117th European Study Group with Industry 23-27 May 2016, Avignon

Mobile networks migration optimization

Problem presented by Matthieu Chardy and Adam Ouorou from Orange Labs

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July 22, 2016

1 Description, general assumptions and notations

1.1 Context and motivations

The Orange group is a worldwide telecommunications operator with around 157.000 employees, 228 business countries and 28 residential countries; mainly in European and African countries. Orange group is both a service provider (owns customers) and a network operator (owns the network). Operations research applications arise at this company in different contexts, like in the design of the networks, in the management of their resources/workforce as well as in the management of customer relationship.

In the last few years we have witnessed a natural growth of data traffic in mobile communications. This growth is a combination of two joint phenomena: the growth of the number of customers and the growth of the Average Usage per User. In a more and more competitive market, Orange has to understand the behavior of customers in order to plan optimally its investment in new mobile generations. Having a deep

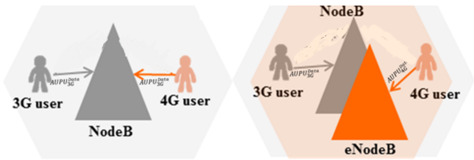

Figure 1: Description of 3G and 4G coverage by NodeB and eNodeB. NodeB base stations can serve 3G and 4G subscribers.

and rigorous analysis of the demand evolution based on the new services/technologies proposed, can be an advantage for Orange compared to other companies. Network economics is a branch of problems related to economic issues in telecommunication networks. In this report, we are interested in investment decision problems for new technologies. Such problems have been treated in the literature but with other targets in mind. For example, the authors in [3] look for network upgrade decision with an objective of revenue maximization in a competitive environment. We propose here an approach based on the quality of service and coverage that aims a cost minimization for migration to new wireless technologies.

In the last years, we have witnessed a fast evolution in the mobile network technologies. Telecommunications operators need to deal with the rollout and management of several generations of mobile networks on the same geographic sites. Dismantling one generation of mobile network is no easy option since operational teams are reluctant to abandon well-functioning (and robust) technologies for new ones without back-up. Moreover, several services may need old(er) technologies (machine-2-machine, roaming, ...). Hence, different technologies must co-exist and Orange group maintains simultaneously 2G and 3G technologies in African affiliates and 2G, 3G and 4G technologies in European affiliates. In this scenario another important issue arrives: the problem of heterogeneous coverage depicted on figure 1. NodeB base stations are able to serve 4G subscribers, whereas the opposite is not possible. The migration of base stations to new wireless generation is a very important operational problem for Orange. A key question is: how to drive the evolution of mobile networks (in terms of capacity and coverage) taking into consideration traffic evolution, new services and potential strategical constraints? By solving the problem presented in this report we hope to shed some light into this question.

The *design of a multi-period master plan for mobile network* (Mobile Master Plan) consists in deciding, for a given set of time points, how to invest on the evolution of network technologies regarding three aspects: densification, coverage extension and user upgrades. An *investment in densification* means the installation of new equipments of a such technology (modules and/or antennas) in order to upgrade the capacity of a location already covered by this technology. An *investment in coverage extension* means the installation of new antennas of a such technology in locations not covered so far by this technology. Finally, an *investment in user upgrades* means the proposition of financial subsidies to customers in order to promote upgrades from their current services (from a 3G subscription to a 4G one for instance). These three kinds of investment decisions must be synchronized: at a given location, the network of a given generation must be dimensioned so that it can handle the traffic resulting from upgrades and new subscriptions. As we could expect, we assume that Mobile Master Plans are driven by cost minimization over the whole time horizon.

A Mobile Master Plan is defined taking into account the specificities of each Orange affiliate (geographic location, population, strategical guidelines). In practice, Mobile Master Plans are designed for a 5-year time horizon with decisions taken for each year in this period.

1.2 Problem description and assumptions

We consider a set $T = \{0, 1, 2, \ldots, \bar{t}\}$ of time points dividing the given time horizon into a set $\mathcal{I} = \{I_1, \ldots, I_{\bar{t}}\}$ of $\bar{t}$ equally-sized time periods where $I_t = [t-1, t)$, for each $t \in \{1, 2, \ldots, \bar{t}\}$. Decisions are taken for each time step. As we have already said, typically the time horizon is a set of $\bar{t}$ years divided into $\bar{t}$ periods of 1 year.

Only two generations, 3G and 4G, of mobile networks (and subscribers) are considered in this work.

At the beginning of the time horizon, the whole territory is fully covered by the 3G network while the 4G network is under extension. The feasibility of the network is only based on data traffic. At the end of each time step, the capacity of the network must be sufficient to the traffic imposed by the current set of all users.

We assume that the operation area is divided in a set $S = \{s_1, \dots, s_n\}$ of telecommunication sites. Each site is either a pure 3G site, providing only 3G service, or a 3&4G site, providing both 3G and 4G services. Actually, a pure 3G site can have 4G users but those behave as 3G users.

Subscribers have a known average traffic that increases with time and that, at each time step, is higher for 4G users. Some traffic allocation engineering rules are considered. In a pure 3G site coverage, any type of subscriber's traffic is conveyed by the 3G network. On the other hand, in a site covered by 3G and 4G technologies, the whole 3G subscribers traffic must be conveyed by the 3G network, as well as the whole 4G subscribers traffic must be conveyed by the 4G network.

The initial number of 3G and 4G subscribers located at each site is given. These values are calculated considering the peaking hour at each site. The increase in the total number of customers is predicted by marketing services for each time period. These values define a set of uncertain parameters of the problem. For each time period, a range of percentages of increase with respect to Orange current number of customers (3G and 4G) is given. We also assume that 33% of the new customers are 3G subscriptions and 67% of the new customers are 4G subscriptions. New customers are assumed to be allocated to sites proportionally to the number of customers at the end of the previous time step.

