

Business Rates Pooling

Problem presented by

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Executive Summary

The Business Rates Retention Scheme came into effect on 1-Apr-2013. It aims to encourage Local Authorities (LAs) to increase their income from business rates in their area by ensuring that they get a financial payoff from the rates they raise locally. One aspect of it is an optional pooling scheme. This aims to encourage nearby LAs to work together by providing a financial incentive for them to be assessed jointly for business rates if they wish. DCLG wish to have ways of helping sets of LAs decide whether or not to pool which includes the unpredictability of the LAs' income from business rates. They also wish to understand better how LAs are likely to form pools.

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1 Introduction

1.1 An outline of the problem

- (1.1.1) The Business Rates Retention Scheme came into effect on 1-Apr-2013. It aims to encourage Local Authorities (LAs) to increase their income from business rates in their area by ensuring that they get a financial payoff from the rates they raise locally.
- (1.1.2) One aspect of it is an optional pooling scheme. This aims to encourage nearby LAs to work together by providing a financial incentive for them to be assessed jointly for business rates if they wish.
- (1.1.3) Pooling is a decision by a set of geographically contiguous LAs to deal with Central Government (CG) as a pool rather than individually. This provides an incentive for them to work together.
- (1.1.4) DCLG would like to provide tools to LAs to help them decide whether or not to pool, and if so in what combinations. They have already provided some tools to LAs for this, but the tools do not allow for the unpredictability of actual business rates income x .
- (1.1.5) There is high volatility in business rates income, x , over time. Modelling should consider a wide range of possible growth scenarios.
- (1.1.6) Data available includes LA baseline funding levels f , LA business rate baselines r , and past data for the LA business rate incomes, x .
- (1.1.7) Some particular questions that can be looked at:
 - Are changes to the particular parameters in the current scheme (such as the safety net or levy, defined in paragraphs 2.2.5 and 2.2.6) required to make pooling more attractive to local authorities?
 - Is there a perfect make up of pools – and are there certain local authority areas that would have benefited who have so far not pooled?
 - Are there areas where pooling (under the current scheme) will never be worth it, and if so what would it take to make it worth it?

2 The Business Rates Retention Scheme

2.1 Key variables

- (2.1.1) Each LA's dealings with CG, whether it joins a pool or not, are determined by 3 quantities each year:
 - Its **baseline funding level**, f , which is a CG estimate of what resources the LA needs to have from the mechanism in order to provide the necessary services to its area for the year;

- Its **business rates baseline**, r , which is a CG estimate of the amount the LA is expected to raise from the mechanism in its area in the year;
- Its actual **business rates income**, x , *i.e.* that the LA receives from the mechanism in its area in the year. The quantity r is the CG estimate of x .

(2.1.2) The quantities f and r are set at the outset of the scheme by CG. In subsequent years f and many of the quantities calculated using f and r will be updated in line with inflation, but r itself will be fixed for the next seven years. The quantities x are difficult to predict because of, for instance, companies going out of business, or relocating, or successfully appealing against their rating assessment.

2.2 The mechanism for billing authorities

(2.2.1) In this section we will consider **billing authorities**, which are LAs which collect business rates (and council tax, though that is not relevant). For the sake of simplicity we will assume that our billing authority belongs to a county council. The information in this section is based on the official documents [1] and [2].

(2.2.2) The actual business rates collected by an LA are some amount $X > x$. Of the amount collected, $X/2$ goes direct to CG for redistribution, $X/10$ goes to the county council that the billing authority is part of, and $X/100$ goes to the fire service, so the x in the pooling problem is 39% of X .

(2.2.3) Each authority with $f > r$ receives an amount $f - r$ from CG. These are referred to as top-up authorities: they need more funding than they are expected to receive from rates, and CG makes up the difference.

(2.2.4) Each authority with $f < r$ gives $r - f$ to CG. These are referred to as tariff authorities: they are expected to receive more from rates than they need, so they contribute the excess to CG.

(2.2.5) This leaves the LA with an amount $l = x + f - r$ (but with a **safety net** from CG which ensures that l is never less than $0.925f$).

(2.2.6) In addition, tariff authorities are subject to a levy rate v , meaning that when $x > r$ they pay a further amount $v(x - r)$ to CG. The quantity $x - r$ is called the **growth**.

(2.2.7) The levy rate is a piecewise linear function of f/r , capped at 50%.

(2.2.8) The levy funds the safety net for other LAs, so protecting them against sharp falls in business rates income.

