Portuguese Study Groups' Reports

Report on "Time Reduction of the Packing Process"

Problem presented by Savana at the

119th European Study Group with Industry

27th June – 1st July 2016

ESTGF and ESEIG, P. Porto

Porto

Portugal

July 29, 2016

Problem presented by:
Jorge Fernandes (Administrador - Savana Calados SA)
Catarina Monteiro (Dept Logistica - Savana Calados SA)

Study group contributors:
Rui Almeida (UBI), José Duque (UBI)
Flora Ferreira (UP/ESTGF), Manuel V.C. Vieira (UNL)

Report prepared by:
Rui Almeida (e-mail: ralmeida@ubi.pt)
José Duque (e-mail: jduque@ubi.pt)
Flora Ferreira (e-mail: flora.ferreira@gmail.com)
Manuel V.C. Vieira (e-mail: mvcv@fct.unl.pt)

Executive summary

Savana is a company in the footwear sector which is over 27 years old and has more than 150 employees. This company specializes in children's footwear of sizes ranging from 18 to 40. Each pair is individually packed in a box which is customized for each client. The ideal size of each box depends on the model of the footwear and the position in which it is placed inside the box. These boxes are ordered from an external supplier that has only certain measures available, so boxes of the same size have to be used for footwear of various sizes and models.

Due to a frequent introduction of new models in the production environment, the box sizes are initially set manually in an experimental procedure (testing), which is often time consuming. Savana challenged ESGI's participants to study their packing process, in order to reduce the variety of box sizes, the empty space inside the boxes and to eliminate the need to perform testing, thereby reducing the time and increasing the efficiency of the packing process.

Furthermore, the footwear ordered by each customer is packed into large boxes, which will henceforth be referred to as containers. With regard to these large boxes, various designs and sizes can be delivered to a single client. The dimensions, weight and forms of these are subject to the customer's specifications. In this context, Savana intends to determine automatically the sizes of the containers to be sent to each customer and how to arrange the individual boxes for each client's order.

This report tells how to automatize and speed up the overall process. It describes how to automatically assign shoes to boxes, and gives a manner to pack the shoe boxes, in such way that permit to reduce the size of the card box.

Savana should be aware that this is not yet a ready to use solution, because more data analysis need to be done, in order to improve and make the method reliable. Furthermore, during implementation it may appear new important challenges.

Contents

1 Introduction

The problem presented by Savana is by far too complex to permit an efficient overall approach (see [1]). Hence, the problem must be split into two subproblems:

1. Assigning footwear to boxes;

2. Packing boxes into containers.

While the first subproblem is not complex, the second one is still too complex to justify a phased approach. Therefore, the following three interconnected ordered decisions should be made:

1. Determine the number of containers to be used;

2. Distribute the boxes within the containers;

3. Calculate the sizes of the containers.

2 Assigning shoes to boxes

2.1 Problem description

As a rule, all shoes/sneakers/gloves are arranged in the boxes as shown in Figure 1 and the boots are arranged as shown in Figure 2.

Figure 1: Arrangement for sneakers ref 17879.

Figure 2: Arrangement for boots ref 17137.

The box size should be chosen so that there is not too much empty space inside the box and at the same time there is no deformation of the footwear due to lack of space, as illustrated in Figure 3.

For practical reasons, for a certain reference, one should not use a large variety of boxes.

It is desirable to develop a process whereby the ideal box sizes can be determined, based on the measurements of the footwear (length, height, ...)

Figure 3: Inappropriate size boxes.

and some parameters (offset or common space, space between footwear and box, ...).

2.2 Current process

The ideal shoes box size may be calculated according to the following formulas:

$$\text{width of box} = (2 \times h) - c + e$$

where h denotes the shoe height, c denotes the space which is common to both shoes, and e denotes the free space between the shoes and the box;

$$\text{length of box} = l + e$$

where l denotes the shoe length, and e denotes the free space between the shoes and the box.

With regard to boots, the ideal box size may be calculated according to the following formulas:

$$\text{width of box} = l + e$$

where l denotes the boot length, and e denotes the free space between the boots and the box;

$$\text{length of box} = (2 \times h) - c + e$$

where h denotes the boot height, c denotes the space which is common to both boots, and e denotes the free space between the boots and the box.