As we have already mentioned, the capacity of the network can be increased in two ways: by installing new antennas into an existing site; by installing new modules in an already installed antenna. The maximum number of modules per antenna of each technology is given. Since the actual policy of Orange is not to invest in old technologies, we assume that only new 4G antennas will be installed. Costs associated with the installation of new antennas as well as new modules are given for each time period.

Current subscribers of a given technology can migrate to a more recent technology service. In the problem treated in this text, this means that, at each time period, a number of 3G subscribers will migrate to the 4G technology. Subsides are offered in order to promote user service upgrades: a subside will be defined for each time period and an upgrade function determines how users respond to the subside offered. More precisely, the upgrade function is the function informing, in percentage, the number of users that respond positively to a given subside and it is defined as a function of both the subside offered and the coverage (i.e., the percentage of 3&4G sites at the end of the previous time period). The upgrade function is assumed to be an increasing function with respect to both the financial subside offered and the coverage.

We denote the problem described in this section as the Uncertain Mobile Master Plan Problem (MMPP).

2 The deterministic case

First, we consider a deterministic version of the Uncertain MMPP, in which we assume that the increase in the total number of users is known for each time period. An ILP formulation is presented in this section for the deterministic version of the problem. The data and variables used to define the formulation are introduced next.

2.1 Input data

Input sets:

$T = \{0, \dots, \bar{t}\}$:	Set of time period indexes.
$S = \{s_1, \dots, s_n\}$:	Set of sites.
$X = \{3, 4\}$:	Set of technologies.

Indexes:

t:	time period.
s:	site.
x:	technology.

Input parameters:

$\forall\ s \in S, \forall\ x \in X$:

$U^0_{s,x}$: total number of xG users in site s at the beginning of the time horizon.
$M^0_{s,x}$: total number of modules xG installed at site s at the beginning of the time horizon.
$a^0_{s,x}$: total number of xG antennas installed at site s at the beginning of the time horizon.

$\forall\ s \in S$:

z^0_s: equal to 1 if both $3G$ and $4G$ technologies are installed at site s at the beginning of the time horizon.

$\forall\ t \in T \setminus \{0\}, \forall\ x \in X$:

$AUPU^t_x$: average usage per user of technology x at time period t.

$\forall\ t \in T \setminus \{0\}$:

$\overline{nU^t}$: upper bound of the percentage of new users of both technologies at time period t.
nU^t: average percentage of new users of both technologies at time period t.
$\underline{nU^t}$: lower bound of the percentage of new users of both technologies at time period t.

$\forall\ x \in X$:

CAP_x: capacity of a xG module in Mb.
NUM_x: maximum number of xG modules per antenna.
$C_{OP,x}$: operation cost of a xG module, paid at each time period after its installation.
C_{AD}: fixed installation cost for a new antenna of technology $4G$.

2.2 Variables

$\forall\ t \in T \setminus \{0\}, \forall\ s \in S, \forall\ x \in X$:

$U^t_{s,x}$: total number of xG users in site s at the end of time period t.
$M^t_{s,x}$: total number of modules xG installed at site s at the end of time period t.

$\forall\ t \in T \setminus \{0\}, \forall\ s \in S$:

z^t_s: equal to 1 if both $3G$ and $4G$ technologies are installed at site s at the end of time period t.

$\forall\ t \in T \setminus \{0\}, \forall\ s \in S$:

y^t_s: total number of new $4G$ antennas installed at site s at the beginning of the time period t.

$\forall\ t \in T \setminus \{0\}$:

sub_t: value of the subsidy, in Euro, offered to $3G$ users at the beginning of time period t.

2.3 Upgrade function

$f(sub_t, P^{34G}_{t-1})$: is the percentage of users that respond positively to the financial subsidy sub_t if the percentage of 3&4G sites at the end of the previous time period is equal to P^{34G}_{t-1}.

2.4 Objective function

We want to minimize the costs involved in the expansion of the communication service offered. It involves three types of costs: (i) costs incurred with the subsidies offered to $3G$ users; costs involved with the expansion of the network, which means, (ii) costs incurred with the installation of news modules for existent antennas and (iii) costs incurred with the installation of new antennas. The first, second and third members of the following objective function are, respectively, related with costs (i), (ii) and (iii).

$$\text{Minimize} \sum_{t\in T\setminus\{0\}} sub_t \sum_{s\in S} U_{s,3}^{t-1} f(sub_t, P_{t-1}^{34G}) + \sum_{t\in T\setminus\{\bar{t}\}} \sum_{s\in S} (M_{s,3}^t C_{OP,3} + M_{s,4}^t C_{OP,4}) + \sum_{t\in T\setminus\{0\}} \sum_{s\in S} y_s^t C_{AD} \quad (1)$$

2.5 Constraints

A limit on the maximum number of $3G$ and $4G$ modules installed at each site must be respected at each time period. Clearly, the number of modules depends on the number of antennas installed.

$$M_{s,3}^t \leq a_{s,3}^0 NUM_3, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (2)$$

$$M_{s,4}^t \leq (a_{s,4}^0 + \sum_{t'\in T\setminus\{0\}:t'\leq t} y_s^{t'}) NUM_4, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (3)$$

The capacity installed at each site must be sufficient to provide services for all users located in this site. Remember that, users from $3G$ technology are always served by $3G$ sites; which means, at the beginning of the time horizon, each site has $3G$ antennas enough to serve the set of current $3G$ users. On the other hand, $4G$ users will be served by $3G$ antennas whenever there is no $4G$ antenna installed in the site.

$$U_{s,3}^t AUPU_3^t + U_{s,4}^t AUPU_4^t (1 - z_s^t) \leq M_{s,3}^t CAP_3, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (4)$$

$$U_{s,4}^t AUPU_4^t z_s^t \leq M_{s,4}^t CAP_4, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (5)$$

As we can expect, we must impose the total number of modules of each technology, at each site, to be non-decreasing on the time.

$$M_{s,x}^t \geq M_{s,x}^{t-1}, \forall\ x \in X, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (6)$$

A site $s \in S$ is a $3\&4G$ site if and only if one $4G$ module is installed at it.

$$z_s^t \leq M_{s,4}^t, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (7)$$

$$z_s^t \geq M_s^t / K, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (8)$$

where K is an upper bound on the number of modules $4G$ that can be installed simultaneously in a site. Usually, a site does not have more than 4 antennas. Since we know the maximum number of modules installed at each antenna, hence we could define $K = 4NUM_4$.