- (2.2.9) The result of this mechanism is that if an LA with quantities (f, r, x) deals with CG individually, then the resulting funds it will have in hand to provide services are a function $P(f, r, x)$, derived from the above rules. We can call this the **payoff function**.
- (2.2.10) If a set of LAs indexed by i deal individually with CG they will each receive a payoff $P(f_i, r_i, x_i)$. If they pool they will collectively receive a payoff of $P(\sum f_i, \sum r_i, \sum x_i)$.
- (2.2.11) Provided that the LAs can agree among themselves how they should share the benefits and responsibilities of the scheme, they will do better to pool if $P(\sum f_i, \sum r_i, \sum x_i) > \sum P(f_i, r_i, x_i)$.
- (2.2.12) Note: P is homogeneous of degree 1 in its arguments. If it were concave it would always be advantageous to pool. If it were convex it would never be advantageous to pool. Since it is neither concave nor convex, the optimal decision varies from case to case.
- (2.2.13) A set of LAs is only allowed to pool if they are geographically contiguous.
- (2.2.14) Although LAs can in theory choose to pool or not each year, they have to decide before they know x for that year, and it is expected that pools will operate for much longer, to recoup the long-term benefits of pooling.
- (2.2.15) A disincentive to rearranging pools often is that LAs would have to come to different financial arrangements with each other about how they share the benefits and responsibilities of the pool.
- (2.2.16) There is no centrally imposed scheme for the arrangements between LAs in a pool.
- (2.2.17) Summarising the above description using mathematical notation, if an LA with quantities (f, r, x) deals with CG individually, then the resulting funds it will have in hand to provide services are a function $P(f, r, x)$ given by

$$P(f, r, x) = \begin{cases} 0.925f & \text{if } x \leq r - 0.075f \\ f + x - r & \text{if } r - 0.075f \leq x \leq r \\ f + (1 - v)(x - r) & \text{if } x \geq r \end{cases} \quad (1)$$

where v is called the levy rate for the authority and is given by

$$v = \begin{cases} \frac{1}{2} & \text{if } f \leq \frac{1}{2}r \\ 1 - f/r & \text{if } \frac{1}{2}r \leq f \leq r \\ 0 & \text{if } f \geq r \end{cases} \quad (2)$$

We refer to P as the payoff function. Actual values of f/r vary between about 0.1 and 5.

- (2.2.18) Note that if $f > r$ then the levy rate is 0, so no top-up LA pays a levy even if $x > r$: the levy can only be positive for tariff authorities.

2.3 Non-billing authorities

- (2.3.1) As well as billing authorities, there are local authorities that do not collect business rates. We will focus our attention on county councils. They participate in this procedure as well, but their parameters are set in a different way.
- (2.3.2) They have a baseline funding level but not a business rates baseline, so are always top-up authorities in terms of the earlier description.
- (2.3.3) Although they do not collect business rates themselves, they receive 10% of the business rates collected by the billing authorities within their area. (This is the $X/10$ that was mentioned earlier.) Therefore their value of x is $10/39$ times the sum of the x_i over local authorities within their area.
- (2.3.4) County councils are themselves allowed to join a pool, and they participate in it on the same basis as the other participants, but with these different parameters, and with their larger geographical area as part of the contiguity rules.
- (2.3.5) A pool containing a county council need not contain all of the billing authorities within that county. A pool containing a county council may contain some billing authorities that are outside that county, provided that they are adjacent to it or connected to it by the pool. A pool may contain more than one county council.

3 Our work

3.1 Overview

- (3.1.1) Our work falls into three broad headings.
- (3.1.2) **Analysis of the two LA/pool case:** In §4 we use analytic methods to explore the simple problem of determining when it's favourable for two LAs (or, equivalently, two pools) to combine, first with point estimates for the business rates incomes and then with probability distributions.
- (3.1.3) **Data processing:** In §5 we convert the available data into easily usable forms, and we explore methods for predicting future values of x from past data.
- (3.1.4) **Simulations and pooling algorithms:** In §6 we use r , f and generated distributions for x to develop possible pooling strategies across the entire

country.

4 When should two individuals pool?

4.1 The set-up

- (4.1.1) Suppose two LAs, A and B , want to know whether pooling would be financially advantageous for them in the next year. Let us write P_A and P_B for the amount of funding A and B would end up getting individually and P_{AB} for the amount they would get if they pooled. We want to know if $P_{AB} - P_A - P_B$ is positive or negative.
- (4.1.2) As f_A , f_B , r_A and r_B are known, the answer only depends on x_A and x_B . We can create a graph with x_A and x_B along the axes and indicate in which regions it is financially advantageous, disadvantageous or neutral to pool. This would be a valuable tool for LAs to decide whether to pool or not.
- (4.1.3) Because P is piecewise linear with its domains of definition determined by $\frac{x-r}{f}$, we introduce $q = \frac{x-r}{f}$ as a dimensionless ‘ x ’ variable. Which formula to use for P depends on where q lies in relation to 0 and $-s$, where $s = 0.075$ is the safety threshold. This allows us to present our results in a way which depends less on the actual values of f_A , f_B , r_A and r_B .
- (4.1.4) Let q_{AB} denote $\frac{x_A+x_B-r_A-r_B}{f_A+f_B}$, the value of q if A and B pool. This is related to q_A and q_B by the formula $q_{AB} = \frac{f_A}{f_A+f_B}q_A + \frac{f_B}{f_A+f_B}q_B$. Hence, q_{AB} is always between q_A and q_B .
- (4.1.5) Figure 1 shows the critical values of q_A , q_B and q_{AB} as dashed lines. This splits the plane up into seventeen regions in which P_A , P_B and P_{AB} are linear functions in q_A , q_B and q_{AB} respectively.

4.2 Results

- (4.2.1) Figure 2 adds red lines marking the boundaries between the regions where $P(f_A + f_B, r_A + r_B, x_A + x_B) - P(f_A, r_A, x_A) - P(f_B, r_B, x_B)$ is positive, negative or zero, under the assumption that $v_A < v_B$. The equations for the red lines are in Appendix A.1.
- (4.2.2) Figure 3 shows the situation when v_A and v_B are equal and positive.
- (4.2.3) Figure 4 shows the situation when v_A and v_B are both zero.