It should be noted that the heights and lengths include the soles of the footwear.

Presently, for each model, the assignment of a certain footwear to a particular box involves the physical measurement of the footwear, and therefore the existence of tests/prototypes for each size of footwear.

2.3 Problem analysis

To have a better understanding of the problem, we were provided with the footwear measures of some existing numbers of two different models, the available box dimensions for a particular customer, and the boxes that are currently used for each number of these two models. This data is given in the table below. From the data available, it appears that there is a linear relationship between the height of the footwear and the common space, as shown in the charts in Figure 4.

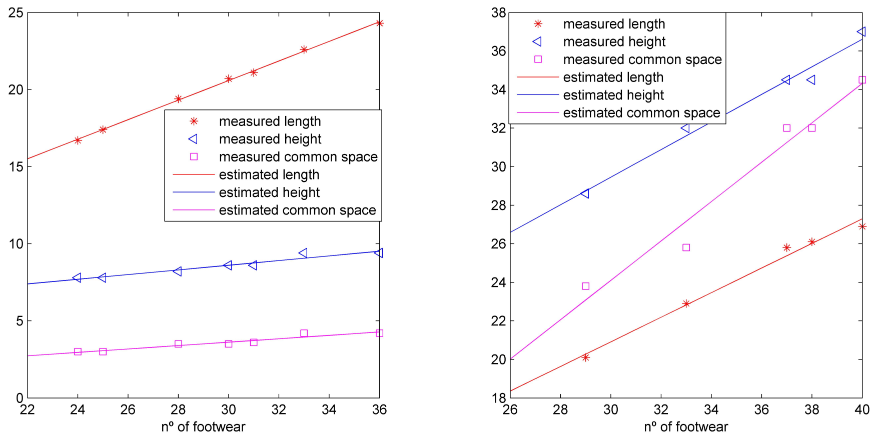

Figure 4: Real measurements and estimated values for sneakers ref 17879 (left) and for boots ref 17137 (right).

There are two clearly distinct models, shoes/sneakers and boots. Preliminary studies indicate that for each type of footwear the pattern holds. Thus, for each new model, after constructing the sample specimen and measuring the height, width and actual common space for this number, it is possible to extrapolate the measures for any number of this model using linear regression. Starting from the formulas for the box sizes, we can calculate the minimum box dimensions for each number, as exemplified in Table 1 for sneakers and in Table 2 for boots.

Based on these values and the existing boxes, the appropriate box for each number can be indicated. To avoid too much free space inside the box, it is further imposed that the space between the footwear and the box is less than 5 cm and that the minimum required "housing" area is at least 70% of the total box area. It is then possible to obtain a matrix with all the admissible boxes for each number, as exemplified in Tables 3 and 4.

shoe size	length of the shoe	width of the shoe	common space estimated	length of the box	width of the box
22	15.5	7.40	2.74	16.5	13.1
23	16.1	7.55	2.85	17.1	13.2
24	16.8	7.70	2.96	17.8	13.4
25	17.4	7.85	3.07	18.4	13.6
26	18.0	8.0	3.18	19.0	13.8
27	18.7	8.15	3.29	19.7	14.0
28	19.3	8.30	3.40	20.3	14.2
29	20.0	8.46	3.51	21.0	14.4
30	20.6	8.61	3.62	21.6	14.6
31	21.2	8.76	3.73	22.2	14.8
32	21.9	8.91	3.84	22.9	15.0
33	22.5	9.06	3.95	23.5	15.2
34	23.1	9.21	4.06	24.1	15.4
35	23.8	9.37	4.17	24.8	15.6
36	24.4	9.52	4.28	25.4	15.8

Table 1: Estimated box sizes to pack sneakers ref 17879

shoe size	length of the boot	width of the boot	common space estimated	length of the box	width of the box
26	18.4	26.6	20.0	34.2	19.4
27	19.0	27.3	21.0	34.6	20.0
28	19.6	28.0	22.1	35.0	20.6
29	20.3	28.7	23.1	35.4	21.3
30	20.9	29.5	24.1	35.8	21.9
31	21.6	30.2	25.1	36.2	22.6
32	22.2	30.9	26.1	36.6	23.2
33	22.8	31.6	27.2	37.0	23.8
34	23.5	32.3	28.2	37.4	24.5
35	24.1	33.0	29.2	37.9	25.1
36	24.7	33.7	30.2	38.3	25.7
37	25.4	34.5	31.3	38.7	26.4
38	26.0	35.2	32.3	39.1	27.0
39	26.7	35.9	33.3	39.5	27.7
40	27.3	36.6	34.3	39.9	28.3

