MULTI-VARIABLE RELATIONSHIPS IN A BATCH ANNEALING PROCESS

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Abstract

Empirical models are developed to predict the mechanical properties of four groups of cold-rolled annealed steels, based on upstream processing variables in the hot-rolling process, annealing conditions, the chemical composition of the steel and product details. Various linear models are considered but the simplest approach of using a multiple linear regression model is considered the most satisfactory. This analytical tool will allow better understanding of the steel making process and the ability to vary input parameters to improve the product. In particular the number of coils which fail mechanical testing may be able to be reduced, with a subsequent fall in production costs.

1. Introduction

Cold-rolled steel is manufactured from hot-rolled coil that has been chemically cleaned on the pickling line before being rolled. Cold rolling reduces the thickness of the steel and at the same time changes its mechanical properties. Specifically, the resulting steel is harder, but less malleable. Some steel is sold as a full hardness product, but other rolls are heated in a controlled atmosphere, that is they are annealed, to increase the formability of the steel. The annealed product is known as cold-rolled annealed steel. In some cases the annealed steel is rolled again after annealing, this process being termed rerolling.

Figure 1 below shows the hot-rolling process. This is followed by cold rolling, possible annealing, and possible rerolling. A schematic diagram

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of the possible processing pathways for cold-rolled steel is shown in Figure 2.

For this project New Zealand Steel Ltd provided a large amount of historical data concerning the mechanical properties of steel produced in their mill. Data was also provided on predictor variables which might affect the mechanical properties of the steel produced. Briefly, the predictor variables comprised three groups: processing variables such as rolling and coiling temperatures; product details and annealing conditions; and the chemical composition of the steel. A schematic outline of the data format in Excel is given in Figure 3. The data provided is discussed in more detail in Section 3. Figures 1, 2 and 3 come from the PowerPoint presentation given to MISG2006 by NZ Steel. Diagrams showing NZ Steel processes are also available from the NZ Steel website at *http://www.nzsteel.co.nz/nz/go/about-new-zealand-steel/operations*.

Crucial to the economic performance of the mill is the ability to control process variables so that the steel produced possesses mechanical properties which conform to a number of international standard specifications. The actual specifications vary according to the particular product. Steel which does not conform to the intended specification must be reprocessed or sold for a lower price, with obvious economic costs to NZ Steel.

NZ Steel set the Study Group the task of developing a mathematical model that would allow them to predict mechanical properties from the predictor variables such as chemistry and process characteristics. Such a model would allow the company to predict properties of products when changes are made to the chemistry or process parameters and hence provide a useful tool to improve the mechanical properties of existing products, to reduce product testing failure rates and for the development of new products. This description is very similar to that of the project presented by NZ Steel to MISG 2005, see [7]. However the current project concerns a different part of NZ Steel's processing, and has an added complication, in that annealing of coils is carried out in batches of nine coils so that the observations provided concerning mechanical properties of steel from different coils cannot be assumed to all be independent. In addition, the Study Group sought to build on the experience gained the previous year and explore the possibility of modelling the response variables as a vector response, rather than individually. That this is desirable can be seen from the relationships between the mechanical variables shown by a pairs plot as in Figure 4. Note that three of the response variables Yield, UTS, and HRB are measures of strength and hardness, while ElongJIS5 is a measure of formability. An inverse relationship between strength or hardness and formability is to be expected. Typically a steel standard imposes simultaneous restrictions on the strength or hardness and the formability, so it is useful to be able to predict them simultaneously.

2. The cold-rolling process

The input material for the cold-rolling process comes from the hot-rolling of the steel which takes it from steel slabs to coils of rolled steel. This is shown in Figure 1 and further described in the 2005 report [7]. The cold-rolling process is shown in Figure 2. There are different pathways to the finished product which depend on the type of steel to be produced. Cold-rolled full hard steel is reduced in gauge in the 4-hi reversing combination mill, and not annealed. Similarly steel for the metal coating line is cold-rolled (in the 6-hi reversing cold mill) and not annealed prior to further processing. This project is not concerned with these steels. Cold-rolled annealed and cold-rolled annealed and rerolled steels are cold-rolled in the 4-hi reversing combination mill, then annealed in the batch annealing furnace. The rerolled steels are then rolled in the 4-hi combination reversing mill. An important point to note for the analysis is that the annealing furnaces hold 9 coils at a time and necessarily the coils which are in the same batch experience similar furnace conditions. In addition, there are three different furnaces which may have different characteristics with consequent effects on the mechanical properties of the steel annealed in them.

3. Historical data

3.1. Mechanical properties

Four response variables have been recorded. These are obtained by carrying out tests which determine various mechanical properties of the individual coils of steel. The four variables are yield strength, coded as Yield, ultimate tensile strength (UTS), elongation (ElongJIS5), and Rockwell hardness (HRB). The first three of these are described in the report on the earlier project from NZ Steel, [7]. Yield and ultimate tensile strength measure in different ways the strength of the steel in terms of its resistance to tensile forces. Elongation measures the ductility of the steel as the proportion of its length by which it can be stretched before breaking. Rockwell hardness is a method of determining hardness of materials such as metals and plastics. A description of hardness measures is available on the website of the UK National Physical Laboratory [11]:

