of the shaft due to lateral bending. To obtain orders of magnitude one can consider the assumed mode shown in Fig 1 in which the dimensions only roughly approximate the real shaft.

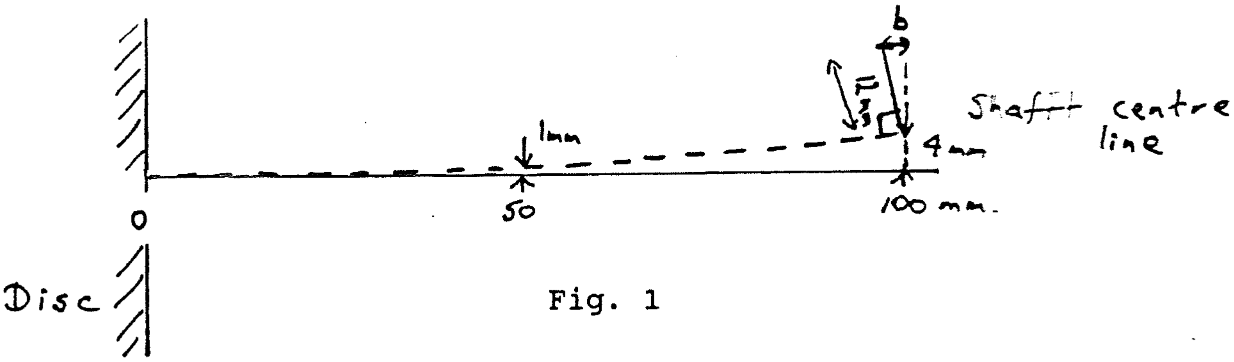


Fig. 1

If a quadratic bending mode $y = x^2$ is assumed the second order change in length is

$$\frac{1}{2}\int_0^{\ell} \left(\frac{dy}{dx}\right)^2 dx.$$

For a lateral displacement of 1 mm at the centre of the shaft (as shown in Fig 1) the second order change of length is about 1/10 mm. However, b, the displacement axially at a radius of 12 mm, shown in Fig 1 is about 1 mm.

This suggests that an axial displacement of the same order of magnitude as the lateral displacement at the shaft centre point could be the result of shaft bending and contact at a certain distance out from the shaft centre line. This distance (12 mm) has been taken to be the approximate radius of the ball bearing race.

Other simple mode shapes may be assumed instead of $y = x^2$ assumed in Fig 1, but rather similar results will be obtained regarding the relative magnitudes of the various displacements.

The experimental results suggest a lateral displacement of 0.1 mm (peak-to-peak) compared with an axial displacement of 0.2 mm (peak to peak).

3. FREQUENCIES

Important clues could be found from a knowledge of resonant frequencies of the shaft and turbine disc, including both the fundamental and higher bending modes. These frequencies were not known except the whirl frequency at about 45 krpm, which one assumes is of the form shown in Fig 2. As instabilities were observed at higher frequencies it is assumed higher modes are involved eg. a tentative shaft bending mode shown in Fig 3. This could cause an axial displacement, b, as was proposed in section 2.