Table 2: Estimated box sizes to pack boots ref 17137

	Box sizes				
No.	21×14×9	23.5×16.5×9.2	25×15×9.5	25×17×9.5	27.5×17.5×10.5
22	1	0	0	0	0
23	1	0	0	0	0
24	1	0	0	0	0
25	1	0	0	0	0
26	1	1	0	0	0
27	0	1	0	0	0
28	0	1	1	1	0
29	0	1	1	1	0
30	0	1	1	1	0
31	0	1	1	1	0
32	0	1	1	1	1
33	0	1	0	1	1
34	0	0	0	1	1
35	0	0	0	1	1
36	0	0	0	0	1

Table 3: Admissible boxes for each size of sneakers ref 17879.

	Box sizes				
No.	36×25×11	40×30×10	40×30×11	43×30×10	45×32×11
26	1	0	0	0	0
27	1	0	0	0	0
28	1	0	0	0	0
29	1	1	1	1	0
30	1	1	1	1	0
31	0	1	1	1	0
32	0	1	1	1	1
33	0	1	1	1	1
34	0	1	1	1	1
35	0	1	1	1	1
36	0	1	1	1	1
37	0	1	1	1	1
38	0	1	1	1	1
39	0	1	1	1	1
40	0	1	1	1	1

Table 4: Admissible boxes for each size of boots ref 17137.

Since for each footwear size we can have different types of boxes, an optimization process is described, whereby the variety of boxes is minimized. The following is a simple algorithm which can be used for this purpose:

Algorithm 1:

- From Table 3 or 4, choose the smallest footwear number that fits well in the least number of boxes;
- From among the boxes that can be used for this size, choose the one which can be used for the greatest amount of different shoe sizes;
- Assign that shoe size to that box;
- Assign that box to all the shoe sizes that are possible;
- Return to the beginning and repeat the process for the shoe sizes that have not been assigned to a box;
- The process ends when all the numbers have been assigned to a box.

The results for the models considered, using the Algorithm 1, are shown in Tables 5 and 6.

Sneakers ref 17879	
No.	box
$22-25$	$21 \times 14 \times 9$
$26-33$	$23.5 \times 16.5 \times 9.2$
$34-36$	$27.5 \times 17.5 \times 10.5$

Table 5: Optimal boxes for each size of sneakers ref 17879.

Boots ref 17137	
No.	box
$26-30$	$36 \times 25 \times 11$
$31-40$	$40 \times 30 \times 10$

Table 6: Optimal boxes for each size of boots ref 17137.

Although the results obtained through this process not differ much of Savana results obtained by a manual process there is a small improvement. In particular, the sneakers with number 32 and 33 may be placed in the middle box ($23.5 \times 16.5 \times 9.2$) instead of being placed in the larger box ($27.5 \times 17.5 \times 10.5$) as is being done currently in Savana.

3 Packing boxes into containers

Given an order indicating the various footwear sizes, such as it is necessary

No.	23	24	25	26	27	28	29	30	31	32	33	34	35	36	Total
Order	2	5	7	15	25	25	25	25	25	25	25	22	20	10	256

to:

1. Determine the number of containers to be used;
2. Distribute the boxes within the containers;
3. Calculate the sizes of the containers.

It should be noted that the container must comply with:

$$(\text{height} + \text{width}) \times 2 + \text{length} \leq 300\,\text{cm}.$$

All footwear of a certain order will be packed as efficiently as possible within the 300 cm limit, according with the formula given above. Each order must be packed in the least possible number of containers. Each container must have a maximum length of 80 cm, and the width and height must each have a maximum of 60 cm. Shoes containers can take no more than 20 boxes, and boot containers no more than 10 boxes.

Note that the boxes must be placed with the label facing up, and with a maximum of 7 boxes in a single row. If the number of boxes is greater than 7 but less than or equal to 14, these must be placed in two consecutive rows.