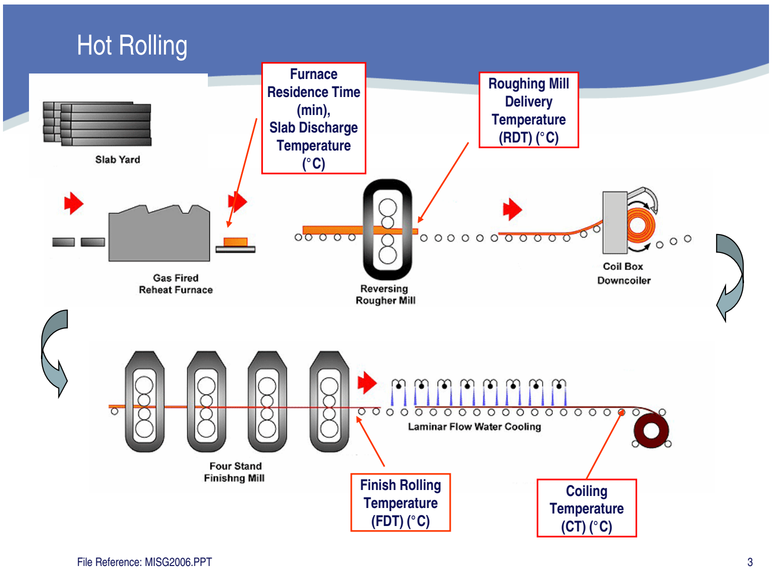


Figure 1. The hot-rolling process

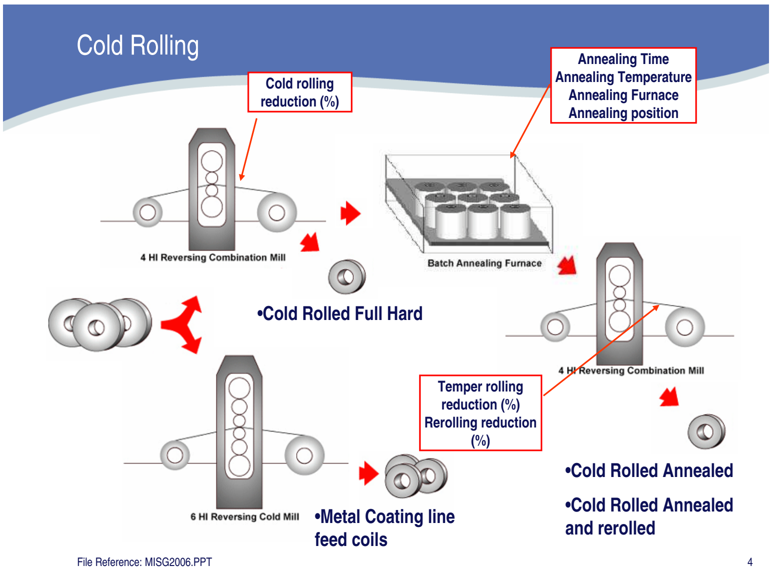


Figure 2. Cold-rolling and annealing

> Hardness is an unusual physical property in that it is the result of a defined measurement procedure and not an intrinsic materials property susceptible to precise definitions in terms of fundamental units of mass, length, and time. In practice, hardness is measured in terms of the size of an impression made on a specimen by an indenter of a specified shape when a specified force is applied for a specified time, the indent being measured after the force has been removed. There are three principal standard methods for expressing the relationship between hardness and the size of the impression, these being Brinell, Rockwell, and Vickers. For practical and calibration reasons, each of these methods is divided into a range of scales, defined by a combination of applied load and indenter morphology, to cover the range of hardness. Recently, with the introduction of instrumented indentation hardness, it has become possible to measure the indent under the applied force.

Rockwell hardness is further explained thus:

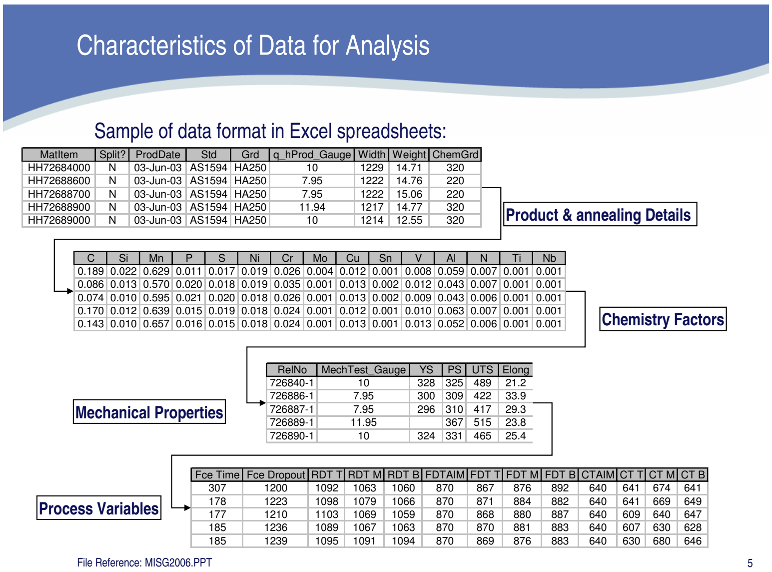
> In the Rockwell hardness test either a 120° diamond cone with a 0.2 mm radius spherical tip or a ball indenter of a specified diameter is pressed into the surface of the test piece using a two step application—preliminary test force F_0 followed by an additional test force F_1. The preliminary test force is applied and maintained for a duration that does not exceed $3s$ and an indenter depth reading is recorded. The increase in force to the total test force F then occurs in between $1s$ and $8s$. This force is maintained for a duration of $4s \pm 2s$, and the additional force is then removed. While the preliminary test force is still applied, a second indenter depth reading is made after a short stabilization time. The Rockwell hardness value is calculated as:
>
> $$\text{Rockwell hardness} = N - h/S$$
>
> where h = the permanent increase in penetration depth at the preliminary test force, in mm; and N and S are constants specific to the scale.

The hardness scale used in the project data set is Rockwell B and measures the penetration depth when a ball indenter is used.

3.2. Predictors

The predictors consist of processing variables and product variables. The product variables form two groups: chemistry factors, and product details and annealing conditions. An outline of these groupings may be seen in Figure 3.

Processing variables refer to processing in the hot-rolling mill. The hot-rolling mill process was the subject of the NZ Steel project presented to MISG 2005, and processing variables were described in the report [7]. Some of the variables have been given slightly different names, and fewer processing variables were including in the data set provided by NZ Steel for this project, so a description of the processing variables follows. All temperatures are in degrees Celsius.



MatItem	Split?	ProdDate	Std	Grd	q_hProd_Gauge	Width	Weight	ChemGrd
HH72684000	N	03-Jun-03	AS1594	HA250	10	1229	14.71	320
HH72688600	N	03-Jun-03	AS1594	HA250	7.95	1222	14.76	220
HH72688700	N	03-Jun-03	AS1594	HA250	7.95	1222	15.06	220
HH72688900	N	03-Jun-03	AS1594	HA250	11.94	1217	14.77	320
HH72689000	N	03-Jun-03	AS1594	HA250	10	1214	12.55	320

C	Si	Mn	P	S	Ni	Cr	Mo	Cu	Sn	V	Al	N	Ti	Nb
0.189	0.022	0.629	0.011	0.017	0.019	0.026	0.004	0.012	0.001	0.008	0.059	0.007	0.001	0.001
0.086	0.013	0.570	0.020	0.018	0.019	0.035	0.001	0.013	0.002	0.012	0.043	0.007	0.001	0.001
0.074	0.010	0.595	0.021	0.020	0.018	0.026	0.001	0.013	0.002	0.009	0.043	0.006	0.001	0.001
0.170	0.012	0.639	0.015	0.019	0.018	0.024	0.001	0.012	0.001	0.010	0.063	0.007	0.001	0.001
0.143	0.010	0.657	0.016	0.015	0.018	0.024	0.001	0.013	0.001	0.013	0.052	0.006	0.001	0.001

RelNo	MechTest_Gauge	YS	PS	UTS	Elong
726840-1	10	328	325	489	21.2
726886-1	7.95	300	309	422	33.9
726887-1	7.95	296	310	417	29.3
726889-1	11.95		367	515	23.8
726890-1	10	324	331	465	25.4

Fce Time	Fce Dropout	RDT T	RDT M	RDT B	FDTAIM	FDT T	FDT M	FDT B	CTAIM	CT T	CT M	CT B
307	1200	1092	1063	1060	870	867	876	892	640	641	674	641
178	1223	1098	1079	1066	870	871	884	882	640	641	669	649
177	1210	1103	1069	1059	870	868	880	887	640	609	640	647
185	1236	1089	1067	1063	870	870	881	883	640	607	630	628
185	1239	1095	1091	1094	870	869	876	883	640	630	680	646