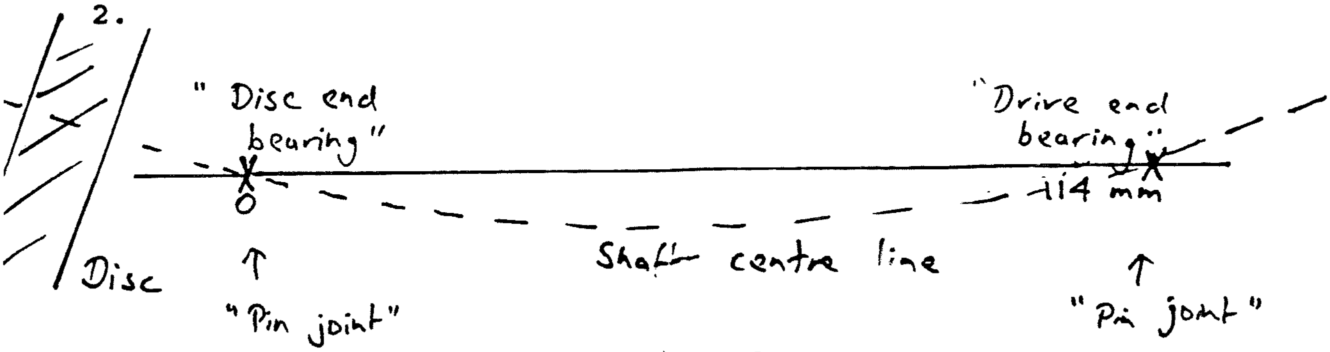


Fig. 2

Careful examination of frequencies (by counting peaks) in Fig 3 of the Dowty handout instability at 76.8 krpm reveal accurate frequencies of 1270 Hz (for shaft rotation) and 485 Hz for the axial vibration. This later is not a neat sub-multiple of 1270, and certainly not ½ as had been suggested in sheet FSD/CCTS/09/5135 (January 1990). It is nearer ⅓ (in fact 1/2.619).

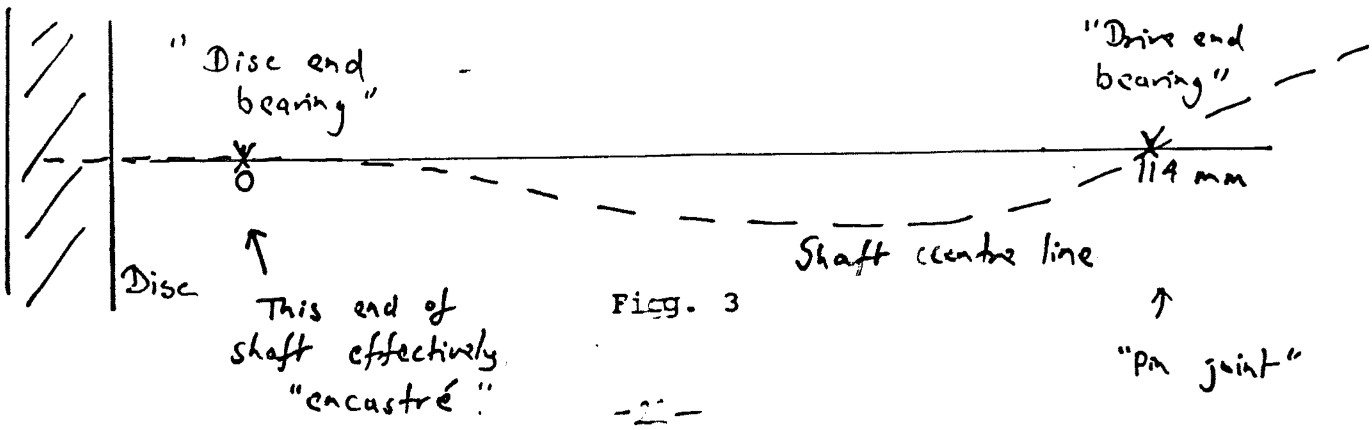
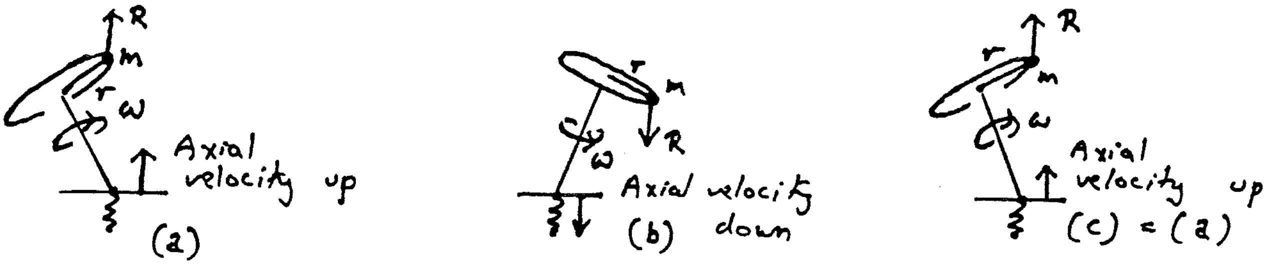