In order to minimize the empty space inside the containers, it is suggested that if the number of boxes is 9, 11, 13, 15, 17, 18, or 19, these should be placed in two containers, as illustrated below.

3.1 Number of containers

Consider the total number NT of footwear in each order. We divide the number NT by 20 (in case of shoes/sneakers) and by 10 (in case of boots):

$$NT = 20 \times Q + R,$$

where Q is the number of containers with 20 shoes (10 in case of boots) and R the remainder of the division, with $0 \leq R < 20$.

- If $R = 0$, then the number of containers needed is $NE = Q$.
- If $R = 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 16$, then $NE = Q + 1$.
- If $R = 9, 11, 13, 15, 17, 18, 19$, then $NE = Q + 2$.

3.2 Distribution of boxes within containers

The notation (NB, NR, NL) is used to indicate how the boxes are placed inside a container; NB the number of boxes in each row, NR the number of rows and NL is the number of levels.

Each of the Q containers is packed according to the arrangement $(5, 2, 2)$. The distribution of the shoe boxes, within the Q containers, is done according with the algorithm below.

Algorithm 2: For each order, a container is filled with boxes in the following way:

- start with an empty container and with the smallest shoes;
- while the container has less than 20 boxes:
  - if the quantity of shoes of the same size is less than the quantity required to fill the space available, then
    - assign this quantity to the container;
    - move to the next shoe size;
  - else
    - assign boxes to fill the available empty space;
    - decrease the quantity of the corresponding shoe size;
    - move to the next container;
- end.

This algorithm was implemented in Excel and the resulting output is shown in Table 7.

Nr.	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**Total**
Order	**2**	**5**	**7**	**15**	**25**	**25**	**25**	**25**	**25**	**25**	**25**	**22**	**20**	**10**	**256**
1	2	5	7	6											20
2				9	11										20
3					14	6									20
4						19	1								20
5							20								20
6							4	16							20
7								9	11						20
8									14	6					20
9										19	1				20
10											20				20
11											4	16			20
12												6	14		20
13													6	10	16

Table 7: Example of an order packing.

The remaining R shoe boxes are distributed, within 1 container or 2 containers, according with the following schemes.

- If $R = 0$, the process is complete. If $R = 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 16$ or 20, the last container is filled according to the following arrangement:

$R = 1, 2, 3, 5, 7$	arrangement	$(R, 1, 1)$
$R = 4, 6, 8, 10, 12, 14$	arrangement	$(R/2, 2, 1)$
$R = 16, 20$	arrangement	$(R/4, 2, 2)$

- If $R = 9, 11, 13, 15, 17, 18$ or19, the last 2 cardboard boxes are filled according to the following arrangements:

$R = 9$	arrangement	$(5, 1, 1) + (4, 1, 1)$
$R = 11$	arrangement	$(5, 1, 1) + (6, 1, 1)$
$R = 13$	arrangement	$(5, 2, 1) + (3, 1, 1)$
$R = 15$	arrangement	$(5, 2, 1) + (5, 1, 1)$
$R = 17$	arrangement	$(6, 2, 1) + (5, 1, 1)$
$R = 18$	arrangement	$(6, 2, 1) + (6, 1, 1)$
$R = 19$	arrangement	$(6, 2, 1) + (7, 1, 1)$

In this last case shoe boxes should be split into two containers such that each container includes shoe boxes as similar as possible. Therefore, an easy adaptation of Algorithm 2 applies here.

3.2.1 Packing method

At this stage, it is already known which shoe boxes are to be packed into the same container. An algorithm which determines precisely how the shoe boxes should be arranged inside the container is now presented. Since the packing is done manually, this algorithm must be simple, thus the container cannot have the optimum size, because otherwise packing would be as difficult as making puzzles.

Algorithm 3:

- Sort boxes by the shoe size number in a non-increasing order;
- Packing starts at the lower right-hand corner, continuing up to the upper right-hand corner, then to the upper left-hand corner and down to the lower left-hand corner (see Figure 5);
- Filling the second level now starts at the lower left-hand corner and continues in a similar way to the process used for filling the first level, but in the opposite direction. (see Figure 6).