Figure 3. Format of data

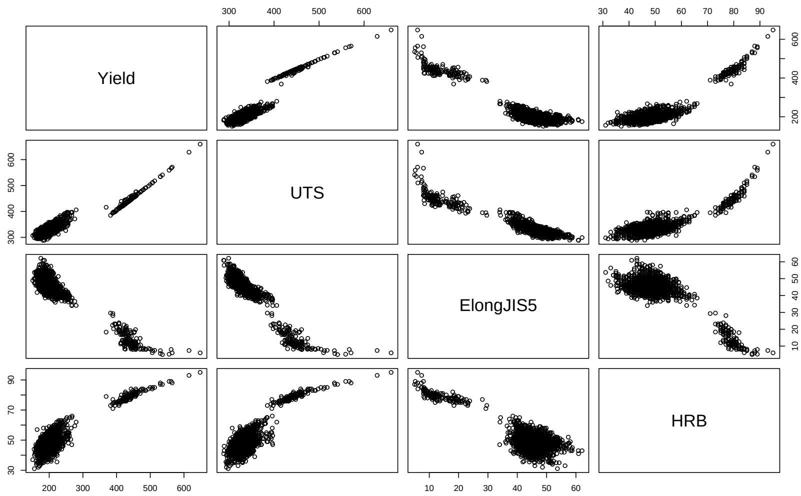


Figure 4. Pairs plot of response variables for the full data set

The hot rolling process is illustrated in Figure 1. The slabs of steel first pass through the *Reheat Furnace*. The temperature at which the slab leaves the Reheat Furnace, previously called the *Dropout Temperature*, is denoted RFT. The time spent in the Reheat Furnace, usually about 3 hours, is denoted FSTime and is given in seconds.

Next the slab passes through the *Roughing Mill*. The temperatures on leaving the Roughing Mill were not provided by NZ Steel in the data set for this project.

The steel then passes through the *Finishing Mill* where the exit temperature, the *Finisher Dispatch Temperature*, is measured at the front, middle and rear of the bar. The variables for these temperature readings are FTHead, FTMid, and FTTail. The steel is finally coiled in the *Downcoiler*. The temperature of the steel is measured before it reaches the downcoiler, again at the front, middle and rear positions, giving variables CTHead, CTMid and CTTail.

Steel performance is affected by chemical composition, most importantly by the carbon composition. To produce a particular grade of steel, the percentage of carbon and possibly other alloying elements is controlled to be within a given range. Some alloying elements appear simply as residual elements due to the varying composition of the input material, which in the case of NZ Steel, is comprised of iron-rich mineral sands. The alloying elements included in the project data set, as percentage by weight, are carbon, silicon, manganese, phosphorus, sulphur, chromium, nitrogen, vanadium and aluminium, denoted respectively C, Si, Mn, P, S, Cr, N, V, Al.

Product details and annealing conditions are a heterogeneous group of variables. Firstly the product (denoted Product) refers to the standard to which the steel must conform. There are various standards used by NZ Steel, but principally they are Australian/New Zealand, Japanese, and American. The source of the particular standard is recorded in the variable Std. The variable Type gives the broad category of the steel, such as cold-rolled annealed (CRA), cold-rolled annealed drawing (CRA drawing) etc. Chemical grade (Grade) specifies the chemical composition of the steel, and mainly reflects the carbon content. Dimensional variables which have been recorded are the gauge at various stages, and width (all in mm), and the weight of the coil in tonnes. Gauge measurements are: HSMGauge, the gauge on leaving the hot-rolling mill; CRGauge, the gauge on leaving the cold-rolling mill, prior to annealing; and FinalGauge, the gauge of the final product. Reductions in gauge brought about by the various rolling operations are recorded as percentages. The reductions recorded are: ColdRRedPcnt, the percentage reduction in gauge from the hot-rolled thickness to the cold-rolled thickness; and TempRedPcnt, the percentage reduction in gauge from the cold-rolled coil to the tempered coil after annealing. There are three furnaces used for annealing, and FurnaceID records which was used for a given coil. Coils are annealed in each of the three furnaces in batches of 9, arrayed in a rectangular grid pattern as shown in Figure 5.

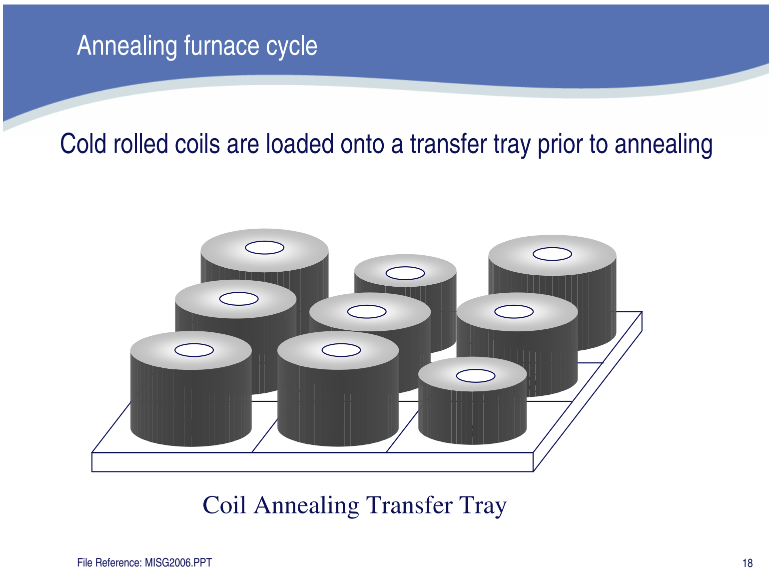


Figure 5. Layout of coils within the annealing furnaces

A number of variables are used to identify batches and coils within batches. Steel is produced and cast in 75 tonne batches, termed heats. Chemistry can vary from heat to heat for a particular chemical grade and the heat is recorded as Heat. PROCORDITM records the batch in which the coil was annealed. The coil identifier is RelNo, and TestDate and StartDateTime denote the date when the mechanical properties were tested, and ostensibly both the time and date when the annealing process commenced, but in reality only the date.

Finally, strength and elongation measurements differ according to the direction in which the test was carried out since the mechanical properties are not isotropic. Most measurements have been carried out in the longitudinal direction, that is parallel to the length of the coil, but some have been carried out at 90° to the length of the coil. TestDir records the test direction as either longitudinal or transverse. The hardness test is not directional, and if only hardness has been tested, which is the case for some products, TestDir is missing.

NZ Steel provided data for 20 cold-rolled coil products, comprising observations on between 100 and 2000 coils for each product, in total around 12,000 coils with 43 variables recorded for each coil.

4. Statistical analysis

Most of the analysis was carried out using the statistical software **R** (see [12]), but some use was also made of Minitab (see [10]). The data was provided in the form of an Excel spreadsheet, and had been checked for outliers and duplicates.

For the NZ Steel project at MISG 2005 a single model was developed for the complete set of data, which comprised products of a number of different types. Whilst this was satisfactory statistically, the model was complex, and writing the computer program to implement the model presented some difficulty. To make implementation easier, for the cold-rolled steel data NZ Steel asked for four separate models to cover different groups of products. The four analysis groups to be used were:

1 **CRA Drawing**
Cold-rolled annealed drawing steel. Defined by Type having the value CRA drawing.

2 **CRA Rerolled**
Cold-rolled annealed rerolled steel. Defined by Type having the value CRArerolled.

3 **CRA Transverse**
Cold-rolled annealed steel with mechanical properties measured across the coil direction. Defined by Type having the value CRA and TestDir the value T.

4 **CRA Longitudinal**
Cold-rolled annealed steel with mechanical properties measured parallel to the coil direction. Defined by Type having the value CRA and TestDir the value L.