Fig. 3

4. **FALSE TRAILS (DUE TO P.C.P)**

4.1 Axial coupling with out of balance mass.



R = axially resolved component of centrifugal force ($mr\omega^2$)

Fig. 4

It is possible to envisage a coupling of an out-of-balance mass m as shown in Fig 4 which generates a centrifugal force. The axially resolved component of this couples with an assumed axial motion in such a way that positive work is done during the cycle at (a) = (c) and (b). The shaft rotation is <u>double</u> the axial vibration frequency. The differential equations describing the motion have periodic coefficients and are therefore of the same type as the celebrated Mathieu equation which has various instability regions (of a complicated nature). It was envisaged in the presentation at Birmingham that the imbalance would be in the turbine disc itself but equally well the imbalance could be in the shaft at the "drive end" and the mode could be as in Figs 1 or 3. (This theory had been ruled out at Birmingham as the disc itself appeared to move only axially and not as shown in Fig 4). A further objection is that the centrifugal force at 95 krpm with a shaft unbalance of 0.06 gm mm is only some 6 N - and the resolved component is insufficient to overcome the axial preload of 1000 N, but could a lightly, damped axial resonance be excited? The resonant frequency of the disc and shaft mass and the axial spring stiffness (taken as 2626 N/mm) would appear to be too low (225 Hz approx.)).

In Birmingham a "Meccano" model had been constructed to demonstrate this phenomenon, but the rotor could not be rotated fast enough to excite this axial vibration.

4.2 <u>Hydrodynamic oil pressure</u>

Another possible means of generating axial forces seemed to be via the centrifugal pressure gradient due to an angular velocity ω (rad/s) generated in the oil (of density ρ) filling the bearings. Integrating outwards from 0 to r a possible axial force of $\frac{1}{4}\pi\rho\omega^2 r^4$ is obtained. This can produce forces of 1000N at radii of 12 mm sufficient to counteract the spring pre-load. However, this idea seems to be ruled out by the nature of the lubrication system in which oil bleeds out into the bearings so that the rotating mass of oil assumed in this theory does not in fact exist.

5. CONCLUSIONS

It seems that much effort by 7 or 8 people at the Dowty Group failed to produce a convincing explanation of this axial vibration. The remaining possibilities seems to be a non-linear investigation along the lines of Zalik's paper on the Jeffcott equations (R.A. Zalik, Quart. of App. Maths 47, No.4 585-599, Dec. 1989), or a more detailed examination of the dynamics of the interaction of the shaft, ball races and axial tensioning springs. Could an axial displacement such as b in Figs 1 and 3 cause a virtual point displacement on the circular springs used to provide axial tension? Could the rotating point contact then set up a resonance in the spring - (Fig 5)?

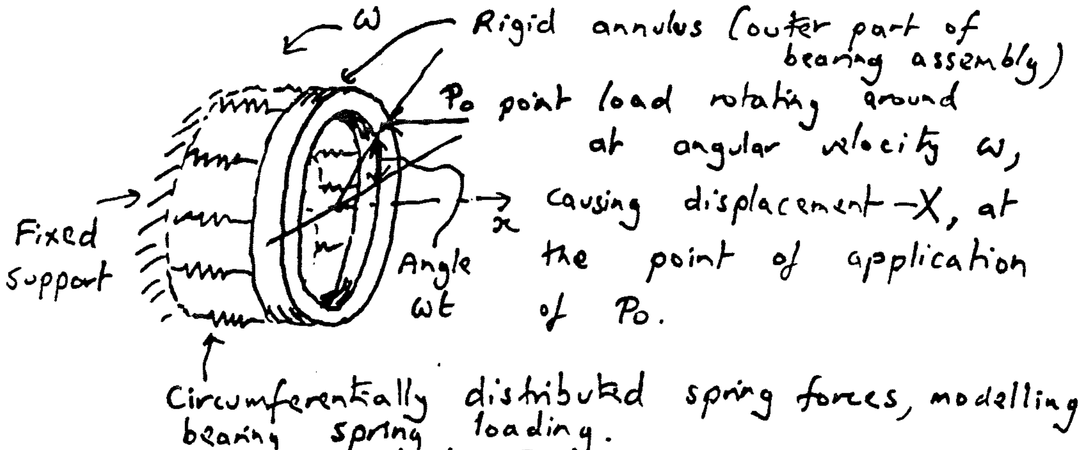


Fig. 5

Such an idea may be rather far-fetched, but, in the absence of more simple explanations, may be worth more detailed study. The appendix makes a start on this study. It should be noted also that there exist papers in the mechanical engineering literature which take into account the dynamical behaviour of the balls in the ball-race - each ball is treated as a minature gyroscope. This kind of detailed analysis is, of course, something that would take weeks, rather than days, to develop fully.