New users of each technology will be added at each time period to the system. Also, $3G$ users will decide to upgrade their services. As a consequence, at each time period, the total number of users of each technology depends on the number of new users entering the system and the number of upgrades obtained with subsidies. Since we are considering in this section a deterministic version of the problem, we assume that the percentage of new users at each time period is known (i.e., it is a certain parameter) and equal to the average value nU^t. Moreover, as we have explained in the introduction, we assume that: 33% of the new users are $3G$, 67% of the new users are $4G$ and that, for each technology, the distribution of the new users in the sites is proportional to the initial number of users in each site.

$$U_{s,3}^t = U_{s,3}^{t-1} + 0.33 nU^t U_{s,3}^{t-1} - f(sub_t, P_{t-1}^{34G}) U_{s,3}^{t-1}, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (9)$$

$$U_{s,4}^t = U_{s,4}^{t-1} + 0.67 nU^t U_{s,4}^{t-1} + f(sub_t, P_{t-1}^{34G}) U_{s,3}^{t-1}, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (10)$$

Variables defining the number of modules and the number of new antennas to be installed are integer.

$$M_{s,x}^t \in \mathbf{Z}, \forall\ x \in X, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \tag{11}$$

$$y_s^t \in \mathbf{Z}, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \tag{12}$$

Variables deciding if a site is $3\&4G$ are binary ones.

$$z_s^t \in \{0, 1\}, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \tag{13}$$

For modeling reasons, we will represent the number of users as non-negative continuous variables. While each set of variables defined here represent exactly the reality (for ex, y_s^t inform us precisely the number of $4G$ antennas to be installed in site s at time period t), variables $U_{s,x}^t$, defined for each $t \in T \setminus \{0\}$, $s \in S$, $x \in X$, give us an approximation for the number of xG users in site s at time period t.

$$U_{s,x}^t \geq 0, \forall\ x \in X, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \tag{14}$$

Finally, variables informing subsidies offered for users are non-negative continuous variables.

$$sub_t \geq 0, \forall\ t \in T \setminus \{0\}. \tag{15}$$

The formulation introduced here is not completely described while function $f(sub_t, P^{34G})$, appearing in (1), (9) and (10), is not defined. The upgrade function is discussed in the next section.

3 Discussing the upgrade function

The function proposed by Orange to explain how customers respond to subsides is

$$f(sub_t, P_{t-1}^{34G}) = 1 - \exp(\alpha * P_{t-1}^{34G} * sub_t) \tag{16}$$

where $\alpha \leq 0$ is a given parameter. We show in Figure 2 this function for $\alpha = -0.01$ and different coverages (5%, 30%, 50% and 90%). We can see that, as stated in the beginning, this function increases with both the subsidy and the coverage. The parameter α modulates how customers respond to small subsides: bigger is $|\alpha|$, faster the function reaches its maximum value (in that case always one) with small subsides.

However, according to our understanding, the idea was to obtain, for a given coverage, an upgrade function with a S-shape; like the sigmoid ($f(x) = \frac{1}{1+\exp^{-x}}$) in Figure 3. Which means that, when subsidies are low, only very few people are willing to move from 3G to 4G; also the upgrade function grows very slowly for low subsides. Inversely, from a given value of subsidy, the function grows very fast as the subsidy increases, reaching very fast its maximum value of one (100% of acceptance to subsidy).

We propose two alternative ways of describing the upgrade function $f(sub_t, P_{t-1}^{34G})$: both of them model the behavior described here and may easily be incorporated in a ILP formulation. The first one is to consider a set of piecewise linear functions as depicted in Figure 4.

Our main idea is to propose a simple function, while keeping the idea of the S-shape. We made the assumption that when coverage is big, we need less subsidies because people tend to follow the crowd. Also, at a certain point, no matter how big is the subsidy, some people will not accept to upgrade; in that case, the function will never reaches the value one. These are the "Laggars" on the Diffusion of innovations model of Rogers [5]. In the contrary, the "Innovators" do not need subsidies and may even pay in order to try as soon as possible the new technology.

The second proposition is to make it even simpler: we extract from the function proposed in Figure 4 a table of values defined for each coverage and for a subset of significant subsides:

In this text, we will report only first experiments made with the upgrade function described by a table of values. Hence, in the following, we revise for this case the model presented in the last section.

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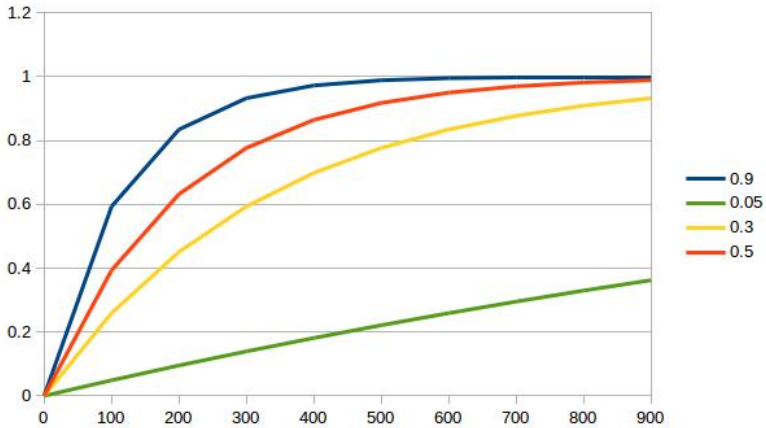

Figure 2: Function $f(sub_t, P_{t-1}^{34G}) = 1 - \exp(\alpha * P_{t-1}^{34G} * sub_t)$ with $\alpha = -0.01$.