4.1. Descriptive statistics

Before attempting to build any models, the data was examined to obtain some idea of distributions and relationships between variables, and to check for outliers or possible erroneous values. Descriptive analyses were carried out for the complete data set and for the four analysis groups individually. Categorical variables such as product and type were tabulated, and pairwise tabulations were also obtained for these categorical variables. Continuous variables were plotted and numerical

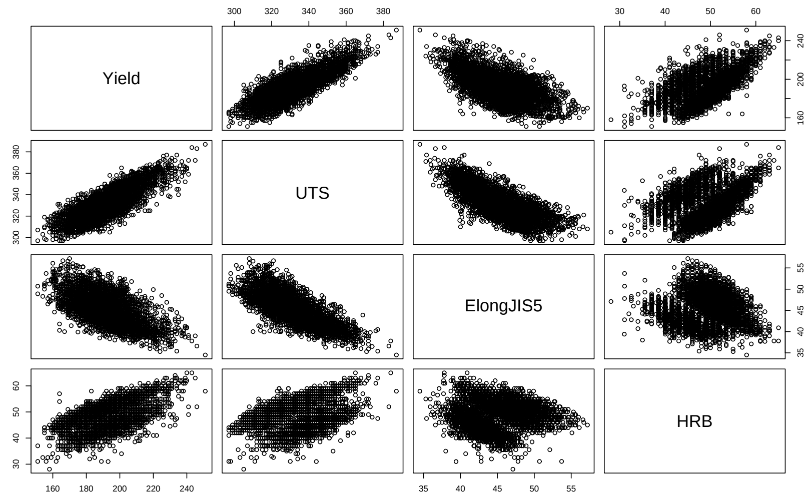


Figure 6. Pairs plot of response variables for CRA Longitudinal analysis group

summaries obtained. The plots used were histograms and pairs plots. A pairs plot is a collection of plots where each pair of variables is plotted one against the other, with the plots being laid out in a grid. They are useful in examining relationships among groups of variables and were popularized by Chambers, Cleveland and others (see [2]).

The relationships between the different mechanical properties were examined using a pairs plot shown in Figure 4. This plot is of the complete data set and it is clear from the existence of two clouds of points in each of the plots in Figure 4 that there are at least two groups in the data. This makes it necessary to identify what characteristics of the data cause the grouping if one is to predict the various mechanical properties adequately. Plots of the data from the analysis groups described above still indicate the presence of subgroups, but with the exception of the CRA Rerolled group the pairs plots show a roughly elliptical cloud of points, suggesting that an assumption of multivariate normality might be reasonable. An example is the pairs plot for the CRA Longitudinal group, Figure 6. In this plot the subplots involving HRB indicate the presence of two groups. This is to be expected since this analysis group is comprised of more than a single product.

The predictor variables were also examined using pairs plots. Pairs plots were obtained for the chemistry variables and the processing variables, for the full data set and the analysis subgroups. These were effective in eliminating further outliers from the data set. At a later stage of the analysis it was discovered there were still duplicate observations in the data set. After these were removed the numbers of observations in the four analysis groups were as given in Table 1. A reference on the consideration of outliers is [1].

Group	**Number of Observations**
CRA Drawing	111
CRA Rerolled	1028
CRA Transverse	2793
CRA Longitudinal	4057

Table 1. Size of analysis groups

4.2. Modelling

A number of models were considered for each analysis group. The simplest approach, which was used in the previous project is to use a linear regression model, an approach we will term the multiple linear regression model. When using this model no model fitting was attempted since eliminating variables using t-tests or F-tests is known to produce biases (see, for example, Harrell [6]). Some model comparisons were made however. For the multiple regression models, two models were fitted, one which included an interaction between the furnace and the position of the coil within the furnace, and one without that interaction term. A test was carried out for the significance of the interaction term and the Akaike Information Criterion (AIC) was used to compare the fit of the two models. See [15] for example, for this approach. The reason for comparing these models was that some of the participants in the Study Group for this project had included an interaction term when fitting the multiple regression model and we wished to determine if it was necessary.

The second approach used was to fit a generalized least squares model which allowed for possible correlations between results for coils annealed in the same batch. The implementation used to fit the generalized least squares model was the package nlme, see [9], which is described in the book [8]. A comparison was made between the generalized least squares model which allowed for correlations between coils in the same batch and the multiple regression model which assumes independent observations. This was done using the likelihood ratio test and AIC. This test was performed to determine if the more complicated generalized least squares model was indeed necessary. For the CRA Drawing group, there are only 111 observations and numerous batches have missing observations for several tray positions. This resulted in a problem when trying to fit a generalized least squares model estimating the correlation within batches between observations in different tray positions. Unfortunately for the CRA Drawing group, this analysis could not be carried out.

Since there are four response variables of interest to NZ Steel, it seems sensible to try and model these responses jointly. Two approaches were considered: seemingly unrelated regressions and partial least squares. Seemingly unrelated regressions (SUR) is an approach often used by econometricians. The **R** package used to implement it was systemfit, see [5]. A description of the methodology may be found in [4]. See also [16] and [3]. SUR models were fitted, but in the present case the estimates from SUR models are the same as from the separate multiple regression models. This is because the same set of predictors is used for each prediction equation and it can be shown theoretically that the prediction equations from SUR are the same as those from fitting separate multiple regression models.

Partial least squares (PLS) looks for a small number of 'latent factors' (linear combinations of the actual qualitative or quantitative predictor variables) that account for much of the variation in the predictor variables and still predict the response variables well. PLS was implemented for some of the data but is more useful for explanation of relationships in terms of underlying latent variables than for prediction, or for cases where the number of predictors exceeds the number of observations. PLS is conceptually difficult also and does not appeal as a technique in a consulting project due to the difficulty of explaining it to a client. Because of these considerations we did not proceed with implementation of this approach. A tutorial on PLS is given in [3]. A web-based introduction, which also describes how to run PLS in the statistics package SAS, is given in [14].

Diagnostic checks were performed on the models obtained and showed that there were no serious violations of the regression assumptions.

Example code for fitting the models described is given in Appendix A. The code is for the CRA Longitudinal analysis group, but similar code was used for the other groups. Code is shown for fitting only one of the four response variables, Yield. For the CRA Rerolled group, the models also include the term TestDir, the indicator for the direction in which testing of the mechanical properties was undertaken.

4.3. Results

For each analysis group three different models have been fitted (but only two for CRA Drawing). In each case, four response variables have been considered. In so far as it is possible to generalize about the re-

	Response Variable			
Model	**Yield**	**UTS**	**ElongJIS5**	**HRB**
Regression without interaction	11.42	8.77	2.03	3.64
Regression with interaction	11.27	8.61	2.02	3.62
Generalized least squares	11.51	8.90	2.05	3.69

Table 2. Standard errors for CRA Longitudinal analysis group

sults, formal tests generally suggested that the multiple regression model with a term for the interaction between TrayPos and Furnace is a better model for predicting the response variables. Besides this, formal testing indicated that there is correlation within batches between observations in different tray positions. Despite these results, what is also apparent is that the estimated standard deviations of the error in the models (which we will henceforth refer to as the standard errors) obtained from the more complex modelling approaches are by and large very similar to those from the simple approach of multiple regression without an interaction term between TrayPos and Furnace. For example in Table 2 the standard errors are given for the CRA Longitudinal analysis group.