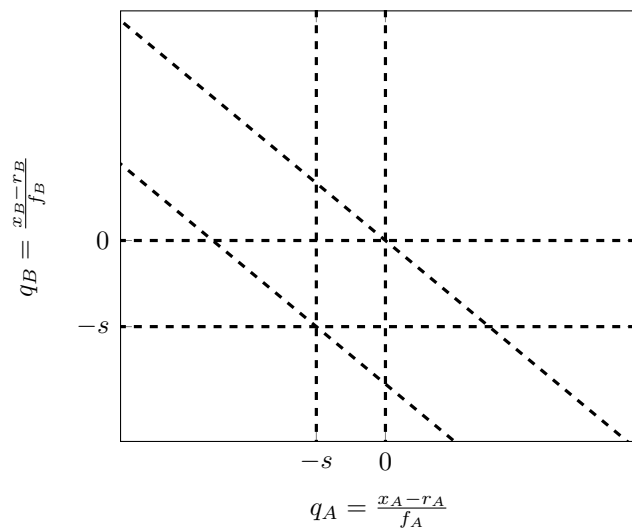


Figure 1: Dashed lines show the critical values of q_A , q_B and q_{AB}

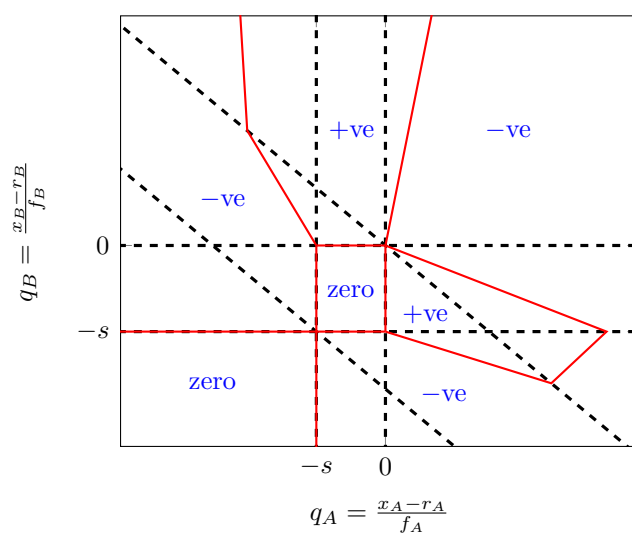


Figure 2: If $v_A < v_B$, what is the sign of $P_{AB} - P_A - P_B$?

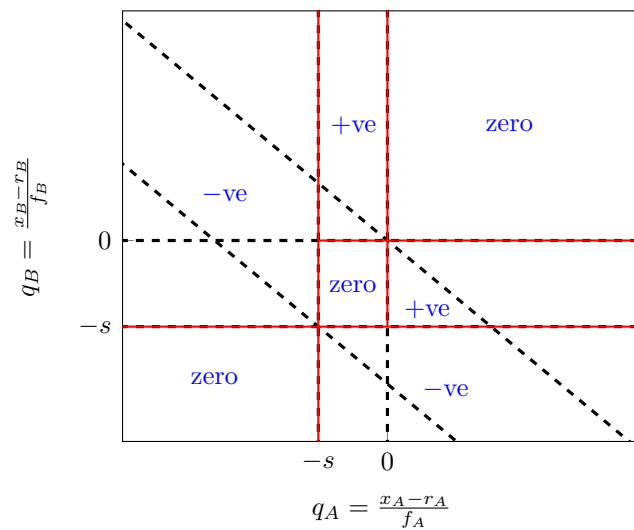


Figure 3: If $v_A = v_B > 0$, what is the sign of $P_{AB} - P_A - P_B$?

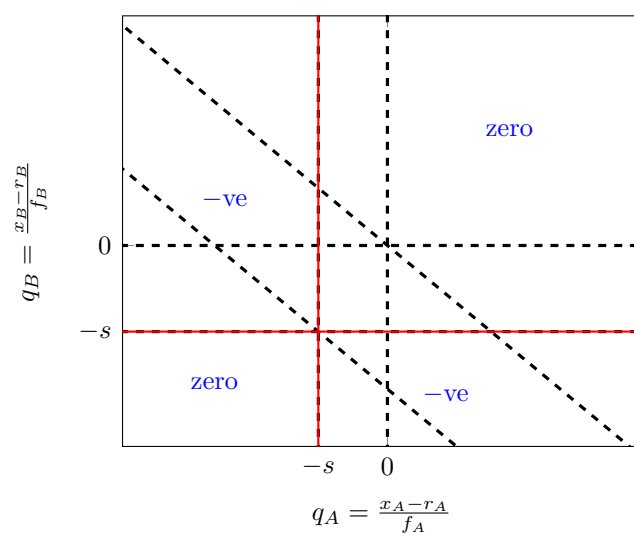


Figure 4: If $v_A = v_B = 0$, what is the sign of $P_{AB} - P_A - P_B$?