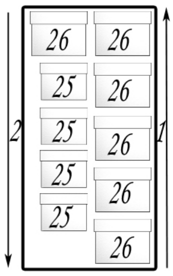

Figure 5: Packing the first level.

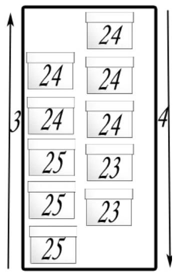

Figure 6: Packing the second level.

3.3 Container sizes

After knowing which boxes are to be assigned to a given container, its size can be accurately estimated.

As an example, consider the following Bisgaard shoes order:

size	23	24	25	26	27	28	29	30	31	32	33	34
order	1	1	1	1	1	1	1	1	1	1	1	1
box size	$21 \times 14 \times 9$			$23.5 \times 16.5 \times 9.2$						$27.5 \times 17.5 \times 10.5$		

According to the position of the boxes (see Figure 7), the optimum container for this order should have the following dimensions:

- length $= 3 \times 10.5 + 3 \times 9.2 + 1 = 60.1$ cm
- width $= 2 \times 16.5 + 1 = 34$ cm
- height $= 27.5 + 1 = 28.5$ cm

  and volume 58236.9 cm^3.

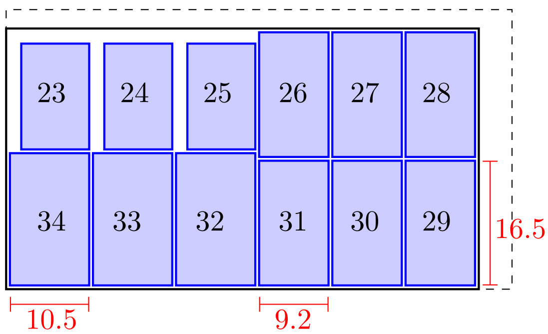

Figure 7: Packing example.

Savana would use a container of volume $67 \times 37 \times 29 = 71891$ cm^3 for this order, and hence this new algorithm permits a volume reduction of 13654.1 cm^3, approximately 13 litres less in volume. In addition, the boxes will be tightly packed, and so it is less likely that the customer will receive a "messy" container.

4 Conclusions and recommendations

This reports indicates how the overall packing process can be automatized and speed up. These ideas must be implemented, tested and tuned. Here is explained the overall idea, but this is not a "ready to use" solution. Savana should be aware that implementation details may still pose important challenges.

The process of assigning shoes to boxes is simple and can be implemented in a computer system. It gives a realistic estimate of the shoe box to be used for each number of the model introduced. This process only uses the measurements obtained from a sample number. There are, however, some observations to be made:

- The linear model seems to fit better to the shoes than to the boots. It is needed to collect more data to be able to validate or not the model and, if not, to think on other models, possibly nonlinear.
- The values suggested for the space between the shoes and the box and for the ratio between the minimum necessary area and the box area are only examples, which can be optimized through several trials.
- The algorithm to minimize the variety of boxes to be used for each model can be improved to contemplate any other practical issues.

The packing algorithm still has room for improvement, specially for the two-floor packing case. However, it is not clear if this is desirable, because the packing is done manually. The packing can not be a hard puzzle to solve.

References

[1] Bortfeldt, A. and Homberger, J., (2013) Constraints in container loading - A state-of-the-art review. EJOR 229(1), 1–20.

[2] Bazaraa, M. S., Jarvis, J. J. and Sherali, H. D. (1990) Linear Programming and Network Flows, John Wiley and Sons, New York.

[3] Ramalhete, M., Guerreiro, J. and Magalhães, A. (1985) Programação Linear, Vol. 1 e 2, McGraw-Hill, Lisboa.

[4] Hillier, F. and Lieberman, G. (1995) Introduction to Operations Research, McGRAW-HILL International Editions, sixth edition.

[5] Tavares, L., Oliveira, R., Themido, I. and Correia, F. (1996) Investigação Operacional, McGraw-Hill.

[6] Burden, R.I. and Faires, J.D., Numerical Analysis, PWS-Kent, Boston, 1988.

[7] Conte, S.D. and Boor, C. de, Elementary Numerical Analysis, Mc Graw-Hill, NY, 1980.

[8] Pina, H., Métodos Numéricos, Mc Graw-Hill, Alfragide, 1995.