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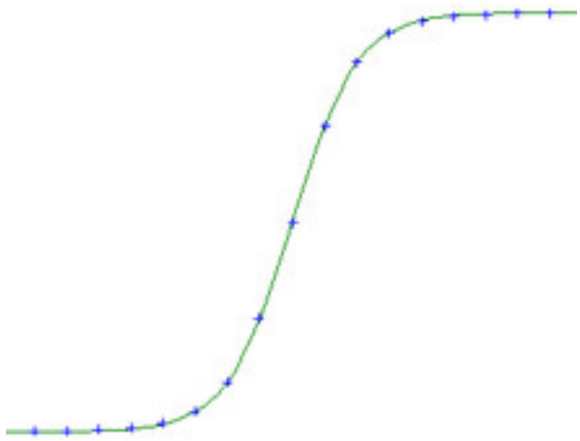


Figure 3: Sigmoid function.

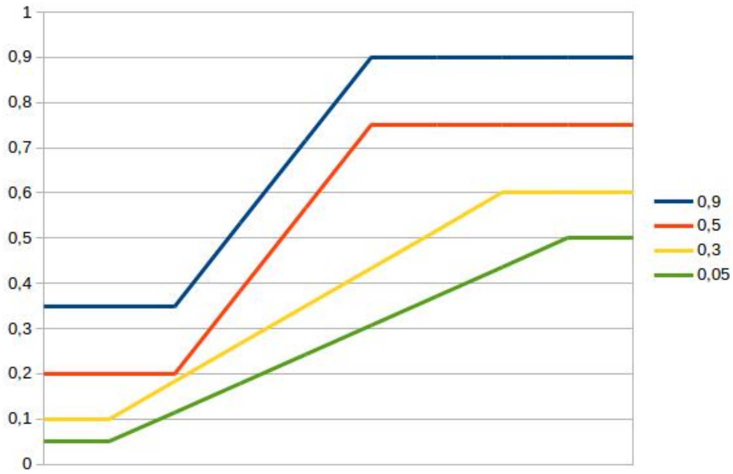


Figure 4: Upgrade function described by piecewise linear functions.

	subsides								
coverage	100	200	300	400	500	600	700	800	900
c1	5	15	20	25	30	40	50	50	50
c2	10	20	25	35	40	50	60	60	60
c3	20	20	35	55	75	75	75	75	75
c4	35	35	35	70	90	90	90	90	90

Table 1: Upgrade (in %)

3.1 Function not depending on the coverage

First, we revise the model presented in the Section 2 by supposing that the upgrade function is described by a table of values and also that it depends only on the subsidy offered. This is an even simpler case where we consider the function defined by one line of Table 1.

We assume to have a set of possible subsidies W. Consider a subsidy $w \in W$. Let us denote by $\overline{sub}_w$ the cost of w (i.e. the subsidy value) and by $\bar{f}_w$ the percentage of $3G$ users that became $4G$ when this subsidy is offered. Also, let us define a set of binary variables δ^t_w, for each $t \in T \setminus \{0\}$, taking value one if the subsidy w is offered at period t; and zero otherwise.

Under this assumption, the objective function (1) takes the form,

$$\text{Minimize} \sum_{t \in T\setminus\{0\}} \sum_{w \in W} \overline{sub}_w \bar{f}_w \delta^t_w \sum_{s \in S} U^{t-1}_{s,3} + \sum_{t \in T\setminus\{\bar{t}\}} \sum_{s \in S} (M^t_{s,3} C_{OP,3} + M^t_{s,4} C_{OP,4}) + \sum_{t \in T\setminus\{0\}} \sum_{s \in S} (y^t_s C_{AD}) \quad (17)$$

and constraints (9) and (10) are modified as follow.

$$U^t_{s,3} = U^{t-1}_{s,3} + 0.33nU^t U^{t-1}_{s,3} - \sum_{w \in W} \bar{f}_w \delta^t_w U^{t-1}_{s,3}, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (18)$$

$$U^t_{s,4} = U^{t-1}_{s,4} + 0.67nU^t U^{t-1}_{s,4} + \sum_{w \in W} \bar{f}_w \delta^t_w U^{t-1}_{s,3}, \forall\ s \in S, \forall\ t \in T \setminus \{0\}. \quad (19)$$

Since, for each time period t, at most one subsidy is offered, the following constraint is added to the formulation.

$$\sum_{w \in W} \delta_w^t \leq 1, \; \forall t \in T \setminus \{0\}. \tag{20}$$

3.2 Function depending on the coverage

We consider now the case in which the upgrade function depends on both the subsidy offered and the coverage. Remember that the coverage at a time period t is defined as the percentage of 3&4G sites at the end of time period $t-1$, i.e., $\frac{\sum_{s \in S} z_s^{t-1}}{|S|}$. Here, in addition to the set of possible subsidies W, we assume to have a set of ranges of coverage C. A range of coverage $c = (l_c, u_c) \in C$ is defined by a lower bound l_c and an upper bound u_c. Consider a subsidy $w \in W$ and a coverage $c \in C$. Let us denote by $\overline{sub}_w$ the cost of w (i.e. the subsidy value). Let us denote by $\bar{f}_{w,c}$ the percentage of $3G$ users that became $4G$ when subsidy w is offered and the coverage belongs to the range (l_c, u_c). We define now a binary variable $\delta_{w,c}^t$, for each $t \in T \setminus \{0\}$, taking value equal to one if, at period t, both the subsidy w is offered and the coverage belongs to the range (l_c, u_c); otherwise the variable take value equal to zero.