Comparing the standard errors for the simplest model, the multiple linear regression model without interaction, to the other models, we observe the following. If the interaction is added, the standard error is reduced, but not by much. If the generalized least squares approach is used, the standard error is increased, but not by much. The increase is expected because this model takes into account that the observations are not independent so the information available concerning the model parameters is less than what is assumed in the regression model.

Examination of R-squared values also indicates that the more complicated models generally do not have notably larger R-squared values.

Finally, we note that the NZ Steel project presenters have indicated that they would prefer a simple model, in particular one without the complication of an interaction between TrayPos and Furnace.

The above discussion suggests that the appropriate model might be the simplest one, however we need to ensure that this approach is statistically valid. Ignoring the possibility of correlations is not a problem: we know from theory that the estimates in this case are unbiased, it is just that the standard error is incorrect. We are only concerned with an appropriate equation for predicting the response, not estimating variability, so we can safely ignore the correlations. The fact that the standard errors change so little between the models suggests that the predictions from the models may be quite similar. Examination of the coefficients in the models suggests this also. Comparison of the predictions obtained from

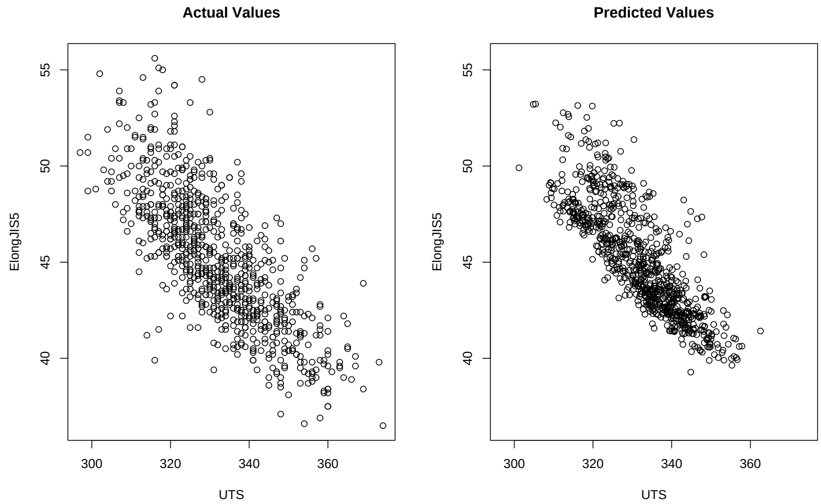


Figure 7. Actual and predicted values for ElongJIS5 versus UTS for the CRA Longitudinal analysis group

the multiple regression models and the generalized least squares models showed that the predictions obtained from the two types of models were very similar.

The other concern we had when eliminating the two possible multivariate approaches, was that although we could predict the individual responses satisfactorily, we might not have a suitable approach for predicting them jointly. To examine this we plotted the predictions from our models compared to the true values for each pair of predictors. Two plots, for the CRA Longitudinal group and the CRA Rerolled group are shown as examples in Figures 7 and 8. It was clear from these plots of pairs of response variables, that the predictions at least captured the appropriate two dimensional relationships between the response variables. Note that in order to be able to see individual points, a sample has been taken of the actual and corresponding predicted values.

5. Conclusions and recommendations

Multiple linear regression models have been developed to predict the mechanical properties of four groups of cold-rolled steel products produced by NZ Steel. The four groups of steels are Cold-Rolled Annealed Drawing steel, Cold-Rolled Annealed Rerolled steel, Cold-Rolled Annealed Transverse steel and Cold-Rolled Annealed Longitudinal steel.

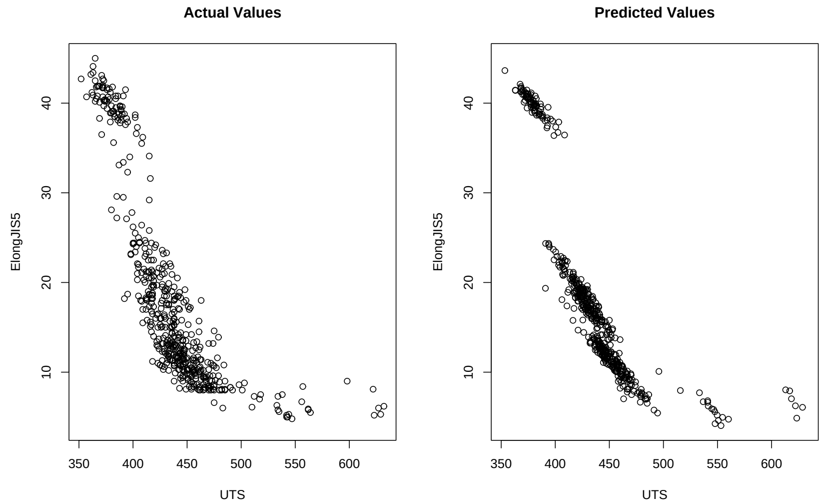


Figure 8. Actual and predicted values for ElongJIS5 versus UTS for the CRA Rerolled analysis group

More complex models were considered but the simple models proposed provide satisfactory predictions and will be easy for NZ Steel to implement. The models proposed may be used to simultaneously predict the mechanical properties which are important to NZ Steel. The actual models are given in Appendix B.

Acknowledgments

The project moderators are David Scott, Ken Russell and Judi Scheffer (student moderator). They were ably assisted by a team of contributors: Park Sang Kyoon, Duk-Soon Oh, Woong Jeung Park, Richard Penny and Reg Rider who worked enthusiastically on the project and provided ideas for the analysis.

The moderators are grateful to Walter Davis of Statistics New Zealand for his suggestion to use the seemingly unrelated regressions methodology.

We thank the NZ Steel representatives, Philip Bagshaw, Andrew Mackay and Michael O'Connor, for presenting this project at MISG 2006. The project was interesting and the project presenters were always willing and able to answer our questions. Some of the text describing the process and the schematic diagrams outlining the process stages and the nature of the data have been taken from the documents provided by NZ Steel.

Appendices

Appendix A: Sample R Code for the CRA Longitudinal Analysis Group

```
### Model responses using linear regression
### Fit linear models for Yield
YieldFit <- lm(Yield~Gauge+factor(ChemGrd)+Width+Weight+
               factor(FurnaceID)+factor(TrayPos)+HeatPeriod+
               C+Si+P+S+Mn+Al+V+N+RFT+FSTime+FTMid+CTMid+
               ColdRRedPcnt+TempRedPcnt,data=steelCRALong)
summary(YieldFit)
YieldFitInt <- lm(Yield~Gauge+factor(ChemGrd)+Width+Weight+
               factor(FurnaceID)*factor(TrayPos)+HeatPeriod+
               C+Si+P+S+Mn+Al+V+N+RFT+FSTime+FTMid+CTMid+
               ColdRRedPcnt+TempRedPcnt,data=steelCRALong)
summary(YieldFitInt)
### Compare models
anova(YieldFit,YieldFitInt)
AIC(YieldFit,YieldFitInt)

### Fit linear models for Yield
### Include correlation
YieldFitCorr <- gls(Yield~Gauge+factor(ChemGrd)+Width+Weight+
                    factor(FurnaceID)+factor(TrayPos)+HeatPeriod+
                    C+Si+P+S+Mn+Al+V+N+RFT+FSTime+FTMid+CTMid+
                    ColdRRedPcnt+TempRedPcnt,
                    correlation=corSymm(form=~TrayPos|PROCORDITM),
                    data=steelCRALong)
summary(YieldFitCorr)
YieldFit <- gls(Yield~Gauge+factor(ChemGrd)+Width+Weight+
               factor(FurnaceID)+factor(TrayPos)+HeatPeriod+
               C+Si+P+S+Mn+Al+V+N+RFT+FSTime+FTMid+CTMid+
               ColdRRedPcnt+TempRedPcnt,
                data=steelCRALong)
summary(YieldFit)
### Compare models
anova(YieldFit,YieldFitCorr)
```

Appendix B: Fitted Models

Note that ChemGrd, FurnaceID and TrayPos are factors. In the models below the coefficient of for example TrayPos(4) represents the contribution to the model when the variable TrayPos takes the value 4. When the variable TrayPos takes another value, there is no contribution to the model from this term. All coefficients are given to 4 significant figures.