4.3 Observations

- (4.3.1) Pooling can only be advantageous if at least one of the potential participants expects $x > r$ (*i.e.* business rates income exceeds baseline).
- (4.3.2) Pooling has no advantages if both of the potential participants have a zero levy rate.
- (4.3.3) If one participant has income over the baseline and the other participant has income between the safety net and the baseline, and $v \neq 0$ for at least one participant, pooling is *always* advantageous if the business rates income for the pool is **less** than the baseline.
- (4.3.4) In plain terms, paragraph (4.3.3) implies that pooling is advantageous if some members of the pool are sufficiently unsuccessful to drag down the income of the pool below the expected income. Hence, there may be perverse incentives in the pooling scheme where pooling is most favourable for LAs that do not expect to collect their forecast business rates.

4.4 Incorporating probability distributions

- (4.4.1) The above analysis is for point predictions, *i.e.* we have a single value for x_A and a single value for x_B .
- (4.4.2) It would be better to use a joint probability distribution for x_A and x_B . The output would be a probability distribution for $(P_{AB} - P_A - P_B)/(P_A + P_B)$.
- (4.4.3) One problem with this approach is that it is hard to come up with a joint distribution for x_A and x_B . The simplest way is to assume that x_A and x_B are independent, so we need a probability distribution for each of them separately. However, this ignores many possible reasons why they might not be independent. For example, they might both depend on the state of the national economy, which would lead to positive correlation, or businesses might move from one LA to the other, in which case they would be negatively correlated.

5 Data processing

5.1 Challenges in data preparation

- (5.1.1) The big challenge with data processing in this project was to bring everything into a consistent form. We needed to:
- Build tables for r_i, f_i and (for the billing authorities only) the historic x values.

- Deal with inconsistent spelling in the data: e.g. & or ‘and’; the apostrophe (or lack thereof) in ‘King’s Lynn’; *etc.*
- Understand and implement the funding differences between the various kinds of LAs: unitary authorities, shire counties, shire districts, London boroughs, fire authorities *etc.*:
 - x_i for *billing authorities* are the business rates collected multiplied by a conversion factor depending on type,
 - x_i for *non-billing authorities* (e.g. counties) are the sum of the income of the constituent billing authorities multiplied by (different) factors.

- (5.1.2) In addition to cleaning up the financial data, it was essential to know which LAs are allowed to pool together. This is encoded in a mathematical object called a graph, which can be thought of as a list of objects and a list saying which pairs of objects are adjacent to each other. In our case the objects are the LAs and two LAs are considered adjacent if they can form a pool of size two. There are two forms of adjacency: two LAs are
- **logically adjacent** if one LA is a county council which contains the other LA, e.g. Gloucestershire and Stroud, or
 - **geographically adjacent** if they are physically next to each other, e.g. Bristol and North Somerset.

5.2 Constructing an adjacency matrix

- (5.2.1) Raw data for logical adjacency was available in the form of a table indicating which billing authorities belong to which county councils. It was fairly straightforward to use this table to create an adjacency matrix.
- (5.2.2) Dealing with geographical adjacency was much harder because the raw data was not available. We constructed our own on the basis of publicly available geographical data, using postcode centroids. For simplicity we only considered geographical adjacency between billing authorities.
- (5.2.3) In Figure 5 the picture on the left is the raw data with all the postcodes in England (and Wales) coloured according to which billing authority they belong to.
- (5.2.4) We established coordinates for each billing authority by taking the centroid of the coordinates for all the postcodes contained in it. From these coordinates we then created a Voronoi diagram, seen on the right of Figure 5 (for England only). This gave us geographical adjacency information that was a reasonable approximation to the truth, and was sufficient for our purposes.

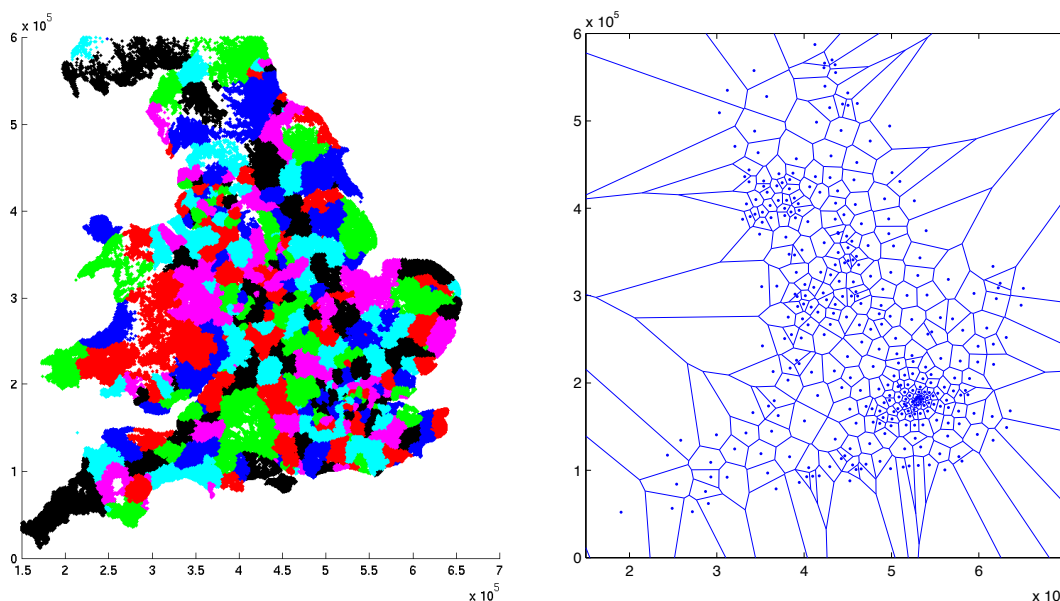


Figure 5: The postcodes and billing authorities of England and, in the left hand picture, Wales.