Under these assumptions, the objective function takes the form

$$\text{Minimize} \sum_{t \in T \setminus \{0\}} \sum_{w \in W} \overline{sub}_w \sum_{c \in C} \bar{f}_{w,c} \delta_{w,c}^t \sum_{s \in S} U_{s,3}^{t-1} + \sum_{t \in T \setminus \{\bar{t}\}} \sum_{s \in S} (M_{s,3}^t C_{OP,3} + M_{s,4}^t C_{OP,4}) + \sum_{t \in T \setminus \{0\}} \sum_{s \in S} (y_s^t C_{AD}) \tag{21}$$

and constraints (18) and (19) are modified as follow.

$$U_{s,3}^t = U_{s,3}^{t-1} + 0.33nU^t U_{s,3}^{t-1} - \sum_{w \in W} \sum_{c \in C} \bar{f}_{w,c} \delta_{w,c}^t U_{s,3}^{t-1}, \quad \forall\, s \in S, \forall\, t \in T \setminus \{0\}, \tag{22}$$

$$U_{s,4}^t = U_{s,4}^{t-1} + 0.67nU^t U_{s,4}^{t-1} + \sum_{w \in W} \sum_{c \in C} \bar{f}_{w,c} \delta_{w,c}^t U_{s,3}^{t-1}, \quad \forall\, s \in S, \forall\, t \in T \setminus \{0\}. \tag{23}$$

As said before, for each time period t, at most one subsidy is offered. The following constraint model this issue and is added to the formulation.

$$\sum_{w \in W} \sum_{c \in C} \delta_{w,c}^t \leq 1, \; \forall t \in T \setminus \{0\}. \tag{24}$$

Finally, the following set of constraints ensure that, for each time period $t \in T \setminus \{0\}$, variables $\delta_{w,c}^t$ are set according to the coverage at the end of time period $t-1$.

$$\sum_{w \in W} \delta_{w,c}^t - 1 \leq u_c - \frac{\sum_{s \in S} z_s^{t-1}}{|S|}, \; \forall t \in T \setminus \{0\}, \forall c \in C. \tag{25}$$

$$1 - \sum_{w \in W} \delta_{w,c}^t \geq l_c - \frac{\sum_{s \in S} z_s^{t-1}}{|S|}, \; \forall t \in T \setminus \{0\}, \forall c \in C. \tag{26}$$

Inequalities (25) state that if $\frac{\sum_{s \in S} z_s^{t-1}}{|S|}$ is greater than the upper bound u_c, then the associated variables $\delta_{w,c}^t$ are forced to be zero. On the other hand, inequalities (26) take into account the value of the lower bound and impose the associated variables $\delta_{w,c}^t$ to be zero whenever $\frac{\sum_{s \in S} z_s^{t-1}}{|S|}$ is less then l_c.

4 Linearizing the formulation

The formulation proposed in this text (for both cases, with or without coverage dependency) is a mixed non-linear formulation. This formulation can be easily linearized by adding new variables and a set of big-M constraints [4]. The non-linear terms of the formulation are listed in the following.

1. $z_s^t U_{s,4}^t$ appearing in constraints (4)–(5);
2. $\delta_w^t U_{s,3}^{t-1}$ appearing in the objective function (17) and in constraints (18)–(19).
3. $\delta_{w,c}^t U_{s,3}^{t-1}$ appearing in the objective function (21) and in constraints (22)–(23).

Each of these terms has the form $m = xY$, consisting of a binary variable x multiplied by a continuous variable Y. They can be linearized by changing all the occurrences of xY by m in the formulation and by adding constraints

$$m \leq Y, \tag{27}$$
$$m \leq Kx, \tag{28}$$
$$m \geq Y + K(x-1), \tag{29}$$

where K is a tight upper bound on the value of Y. For our formulation, we need to define upper bounds $K_{s,x}^t$ on $U_{s,x}^t$ which can be taken as follow.

$$K_{s,3}^t = U_{s,3}^0(1 + 0.33\overline{nU^t})^t \tag{30}$$
$$K_{s,4}^t = (U_{s,3}^0 + U_{s,4}^0)(1 + \overline{nU^t})^t \tag{31}$$

Notice that a tighter upper bound $K_{s,x}^t$ can be defined by taking into account upper bounds on the upgrade function used.

5 Computational results

In this section, we present a very preliminary evaluation of the aforementioned model. Our first goal is to understand what are the computational limitations. Furthermore, we would like to thoroughly look into the exams and infer valuable application-wise information. The mathematical formulation has been implemented in *Python* programming language, making use of the *SageMath* open-source mathematics software. The tests have been carried out on a machine with *Intel i7 CPU* and *4GB RAM.* The solver that has been used is *CPLEX* from *IBM.*

5.1 Instances

The results that we present are based on an instance provided by Orange and it consists of 461 sites. The initial coverage, that is the number of 3&4G sites over the total number of sites, is 27%. The total number of 3G and 4G users is 280447 and 117274, respectively, the installed modules are 2703 and 207 for 3G and 4G, respectively.

In the following, we present the values informed by Orange and that have been used for each input parameter. In Table 2, we report the estimated number of new users entering the network at each time period t, as well as the average usage per user $AUPU$ for both 3G and 4G technologies. We must note that since we have implemented a deterministic model, we have chosen without loss of generality to consider the average values nU^t.

For confidentiality reasons, input data related to cost and capacity are not given here.

t	0	1	2	3	4	5
$\overline{nU^t}$	0%	4%	8%	11%	16%	18%
nU^t		3%	6%	9%	13%	16%
$\underline{nU^t}$	0%	2%	4%	7%	10%	14%
$AUPU_3^t$	0.0106	0.0112	0.0120	0.0132	0.0149	0.0178
$AUPU_4^t$	0.0233	0.0317	0.0431	0.0586	0.0797	0.1084

Table 2: Estimated number of new users entering the system and average usage per user of both technologies 3G and 4G, for each time period t.