CRA Drawing

$$
\begin{aligned}
\mathsf{Yield} = {} & 18.83 + 7.872\mathsf{Gauge} - 0.7395\mathsf{ChemGrd}(110) + 0.04164\mathsf{Width} \\
& -1.220\mathsf{Weight} + 8.302\mathsf{FurnaceID}(2) + 0.3659\mathsf{FurnaceID}(3) \\
& -2.694\mathsf{TrayPos}(2) + 0.5576\mathsf{TrayPos}(3) + 0.6224\mathsf{TrayPos}(4) \\
& +8.907\mathsf{TrayPos}(5) + 1.221\mathsf{TrayPos}(6) - 0.3074\mathsf{TrayPos}(7) \\
& +8.655\mathsf{TrayPos}(8) + 5.027\mathsf{TrayPos}(9) - 0.01409\mathsf{HeatPeriod} \\
& +279.8\mathsf{C} + 451.8\mathsf{Si} + 693.0\mathsf{P} - 55.39\mathsf{S} + 134.8\mathsf{Mn} - 268.4\mathsf{Al} \\
& +360.4\mathsf{V} + 992.3\mathsf{N} + 0.05245\mathsf{RFT} - 0.00008396\mathsf{FSTime} \\
& -0.03414\mathsf{FTMid} - 0.04441\mathsf{CTMid} + 1.506\mathsf{ColdRRedPcnt}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{UTS} = {} & 270.4 - 23.51\mathsf{Gauge} - 7.227\mathsf{ChemGrd}(110) + 0.02772\mathsf{Width} \\
& -0.2837\mathsf{Weight} + 5.192\mathsf{FurnaceID}(2) + 0.9390\mathsf{FurnaceID}(3) \\
& -0.2477\mathsf{TrayPos}(2) - 1.793\mathsf{TrayPos}(3) - 2.820\mathsf{TrayPos}(4) \\
& +3.129\mathsf{TrayPos}(5) - 1.170\mathsf{TrayPos}(6) - 0.6993\mathsf{TrayPos}(7) \\
& +6.458\mathsf{TrayPos}(8) + 1.687\mathsf{TrayPos}(9) - 0.01701\mathsf{HeatPeriod} \\
& +562.2\mathsf{C} - 206.8\mathsf{Si} + 1032\mathsf{P} - 325.6\mathsf{S} + 83.44\mathsf{Mn} - 100.6\mathsf{Al} \\
& +792.2\mathsf{V} + 972.0\mathsf{N} + 0.06282\mathsf{RFT} - 0.00003020\mathsf{FSTime} \\
& -0.02207\mathsf{FTMid} + 0.07574\mathsf{CTMid} - 0.7443\mathsf{ColdRRedPcnt}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{ElongJIS5} = {} & 62.56 + 0.9919\mathsf{Gauge} - 0.005118\mathsf{ChemGrd}(110) - 0.003922\mathsf{Width} \\
& +0.1423\mathsf{Weight} - 0.2704\mathsf{FurnaceID}(2) + 0.2016\mathsf{FurnaceID}(3) \\
& +0.3787\mathsf{TrayPos}(2) - 0.9329\mathsf{TrayPos}(3) - 0.7639\mathsf{TrayPos}(4) \\
& -0.6368\mathsf{TrayPos}(5) - 1.048\mathsf{TrayPos}(6) + 0.4639\mathsf{TrayPos}(7) \\
& -0.2238\mathsf{TrayPos}(8) - 1.586\mathsf{TrayPos}(9) + 0.002579\mathsf{HeatPeriod} \\
& -10.20\mathsf{C} - 41.96\mathsf{Si} - 119.0\mathsf{P} - 184.5\mathsf{S} - 26.55\mathsf{Mn} + 89.66\mathsf{Al} \\
& -223.8\mathsf{V} - 115.1\mathsf{N} - 0.005502\mathsf{RFT} + 0.00001020\mathsf{FSTime} \\
& +0.03081\mathsf{FTMid} - 0.02180\mathsf{CTMid} - 0.3123\mathsf{ColdRRedPcnt}
\end{aligned}
$$

$$\begin{aligned}
\mathsf{HRB} \;=\;& 10.89 - 14.44\mathsf{Gauge} - 2.312\mathsf{ChemGrd}(110) + 0.01330\mathsf{Width} \\
& -0.5006\mathsf{Weight} + 2.247\mathsf{FurnaceID}(2) + 0.7561\mathsf{FurnaceID}(3) \\
& -0.3688\mathsf{TrayPos}(2) + 0.3205\mathsf{TrayPos}(3) + 0.8581\mathsf{TrayPos}(4) \\
& +3.720\mathsf{TrayPos}(5) + 0.7283\mathsf{TrayPos}(6) - 0.2941\mathsf{TrayPos}(7) \\
& +1.266\mathsf{TrayPos}(8) + 1.902\mathsf{TrayPos}(9) - 0.004822\mathsf{HeatPeriod} \\
& +106.1\mathsf{C} - 13.42\mathsf{Si} + 285.4\mathsf{P} + 43.93\mathsf{S} + 49.41\mathsf{Mn} - 90.60\mathsf{Al} \\
& -70.02\mathsf{V} + 549.3\mathsf{N} + 0.01349\mathsf{RFT} - 0.000007709\mathsf{FSTime} \\
& +0.1127\mathsf{FTMid} - 0.005384\mathsf{CTMid} - 1.081\mathsf{ColdRRedPcnt}
\end{aligned}$$

CRA Rerolled

$$\begin{aligned}
\mathsf{Yield} \;=\;& 192.4 - 33.66\mathsf{Gauge} - 157.2\mathsf{ChemGrd}(210) - 3.594\mathsf{ChemGrd}(320) \\
& -17.29\mathsf{ChemGrd}(810) + 8.981\mathsf{ChemGrd}(904) - 0.01553\mathsf{Width} \\
& +2.137\mathsf{Weight} + 8.162\mathsf{TestDir}(\mathsf{T}) + 8.878\mathsf{FurnaceID}(2) \\
& +4.225\mathsf{FurnaceID}(3) + 9.580\mathsf{TrayPos}(2) - 0.3675\mathsf{TrayPos}(3) \\
& -4.378\mathsf{TrayPos}(4) + 2.074\mathsf{TrayPos}(5) - 5.387\mathsf{TrayPos}(6) \\
& -4.022\mathsf{TrayPos}(7) + 4.204\mathsf{TrayPos}(8) - 4.867\mathsf{TrayPos}(9) \\
& -0.01199\mathsf{HeatPeriod} + 367.4\mathsf{C} - 387.9\mathsf{Si} + 1690\mathsf{P} - 562.3\mathsf{S} \\
& +72.53\mathsf{Mn} - 536.5\mathsf{Al} + 804.6\mathsf{V} + 4441\mathsf{N} + 0.1103\mathsf{RFT} \\
& +0.0001737\mathsf{FSTime} + 0.109\mathsf{FTMid} + 0.02980\mathsf{CTMid} \\
& -1.489\mathsf{ColdRRedPcnt} + 4.110\mathsf{TempRedPcnt}
\end{aligned}$$