Professor P.C. Parks

APPENDIX

Fig 6 shows a model of a ring representing the outer part of the spring loaded bearing supported by a uniform circumferential distribution of springs.

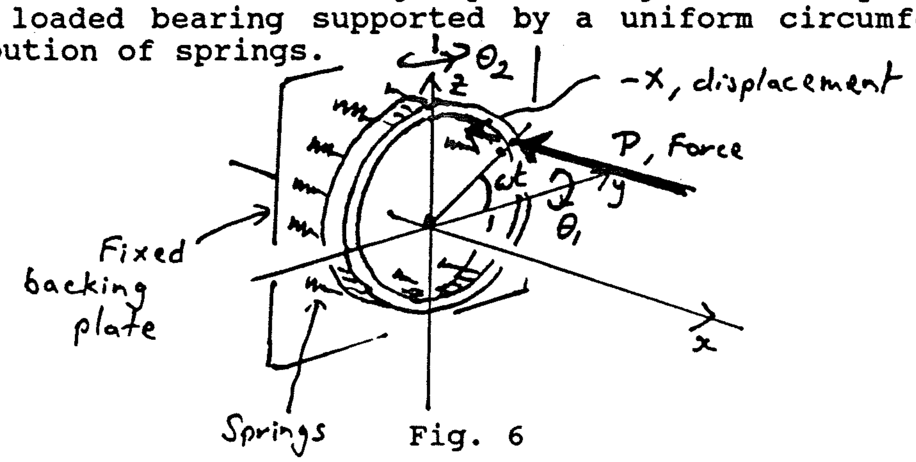


Fig. 6

A moving point load P giving rise to a displacement -X is considered. An axial displacement of the ring plus small angular displacements θ_1 and θ_2 about the y- and z- axes make up the total ring motion.

The equations of motion of the ring are with zero damping

$$
\begin{aligned}
m\ddot{x} + Kx &= -P \\
I\ddot{\theta}_1 + k\theta_1 &= -Pr\sin\omega t \\
I\ddot{\theta}_2 + k\theta_2 &= Pr\cos\omega t \\
X &= x + r\sin\omega t\,\theta_1 - r\cos\omega t\,\theta_2
\end{aligned}
$$

Now if k* = stiffness per unit distance around the circumference

$$k = \int_0^{2\pi} k^* r^3 \sin^2\theta\, d\theta = \pi k^* r^3$$

$$K = \int_0^{2\pi} k^* r\, d\theta = 2\pi k^* r$$

and $k = \dfrac{r^2 K}{2}$.

Assume that $I = \dfrac{mr^2}{2}$ then

$$
\left.
\begin{aligned}
m\ddot{x} + Kx &= -P \\
r(m\ddot{\theta}_1 + K\theta_1) &= -2P\sin\omega t \\
r(m\ddot{\theta}_2 + K\theta_2) &= 2P\cos\omega t
\end{aligned}
\right\} \qquad \text{(A1)}
$$

and $X = x + r\sin\omega t\ \theta_1 - r\cos\omega t\ \theta_2$

If P is constant we obtain

$$X = -\frac{P}{K} - \frac{2P\sin^2\omega t}{(-m\omega^2 + K)} - \frac{2P\cos^2\omega t}{(-m\omega^2 + K)} = \frac{-P}{K}\left\{1 + \frac{2}{(1 - (\omega/\omega_o)^2)}\right\}$$

where $\omega_o^2 = K/M$. Thus

$$\frac{P}{(-X)} = K \left\{ \frac{1 - (\omega/\omega_o)^2}{3 - (\omega/\omega_o)^2} \right\}$$

A fuller treatment adding a damping term into each differential equation gives

$$\frac{P}{(-X)} = K \frac{[1 - (\omega/\omega_o)^2]^2 + 4\zeta^2 (\omega/\omega_o)^2}{[1 - (\omega/\omega_o)^2]^2 + 2\,[1 - (\omega/\omega_o)^2] + 4\zeta^2 (\omega/\omega_o)^2}$$

where ζ is the "damping ratio" in conventional notation.