Regarding the upgrade function, we have considered the four lines of Table 1 on the formulation described in Subsection 3.1. Thus, we have 4 different functions that can represent the customer behavior with respect to different percentages of coverage. As described in the formulation (see Subsection 3.1), we have considered each of the 4 cases isolated to obtain our first results. This has been done as a first step, in order to look in depth at each function independently and extrapolate more precise information. As we can expected, when considering a given coverage we can eliminate from the formulation a set of subsidies that give the maximum value of the function; for example, for coverage c_4 subsides 600 to 900 can be eliminated. Even with this reduction, we were not able to solve the problem after 3 hours of computation, when more than 5 subsides were considered. Hence, we decided to consider the subset of subsides in Table 3. Four different upgrade functions, denoted f^1, f^2, f^3, and f^4, which have been considered for the experiments are depicted in Table 3. All these values of $\bar{f}_w$ for each function f_i and subsidy $\overline{sub}_w$, are reported in Table 3.

$\overline{sub}_w$	100	200	300	400	500
f_1	5	15	20	25	30
f_2	10	20	25	35	40
f_3	20	20	35	55	75
f_4	35	35	35	70	90

Table 3: Value of $\bar{f}_w$ for each function f_i and subsidy $\overline{sub}_w$.

5.2 Numerical results

Following, in Table 4 we can see the final results that have been acquired by solving the program for each upgrade function. In all the computations, we have imposed a plausible time limit of 60 minutes. Since the time limit was not sufficient for the program to terminate, on the second column we present the *optimality gap* as reported by the solver. Finally, in the last columns we present the total number of installed 4G antennas (#Ant), the total number of users (#User) and the total number of modules (#Mod) at the end of the time horizon for both 3G and 4G technologies.

				3G		4G	
	time	gap	#Ant	#User	#Mod	#User	#Mod
f_1	60	3.31	235	234698	2892	243438	872
f_2	60	2.48	219	203850	2934	274285	862
f_3	60	2.06	287	125639	2884	352497	1058
f_4	60	1.12	327	60706	2906	417429	1189

Table 4: Numerical results for each upgrade function.

We can see in graphics 5–8 the subsides that have been chosen at each time period, for each of the 4 functions considered, as well as the evolution on both the number of users and modules of each technology.

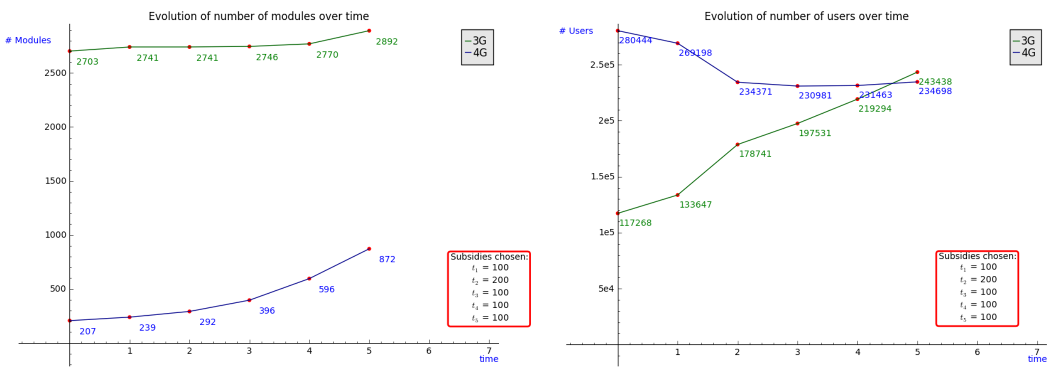
Looking into Table 4 and graphics 5–8 we note the following facts:

→ There is a huge increase in 4G modules. Starting from 207 we end up with at least 872, meaning a 400% − 500% increase. On the other hand there is a very small increase in the number of 3G modules. Starting from 2703 we reach only up to 2934, showing a less than 10% increase. This behavior can be explained if we consider the capacity constraints that have been imposed on the 3G antennas. As we have mentioned above, since the aim of Orange is to invest on new technologies the limit has been set to only the current 1 existing antenna at each site. Also, many of the sites are full, with respect to the modules, even from the initial time period. So, there is a small increase in 3G modules and a continuous growing number of 4G modules, explaining this behavior.

→ Almost in every upgrade function and at each time period the choice that minimizes the total cost is the one with the smallest subsidy. This can be justified if we look into the percentages of 3G users that a given f_i transforms in comparison with the percentage of new users nU^t arriving in the system at each time period. Apart from f_1 and f_2, for $\overline{sub}_1$, we observe that the percentage of new users is strictly smaller than the upgrade provided by function f_i. Thus, all the 3G users can be satisfied by the respective equipment already installed. Thus, as long as the percentage of new arrivals in 3G, which is $\frac{nU^t}{3}$, is smaller than the percentage of users that transform to 4G, the minimum subsidy will always be chosen. Apart from the bigger number of people that will take the subsidy also potentially more 4G equipment has to be installed, which comes with a cost.

→ As the function f_i transforming users from 3G to 4G increases, the cost increases as well (apart from f_1 and f_2 where the difference is insignificant). This comes as a result from something we have already mentioned. If we observe f_3 and f_4, even the minimum possible subsidy $\overline{sub}_1$ produces a big increase in the set of users. The overall cost will be increased for 2 main reasons. Since f_i is relatively bigger, even by looking into the subsidies, the value of the objective function will grow. Furthermore, the cost of the equipment to satisfy the increasing capacity generated by migration is significant. This is obvious from the results where, e.g. in f_1 in the end of the time horizon we have 872 4G modules whereas in f_4 we have 1189. On the other hand, since we have assumed that we cannot decommission any 3G equipment, large values of f_i will mean that a growing number of 3G equipment will be maintained but will be underused.