$$\begin{aligned}
\mathsf{UTS} \;=\;& 316.3 - 32.23\mathsf{Gauge} - 10.19\mathsf{ChemGrd}(210) + 39.44\mathsf{ChemGrd}(320) \\
& -16.12\mathsf{ChemGrd}(810) + 51.95\mathsf{ChemGrd}(904) - 0.009045\mathsf{Width} \\
& +1.856\mathsf{Weight} + 9.207\mathsf{TestDir}(\mathsf{T}) + 6.801\mathsf{FurnaceID}(2) \\
& +3.191\mathsf{FurnaceID}(3) + 9.189\mathsf{TrayPos}(2) - 0.5508\mathsf{TrayPos}(3) \\
& -4.528\mathsf{TrayPos}(4) + 1.425\mathsf{TrayPos}(5) - 4.657\mathsf{TrayPos}(6) \\
& -4.723\mathsf{TrayPos}(7) + 4.066\mathsf{TrayPos}(8) - 4.131\mathsf{TrayPos}(9) \\
& -0.01555\mathsf{HeatPeriod} + 292.9\mathsf{C} - 134.1\mathsf{Si} + 1580\mathsf{P} \\
& -572.7\mathsf{S} + 34.23\mathsf{Mn} - 320.2\mathsf{Al} + 882.2\mathsf{V} + 3863\mathsf{N} \\
& +0.1021\mathsf{RFT} + 0.00009255\mathsf{FSTime} + 0.01343\mathsf{FTMid} \\
& -0.003026\mathsf{CTMid} - 1.467\mathsf{ColdRRedPcnt} + 4.165\mathsf{TempRedPcnt}
\end{aligned}$$

$$\begin{aligned}\mathsf{ElongJIS5} = {} & 8.427 + 7.487\mathsf{Gauge} + 19.06\mathsf{ChemGrd(210)} + 0.2297\mathsf{ChemGrd(320)} \\ & +2.352\mathsf{ChemGrd(810)} + 1.396\mathsf{ChemGrd(904)} - 0.002671\mathsf{Width} \\ & -0.1874\mathsf{Weight} - 5.303\mathsf{TestDir(T)} - 0.3325\mathsf{FurnaceID}(2) \\ & -0.2409\mathsf{FurnaceID}(3) - 0.8703\mathsf{TrayPos}(2) - 0.1660\mathsf{TrayPos}(3) \\ & +0.2097\mathsf{TrayPos}(4) + 0.1729\mathsf{TrayPos}(5) + 0.4504\mathsf{TrayPos}(6) \\ & +0.5942\mathsf{TrayPos}(7) - 0.5070\mathsf{TrayPos}(8) + 0.4469\mathsf{TrayPos}(9) \\ & +0.002165\mathsf{HeatPeriod} + 3.532\mathsf{C} - 8.556\mathsf{Si} - 251.4\mathsf{P} \\ & +127.2\mathsf{S} - 1.155\mathsf{Mn} + 33.27\mathsf{Al} - 186.3\mathsf{V} - 594.8\mathsf{N} \\ & -0.01178\mathsf{RFT} - 0.00001509\mathsf{FSTime} + 0.01834\mathsf{FTMid} \\ & -0.007460\mathsf{CTMid} + 0.2327\mathsf{ColdRRedPcnt} - 0.3704\mathsf{TempRedPcnt}\end{aligned}$$

$$\begin{aligned}\mathsf{HRB} = {} & 65.77 - 0.1280\mathsf{Gauge} - 21.61\mathsf{ChemGrd(210)} - 6.972\mathsf{ChemGrd(320)} \\ & -1.807\mathsf{ChemGrd(810)} - 12.75\mathsf{ChemGrd(904)} + 0.01031\mathsf{Width} \\ & +0.2995\mathsf{Weight} + 2.777\mathsf{TestDir(T)} + 0.4480\mathsf{FurnaceID}(2) \\ & +0.5481\mathsf{FurnaceID}(3) + 0.7414\mathsf{TrayPos}(2) - 0.1203\mathsf{TrayPos}(3) \\ & -0.6823\mathsf{TrayPos}(4) + 0.1694\mathsf{TrayPos}(5) - 0.8143\mathsf{TrayPos}(6) \\ & -0.3443\mathsf{TrayPos}(7) + 0.6945\mathsf{TrayPos}(8) - 0.7665\mathsf{TrayPos}(9) \\ & -0.002774\mathsf{HeatPeriod} + 35.29\mathsf{C} + 3.656\mathsf{Si} + 222.2\mathsf{P} - 64.97\mathsf{S} \\ & +24.37\mathsf{Mn} - 44.16\mathsf{Al} + 171.5\mathsf{V} + 425.7\mathsf{N} + 0.01688\mathsf{RFT} \\ & +0.00001191\mathsf{FSTime} - 0.02553\mathsf{FTMid} - 0.007215\mathsf{CTMid} \\ & -0.2063\mathsf{ColdRRedPcnt} + 0.3934\mathsf{TempRedPcnt}\end{aligned}$$

CRA Transverse

$$\begin{aligned}\mathsf{Yield} = {} & -153.2 + 17.31\mathsf{Gauge} + 2.469\mathsf{ChemGrd}(110) - 3.924\mathsf{ChemGrd}(117) \\ & -5.957\mathsf{ChemGrd(810)} + 2.914\mathsf{ChemGrd}(901) + 0.01841\mathsf{Width} \\ & +1.359\mathsf{Weight} + 4.156\mathsf{FurnaceID}(2) + 1.614\mathsf{FurnaceID}(3) \\ & +6.648\mathsf{TrayPos}(2) + 2.290\mathsf{TrayPos}(3) - 0.7916\mathsf{TrayPos}(4) \\ & +4.520\mathsf{TrayPos}(5) - 1.356\mathsf{TrayPos}(6) - 2.064\mathsf{TrayPos}(7) \\ & +3.760\mathsf{TrayPos}(8) - 1.708\mathsf{TrayPos}(9) - 0.01856\mathsf{HeatPeriod} \\ & +7.549\mathsf{C} + 129.5\mathsf{Si} + 556.4\mathsf{P} - 500.2\mathsf{S} + 7.050\mathsf{Mn} - 508.4\mathsf{Al} \\ & +1915\mathsf{V} + 3448\mathsf{N} + 0.1461\mathsf{RFT} + 0.0002614\mathsf{FSTime} \\ & +0.03898\mathsf{FTMid} + 0.07089\mathsf{CTMid} + 0.7458\mathsf{ColdRRedPcnt} \\ & +22.37\mathsf{TempRedPcnt}\end{aligned}$$