If we suppose that (-X) is constant then the behaviour of P as a function of $(\omega/\omega_o)^2$ is sketched in Fig 7.

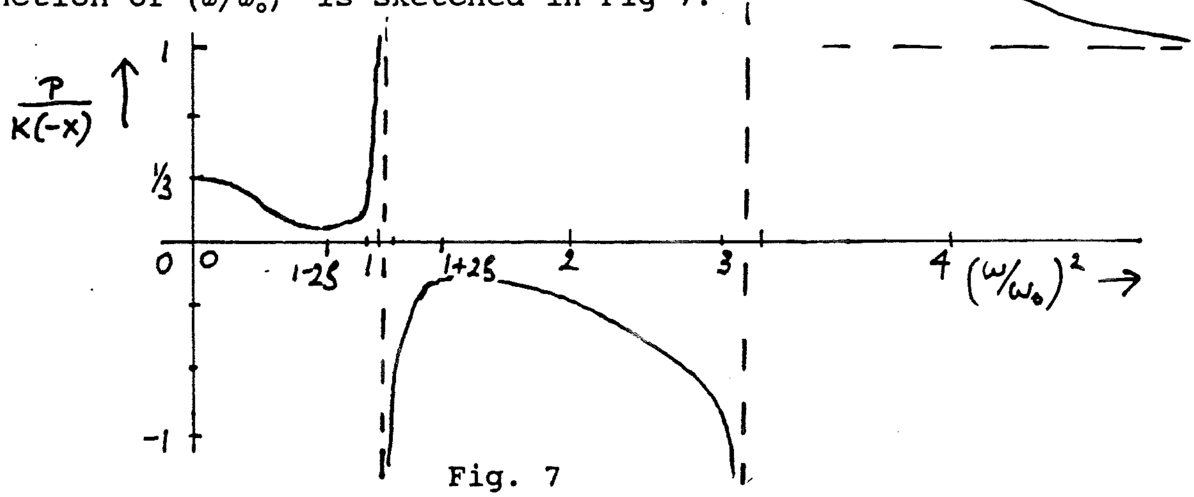


Fig. 7

Of course P and (-X) are also related via the dynamics of the axis whirling equations leading to a closed "loop block diagram" shown in Fig 8.

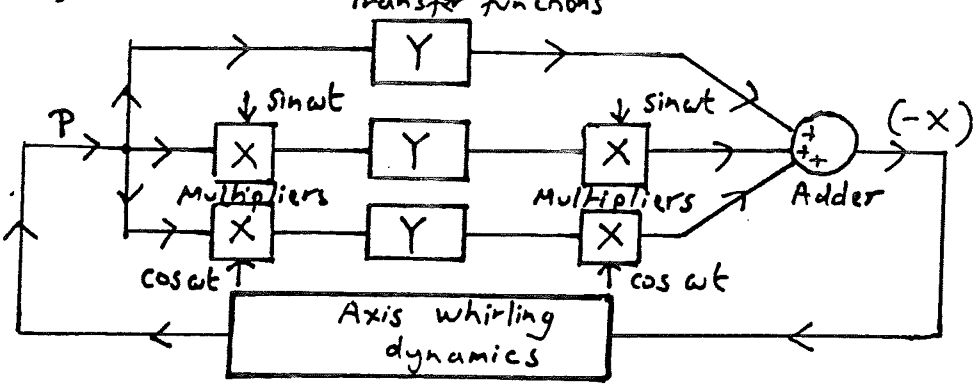


Fig. 8

Here Y is the standard second order transfer function

$$\frac{1}{K\left(1 + \frac{2\zeta D}{\omega_o} + \frac{D^2}{\omega_o^2}\right)},$$

where $D \equiv d/dt$.