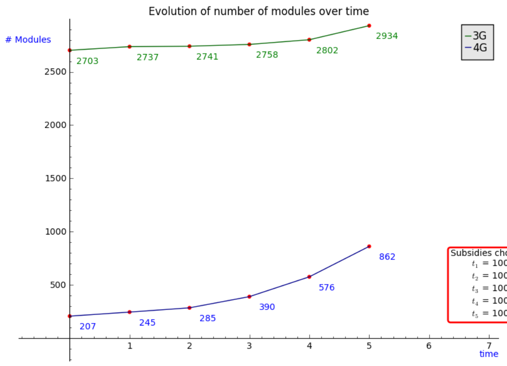
For further understanding of the results and the observations, the following figures should be considered together with Table 4. The detailed curves with the evolution over the time periods in the number of users and modules of each technology are depicted.



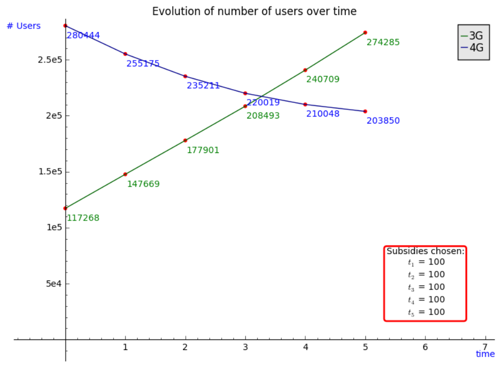
(a) Number of modules

(b) Number of users

Figure 5: Number of modules and users for each period t with upgrade function f_1.

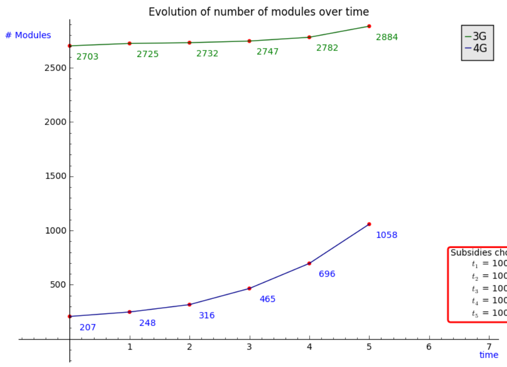


(a) Number of modules

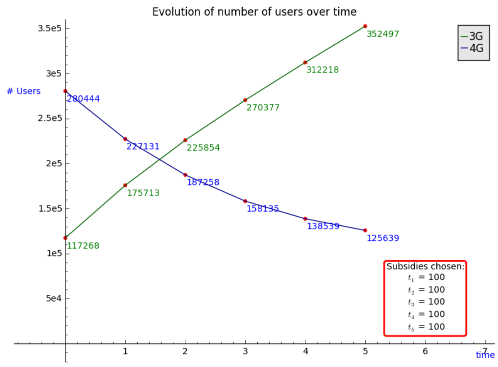


(b) Number of users

Figure 6: Number of modules and users for each period t with upgrade function f_2.

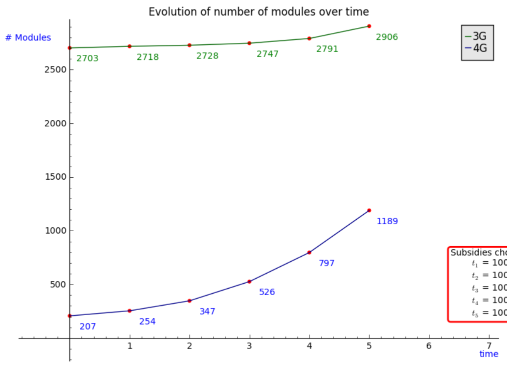


(a) Number of modules

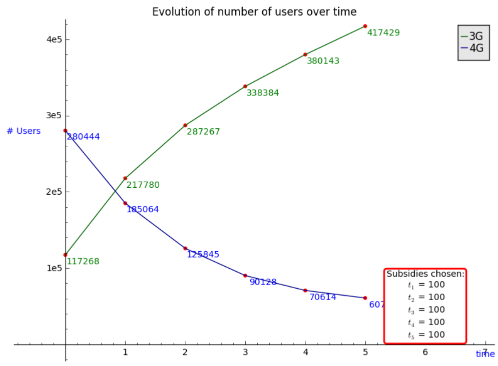


(b) Number of users

Figure 7: Number of modules and users for each period t with upgrade function f_3.



(a) Number of modules



(b) Number of users

Figure 8: Number of modules and users for each period t with upgrade function f_4.

6 The robust case

In this section, we present a preliminary discussion on an uncertain version of the MMPP, i.e., when we do not know the percentage of new users added to the model at each time point $t \in T$. For each $t \in T \setminus \{0\}$, let us define the input data:

$\hat{nU}^t \in [\underline{nU^t}, \overline{nU^t}]$, the percentage of new users of both technologies at time period t.

In this work, we adopt a robust min-max criterion to assess the cost of feasible solutions to the problem. This means that we look for a solution that is feasible for each possible attribution of the uncertain parameters, which we call here-on a scenario, and that minimizes the cost of the maximum cost evaluation of such solution over all possible scenarios. The set of all possible scenarios, which means the realizations of the uncertain parameter against which one must be protected is called the uncertainty set.

Clearly, the formulation of such robust optimization model is connected with the definition of this uncertainty set and this definition depends on the suppositions made on the problem being solved. The sets $[\overline{nU^t}, \underline{nU^t}]$, $t \in T \setminus \{0\}$, describe the uncertainty set in our problem. We are assuming here that each uncertain parameter belongs to an interval and the uncertainty set is obtained by the Cartesian product of all such intervals, such uncertainty model is denominated *box uncertainty set* and is considered for example in [1, 2].