5.3 Approaches to modelling x_t

- (5.3.1) Since x for a single authority can be expected to increase exponentially over time due to inflation, an appropriate stochastic model for x might be

$$x_t = A \exp\{(\mu - 0.5\sigma^2)t + \sigma W_t\},$$

where t is time, W_t is a Wiener process, and A , μ and σ are all constants that need to be determined.

- (5.3.2) However, there are only nine historic data points (2003/04 to 2011/12) and the data are often very noisy, as illustrated by Figure 6. We therefore decided that a complicated model would give very little benefit over a very simple model, and opted to use linear regression.
- (5.3.3) The probability distribution we used for the business rates income from each LA was a normal distribution with mean given by the linear regression and variance given by the residuals in the linear regression.
- (5.3.4) We assumed that these distributions were independent. As discussed before this is a simplifying assumption which is not necessarily justified, and it would be better to model the dependence between the LAs.

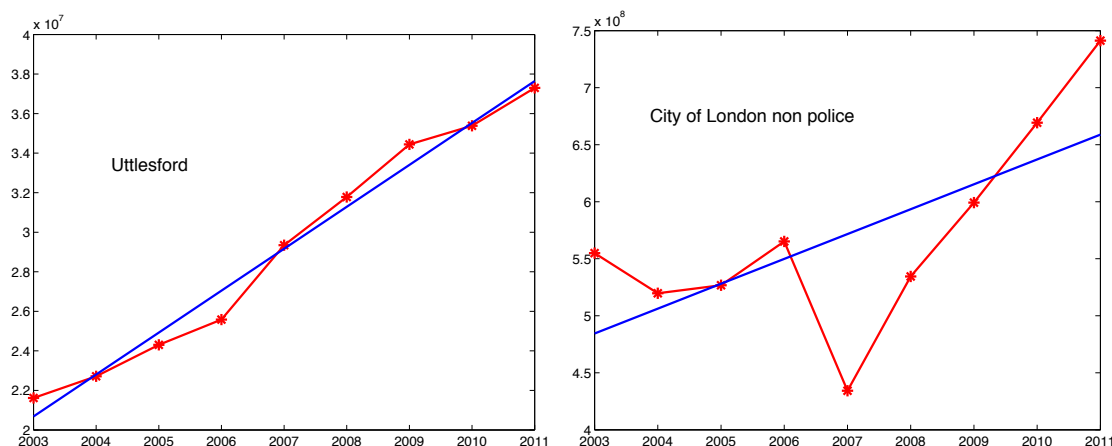


Figure 6: The contrasting business rates income trends of Uttlesford and the City of London.

6 Pooling

6.1 Setting up the problem

- (6.1.1) We now turn to the study of how LAs are likely to form pools.
- (6.1.2) The question we considered was the optimal graph partition problem: how should the LAs form pools in order to maximise their payoff? Academics from two universities have expressed interest in aspects of this problem possibly forming part of an MSc project.
- (6.1.3) At this point it is worth mentioning an optimal solution that can be spotted by eye.
- (6.1.4) If no LA is going to need the safety net, the maximum amount of funding can be achieved (albeit non-uniquely) by minimising the total tariff. The levy rate for a pool made of every LA is zero because

$$\sum f > \sum r,$$

so no matter how large the business rates incomes are the pool will never have to pay a tariff.

- (6.1.5) However, this is not a realistic solution, as CG has to approve every pool and the guidelines state that there should be a clear rationale for the geographic coverage of proposed pools. It may be necessary to add an extra condition to the problem to stop the formation of unrealistically large pools.
- (6.1.6) Considering every possible set of pools would take prohibitively long, and

finding a more efficient algorithm that is guaranteed to find the best possible answer is very difficult. A common approach in such situations is to use a heuristic algorithm which produces a good result in a reasonable time.

(6.1.7) We used the following very simple heuristic algorithm:

1. Initialise each pool to contain a single LA.
2. Pick a pool at random, and find all of its neighbouring pools.
3. Choose the neighbour that leads to the largest expected value of $(P_{AB} - P_A - P_B)/(P_A + P_B)$.
4. If this is above a certain (nonnegative) threshold θ then combine the two pools.
5. Return to Step 2, and repeat until convergence is reached.

(6.1.8) The random choice in step 2 means that the algorithm will give a different result each time it is run.

(6.1.9) In step 3 we use the expected value of $(P_{AB} - P_A - P_B)/(P_A + P_B)$ for the next year. This could easily be adapted to include future years. As discussed before, we could also replace the expected value with a different function of the probability distribution for $(P_{AB} - P_A - P_B)/(P_A + P_B)$, for example to be more risk averse. See Figure 7 for an example of this distribution.