A very crude model for the axis whirling dynamics is that $P = K^*(-X)$ where K^* is a probably "large" stiffness. Feeding this relationship back into the equations (A1) (with the damping term added):

$$m\ddot{x} + c\dot{x} + Kx = -K^* (x + r\sin\omega t\,\theta_1 - r\cos\omega t\,\theta_2)$$

$$r(m\ddot{\theta}_1 + c\dot{\theta}_1 + k\theta_1) = -2K^* (x + r\sin\omega t\theta_1 - r\cos\omega_2)\sin\omega t$$

$$r(m\ddot{\theta}_2 + c\dot{\theta}_2 + k\theta_2) = -2K^* (x + r\sin\omega t\theta_1 - r\cos\omega_2)\cos\omega t$$

The left hand sides of the second and third equations may be written

$$r(m\ddot{\theta}_1 + c\dot{\theta}_1 + (K + 2K^* \sin^2\omega t)\theta_1) = \ldots.$$

$$r(m\ddot{\theta}_2 + c\dot{\theta}_2 + (K + 2K^* \cos^2\omega t)\theta_2) = \ldots.$$

which take the form of the damped Mathieu equation on replacing $2\sin^2\omega t$ and $2\cos^2\omega t$ by $1 - \cos 2\omega t$ and $1 + \cos 2\omega t$ respectively. Here is a new possible cause of instability in θ_1, θ_2 (and consequently X) which meets further more detailed investigation.

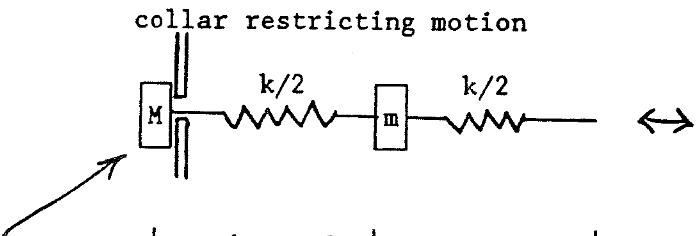
D. C. P.

31 May 1990.

DOWTY: the possibility of ' chattering'

A important piece of evidence concerning the onset of 'chatter' is that it first appears as the shaft begins to whirl with an elliptic path. Once the whirling is elliptic the effective axial length of the shaft oscillates harmonically and thus there is a time varying restoring force due to the springs. There is a natural frequency for the shaft/spring system and therefore resonance can occur if this coincides with the frequency of 'shaft length vibration' (which is twice the whirl frequency). Larger amplitude oscillations are possible causing chatter at the disc end of the shaft.

A simple model which may exhibit this phenomenon is described below. Note that the subtleties of beam elasticity have been approximated by a mass/spring system.

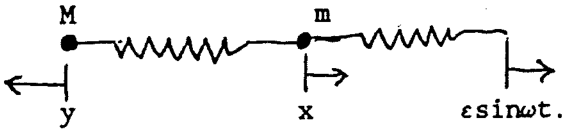


Large mass, modelling disc.	spring and small mass, modelling the elastic shaft.	spring, modelling the bearing/ spring.	forced oscillation, modelling change in length of shaft (and spring).

Both springs have been assumed to have the same spring constant for simplicity.

Coordinate system:

x and y measured from rest positions.



$$-M\ddot{y} - T_0 + (k/2)(x+y) \quad \text{or } y = 0$$

↑ rest tension.

$$m\ddot{x} = -\frac{k}{2}(x+y) + \frac{k}{2}(\varepsilon\sin\omega t - x).$$

so $$m\ddot{x} = -kx - \frac{ky}{2} + \frac{k\varepsilon}{2}\sin\omega t.$$

Resonance when $\omega^2 = k/m$.

Note that resonance depends upon the mass of the shaft not the disc/shaft combination. Away from resonance, $y = 0$, and $x = O(\varepsilon)$. As resonance occurs x becomes large and when $x = -2T_0/k$ the mass M begins to move and chatter occurs, the problem is nonlinear - an impact oscillator. The next stage is to construct a more accurate model bringing into play the elastic properties of the shaft.

PW.