We need to introduce additional notations before we can proceed. Let us define

$$nU = \times_{t \in T\setminus\{0\}} [\overline{nU^t}, \underline{nU^t}].$$

We will denote by nU^t the projection of nU in the space defined by the components corresponding to $t' \leq t$. Also, let us define $\hat{nU}_t = (\hat{nU}^1, \hat{nU}^2, \ldots, \hat{nU}^t)$, for $t \in T \setminus \{0\}$ and $\hat{nU}_0 = 1$.

In the robust version of our problem, variables U, M, y, z, and δ, are adjustable ones, i.e, they consist of "wait and see" decisions and define a set of decisions that depend on the uncertain parameters. One must observe that in the problem defined here all variables are adjustable ones, i.e., there is no first-stage decision. More precisely, for a given $t \in T \setminus \{0\}$, variables $U^t_{s,x}$, $M^t_{s,x}$, y^t_s, z^t_s, and $\delta^t_{w,c}$[1], for each possible value of s, x, w and c, depend on the vector of uncertain parameters $\hat{nU}_t \in nU^t$ and became functions of the uncertain parameters. As a consequence, the robust counterpart formulation of our problem looks for a set of functions that minimizes the expansion cost in the worst case scenario, i.e., the maximum value of the objective function achieved over the set of uncertain parameters. The Min-Max Adjustable Robust Counterpart formulation is as follows.

$$\begin{aligned}
\text{ARC-MMPP}: \min_{U(.),M(.),y(.),z(.),\delta(.)} \quad & \max_{\hat{nU} \in nU} \sum_{t \in T\setminus\{0\}} \sum_{w \in W} \overline{sub}_w \bar{f}_w \delta^t_w(\hat{nU}_t) \sum_{s \in S} U^{t-1}_{s,3}(\hat{nU}_{t-1}) \\
& + \sum_{t \in T\setminus\{\bar{t}\}} \sum_{s \in S} (M^t_{s,3}(\hat{nU}_t) C_{OP,3} + M^t_{s,4}(\hat{nU}_t) C_{OP,4}) \\
& + \sum_{t \in T\setminus\{0\}} \sum_{s \in S} y^t_s(\hat{nU}_t) C_{AD},
\end{aligned} \tag{32}$$

subject to constraints (2)–(8),(11)–(14) and (18)–(20) when the adjustable variables are replaced by the respective functions of the uncertain parameter.

Remark 6.1. Let us denote by ARC-MMPP($\hat{nU}$) the ARC-MMPP obtained when the uncertain parameter is fixed and equal to $\hat{nU}$. Let $(U', M', y', z', \delta')$ be the optimal solution of the ARC-MMPP($\overline{nU}_{\bar{t}}$) when the uncertain parameter is fixed and equal to $\overline{nU}_{\bar{t}} = (\overline{nU^1}, \overline{nU^2}, \ldots, \overline{nU^{\bar{t}}})$. Let also v' be the associated value

[1] In this section we are considering the formulation discussed in Section 3.1; clearly the same conclusions are valid for the formulation in Section 3.2.

of the objective function. The value v' is an upper bound for the optimal solution of the ARC-MMPP($\hat{nU}$), $\hat{nU} \in nU \setminus \overline{nU}_{\bar{t}}$.

Consider a vector $\hat{nU} \in nU \setminus \overline{nU}_{\bar{t}}$. A feasible solution for ARC-MMPP($\hat{nU}$) is given by $(M(.), y(.), z(.), \delta(.)) = (M', y', z', \delta')$ and function $U(.)$ defined according to constraints (18) and (19). This feasible solution gives the same value of costs (ii) and (iii) given by the optimal solution $(U', M', y', z', \delta')$ of ARC-MMPP($\overline{nU}_{\bar{t}}$). Moreover, cost (i) must be less than or equal to the one obtained by $(U', M', y', z', \delta')$ since the number of 3G users defined by $U(.)$ will be necessarily less than or equal to the number of 3G users defined by U'.

The consequence of Remark 6.1 is that, as the problem is announced today, the optimal solution of ARC-MMP, i.e., the worst case scenario for the robust version of the MMPP, is given by the deterministic solution of the MMPP defined with $\overline{nU}_{\bar{t}} = (\overline{nU^1}, \overline{nU^2}, \ldots, \overline{nU^{\bar{t}}})$.

7 Perspectives

Considering the preliminary experiments reported in Section 5 and the discussion presented in Section 6, we point out some topics to be discussed with the Orange team. These discussions can give us fruitful directions to continue this research.

- Deterministic Formulation:
  - Discuss if other constraints and costs/gains should be considered.
  - The current formulation can be improved: other linearization techniques can be used; additional constraints can be included in order to strength the formulation.
  - Additional experiments need to be realized in a high power machine.
- Upgrade Function:
  - Discuss if the proposed upgrade functions are appropriated to model the telecommunications market.
  - An ILP formulation can be described by using an approximation of the piecewise linear function proposed.
- Robust Formulation:
  - Other uncertainty sets must be considered as the budget uncertainty set.
  - The inclusion of new constraints related with the coverage and/or new costs/gain will totally change our conclusions about the use of the box uncertainty set.
  - Experiments on the Robust versions of the problem need to be realized (of course, it does only make sense if other uncertainty sets and/or new constraints are considered).

  Also, another interesting perspective could be to consider a non-cooperative scenario between service providers. In this context, subscribers may have an interest to change to another provider based on different criteria like quality, coverage, subsidies. Therefore, the decisions of the service provider depends on the decision of the other service provider, and a war of prices scenario may appear. Then, looking at the existence of equilibrium strategy/decision is an important aspect for optimizing long term investment decision for a service provider like Orange, in a competitive market like the telecommunication one.

8 Acknowledgements

We would like to kindly thanks Matthieu Chardy and Adam Ouorou, from Orange Labs, for having accepted to present this problem on the 117th ESGI, held in Avignon, and for providing us with some data representing a real case of the problem.

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