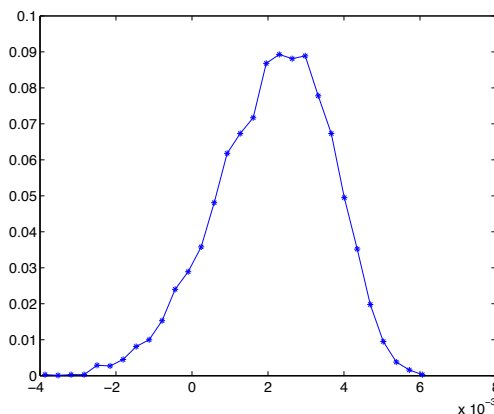


Figure 7: Distribution of $\frac{P_{AB}-P_A-P_B}{P_A+P_B}$ for Croydon to join with the GLA.

6.2 Videos of pooling simulations

(6.2.1) We ran our algorithm three times with different values of θ . The results were recorded in the videos `PoolThresh0.avi`, `PoolThresh1.avi` and `PoolThresh2.avi`, whose final frames are shown in Figures 8, 9 and 10. Only billing authorities are shown.

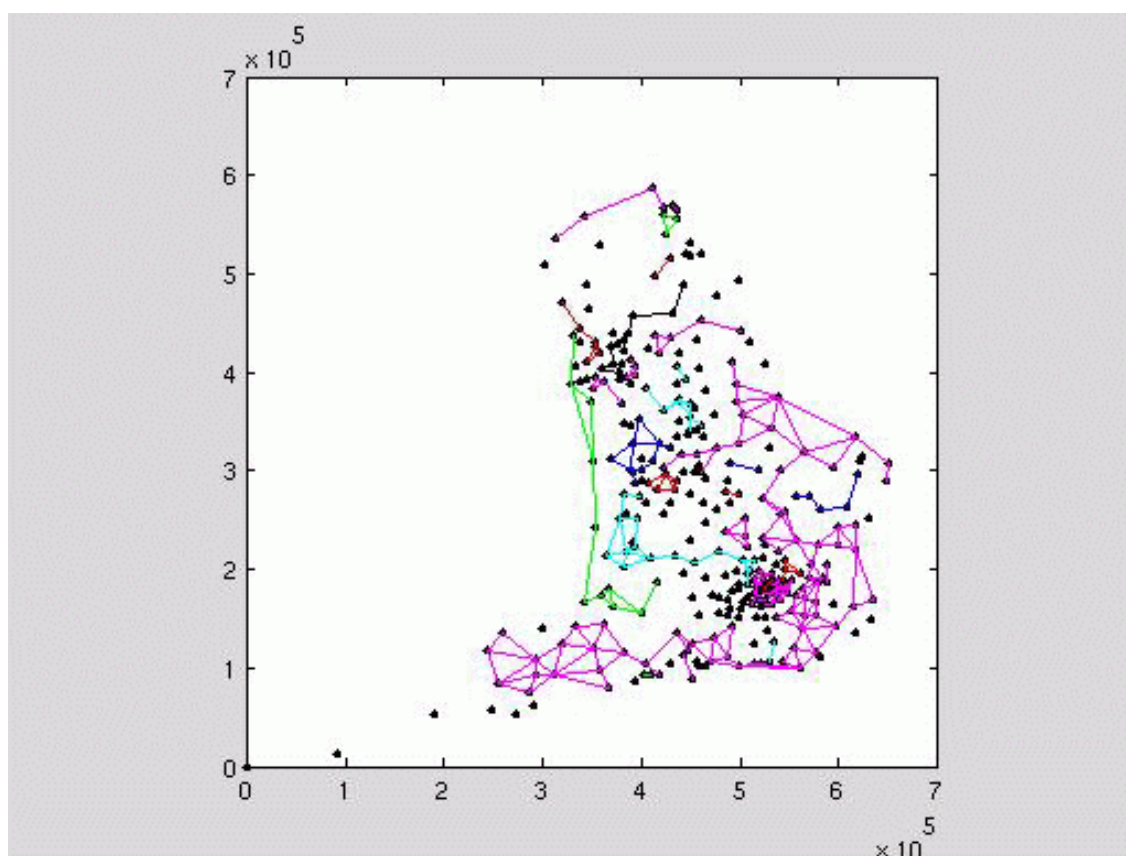


Figure 8: The final frame of the video `PoolThresh0.avi`, which shows the development of pools when our algorithm was run with $\theta = 0$. Note that one of the pools stretches from the South West, along the south coast to the South East, and up the east coast to the North East. This is clearly unrealistic.

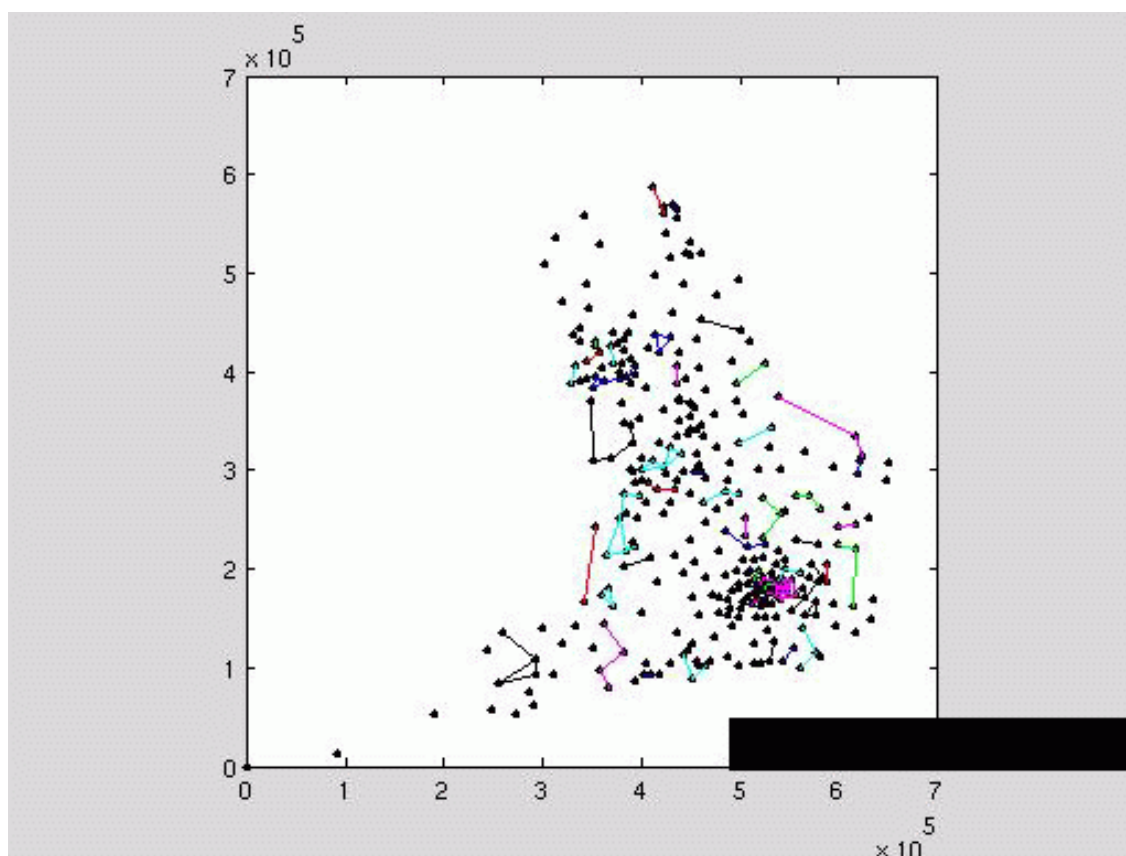


Figure 9: The final frame of the video `PoolThresh1.avi`, which shows the development of pools when our algorithm was run with $\theta = 0.001$. As expected, fewer LAs have joined pools than in the previous run with $\theta = 0$.

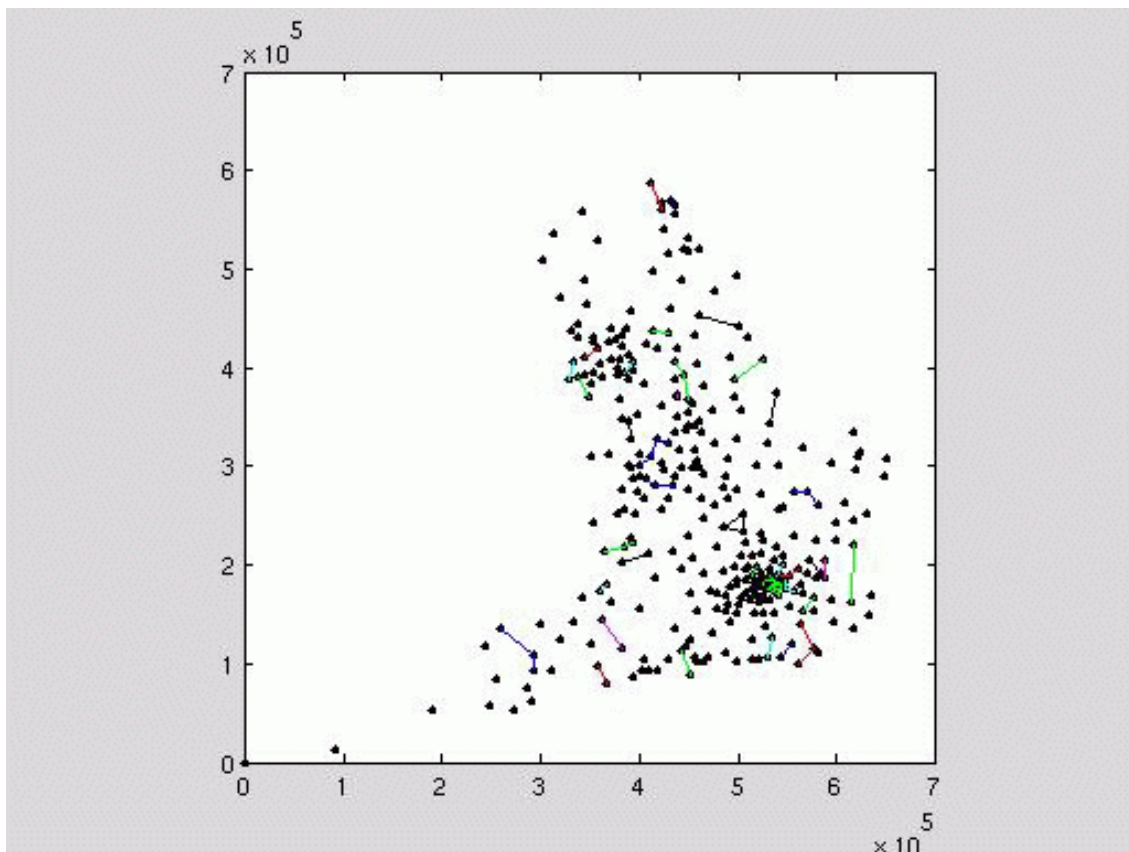


Figure 10: The final frame of the video `PoolThresh2.avi` shows the development of pools when our algorithm was run with $\theta = 0.002$. As expected, fewer LAs have joined pools than in the previous run with $\theta = 0.001$.

7 Conclusions

7.1 Conclusions

- (7.1.1) For two pools with fixed f and r , there are (relatively) simple conditions on x that indicate when they would benefit from combining.
- (7.1.2) Pairwise algorithms can be used as a heuristic to obtain ‘good’ pools.
- (7.1.3) When the threshold θ is too small our pairwise algorithm produces very large pools.

7.2 Further work

- (7.2.1) Look at the historic data to see how dependent the values of x_i are for adjacent LAs.
- (7.2.2) Improve on our pooling algorithm, *e.g.* by considering more possible pairings before choosing the one that increases the total payoff the most.
- (7.2.3) Explore ways of keeping pools realistic beyond our use of a non-zero threshold θ . Perhaps it would make sense to score potential pools using a suitable function of the increase in payoff from forming the pool and the number of LAs in the resulting pool.
- (7.2.4) Consider whether our pooling algorithm is missing some possible pools by only looking at two pools at a time. Is it possible to have $n > 2$ pools with quantities such that it doesn’t make sense for any pair of them to pool, but it does make sense for them all to pool?
- (7.2.5) Consider functions of the probability distribution for $\frac{P_{AB}-P_A-P_B}{P_A+P_B}$ other than the expected value. It might be helpful to suggest some to LAs which are risk adverse, for example.
- (7.2.6) Within a pool, how should the payoff be split between the LAs? A simple approach would be to split it in proportion to the payoff each LA would have received if it hadn’t pooled. However, it would be interesting to consider what bargaining power each LA in the pool might have by looking at the payoff the pool would expect without it.

A Appendices

A.1 Equations of plots

- (A.1.1) Figure 11 is a copy of Figure 2 with the red line segments which are not parallel to the q_A or q_B axis labelled. Below we list the equations of each

of the labelled line segments. We use v_{AB} to denote the value of v for A and B if they pool, *i.e.* the value of v calculated using $f = f_A + f_B$ and $r = r_A + r_B$ in equation (2).

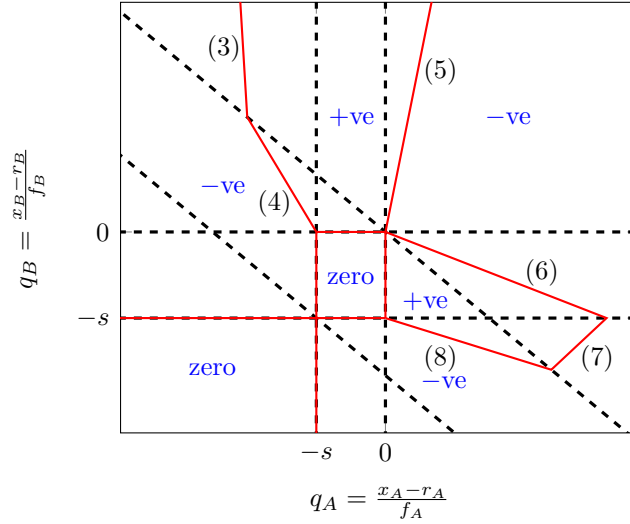


Figure 11: If $v_A < v_B$, what is the sign of $P_{AB} - P_A - P_B$?

$$q_A = -\frac{v_B - v_{AB}}{1 - v_{AB}} \frac{f_B}{f_A} q_B - \frac{s}{1 - v_{AB}} \quad (3)$$

$$q_A = -v_B \frac{f_B}{f_A} q_B - s \quad (4)$$

$$q_B = \frac{v_{AB} - v_A}{v_B - v_{AB}} \frac{f_A}{f_B} q_A \quad (5)$$

$$q_B = -(v_{AB} - v_A) \frac{f_A}{f_B} q_A \quad (6)$$

$$q_B = \frac{v_{AB} - v_A}{1 - v_{AB}} \frac{f_A}{f_B} q_A - \frac{s}{1 - v_{AB}} \quad (7)$$

$$q_B = -v_A \frac{f_A}{f_B} q_A - s \quad (8)$$

(A.1.2) Note that when $\frac{1}{2}r_A \leq f_A \leq r_A$ and $\frac{1}{2}r_B \leq f_B \leq r_B$, the identity $v_{AB} = \frac{r_A}{r_A + r_B} v_A + \frac{r_B}{r_A + r_B} v_B$ holds, from which it follows that equation (5) is equivalent to

$$\frac{x_A}{r_A} = \frac{x_B}{r_B} \quad (9)$$

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