$$
\begin{aligned}
\mathsf{UTS} = {} & 124.1 - 0.8591\mathsf{Gauge} + 2.174\mathsf{ChemGrd}(110) - 3.599\mathsf{ChemGrd}(117) \\
& +0.9762\mathsf{ChemGrd}(810) + 3.828\mathsf{ChemGrd}(901) + 0.008121\mathsf{Width} \\
& +1.618\mathsf{Weight} + 3.749\mathsf{FurnaceID}(2) + 2.013\mathsf{FurnaceID}(3) \\
& +6.083\mathsf{TrayPos}(2) + 2.182\mathsf{TrayPos}(3) - 0.5415\mathsf{TrayPos}(4) \\
& +3.905\mathsf{TrayPos}(5) - 0.9550\mathsf{TrayPos}(6) - 1.706\mathsf{TrayPos}(7) \\
& +3.315\mathsf{TrayPos}(8) - 1.431\mathsf{TrayPos}(9) - 0.01648\mathsf{HeatPeriod} \\
& +263.6\mathsf{C} - 34.07\mathsf{Si} + 878.3\mathsf{P} - 393.9\mathsf{S} + 72.57\mathsf{Mn} - 323.2\mathsf{Al} \\
& +1744\mathsf{V} + 2127\mathsf{N} + 0.08632\mathsf{RFT} + 0.0001710\mathsf{FSTime} \\
& +0.008018\mathsf{FTMid} + 0.08372\mathsf{CTMid} + 0.09105\mathsf{ColdRRedPcnt} \\
& -0.1623\mathsf{TempRedPcnt}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{ElongJIS5} = {} & 96.36 - 0.3619\mathsf{Gauge} - 0.5140\mathsf{ChemGrd}(110) + 0.5449\mathsf{ChemGrd}(117) \\
& +0.8470\mathsf{ChemGrd}(810) - 0.4827\mathsf{ChemGrd}(901) - 0.003359\mathsf{Width} \\
& -0.2532\mathsf{Weight} - 0.5667\mathsf{FurnaceID}(2) - 0.1286\mathsf{FurnaceID}(3) \\
& -1.282\mathsf{TrayPos}(2) - 0.5030\mathsf{TrayPos}(3) - 0.009166\mathsf{TrayPos}(4) \\
& -1.130\mathsf{TrayPos}(5) + 0.09319\mathsf{TrayPos}(6) + 0.3112\mathsf{TrayPos}(7) \\
& -0.9060\mathsf{TrayPos}(8) + 0.2323\mathsf{TrayPos}(9) + 0.004282\mathsf{HeatPeriod} \\
& -28.32\mathsf{C} + 0.8199\mathsf{Si} - 108.9\mathsf{P} - 1.160\mathsf{S} - 0.7636\mathsf{Mn} + 102.7\mathsf{Al} \\
& -285.9\mathsf{V} - 608.7\mathsf{N} - 0.01707\mathsf{RFT} - 0.00002462\mathsf{FSTime} \\
& -0.004798\mathsf{FTMid} - 0.01379\mathsf{CTMid} - 0.2000\mathsf{ColdRRedPcnt} \\
& -1.068\mathsf{TempRedPcnt}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{HRB} = {} & -38.37 + 3.645\mathsf{Gauge} + 0.9709\mathsf{ChemGrd}(110) + 1.775\mathsf{ChemGrd}(117) \\
& -0.6527\mathsf{ChemGrd}(810) - 0.02918\mathsf{ChemGrd}(901) + 0.009440\mathsf{Width} \\
& +0.4601\mathsf{Weight} + 1.080\mathsf{FurnaceID}(2) + 0.3862\mathsf{FurnaceID}(3) \\
& +1.790\mathsf{TrayPos}(2) + 0.2934\mathsf{TrayPos}(3) - 0.4541\mathsf{TrayPos}(4) \\
& +1.786\mathsf{TrayPos}(5) - 0.5970\mathsf{TrayPos}(6) - 0.5179\mathsf{TrayPos}(7) \\
& +1.133\mathsf{TrayPos}(8) - 0.5915\mathsf{TrayPos}(9) - 0.009231\mathsf{HeatPeriod} \\
& +33.18\mathsf{C} + 40.35\mathsf{Si} + 195.4\mathsf{P} - 79.51\mathsf{S} + 15.77\mathsf{Mn} - 106.2\mathsf{Al} \\
& +281.8\mathsf{V} + 980.2\mathsf{N} + 0.01643\mathsf{RFT} + 0.00008381\mathsf{FSTime} \\
& +0.07200\mathsf{FTMid} - 0.001427\mathsf{CTMid} - 0.09760\mathsf{ColdRRedPcnt} \\
& -1.345\mathsf{TempRedPcnt}
\end{aligned}
$$

CRA Longitudinal

$$\begin{aligned}\mathsf{Yield} \;=\; & 41.86 + 1.899\mathsf{Gauge} + 4.412\mathsf{ChemGrd}(110) - 1.615\mathsf{ChemGrd}(117)\\ & -9.657\mathsf{ChemGrd}(810) + 16.01\mathsf{ChemGrd}(901) + 0.01359\mathsf{Width}\\ & +1.504\mathsf{Weight} + 3.661\mathsf{FurnaceID}(2) + 0.5215\mathsf{FurnaceID}(3)\\ & +7.542\mathsf{TrayPos}(2) + 2.657\mathsf{TrayPos}(3) - 2.450\mathsf{TrayPos}(4)\\ & +4.901\mathsf{TrayPos}(5) - 1.414\mathsf{TrayPos}(6) - 2.845\mathsf{TrayPos}(7)\\ & +5.396\mathsf{TrayPos}(8) - 1.492\mathsf{TrayPos}(9) - 0.02567\mathsf{HeatPeriod}\\ & +2.558\mathsf{C} + 16.05\mathsf{Si} + 480.3\mathsf{P} - 399.1\mathsf{S} + 46.45\mathsf{Mn} - 445.9\mathsf{Al}\\ & +1358\mathsf{V} + 3003\mathsf{N} + 0.1225\mathsf{RFT} + 0.0001667\mathsf{FSTime}\\ & +0.09031\mathsf{FTMid} - 0.1573\mathsf{CTMid} + 0.09511\mathsf{ColdRRedPcnt}\\ & -1.070\mathsf{TempRedPcnt}\end{aligned}$$

$$\begin{aligned}\mathsf{UTS} \;=\; & 245.5 - 0.5920\mathsf{Gauge} + 9.086\mathsf{ChemGrd}(110) + 6.227\mathsf{ChemGrd}(117)\\ & +3.118\mathsf{ChemGrd}(810) + 21.42\mathsf{ChemGrd}(901) + 0.009554\mathsf{Width}\\ & +1.264\mathsf{Weight} + 2.909\mathsf{FurnaceID}(2) + 0.4133\mathsf{FurnaceID}(3)\\ & +6.198\mathsf{TrayPos}(2) + 2.320\mathsf{TrayPos}(3) - 2.512\mathsf{TrayPos}(4)\\ & +4.379\mathsf{TrayPos}(5) - 1.694\mathsf{TrayPos}(6) - 2.776\mathsf{TrayPos}(7)\\ & +4.166\mathsf{TrayPos}(8) - 1.581\mathsf{TrayPos}(9) - 0.02014\mathsf{HeatPeriod}\\ & +257.8\mathsf{C} - 77.05\mathsf{Si} + 933.1\mathsf{P} - 351.7\mathsf{S} + 54.89\mathsf{Mn} - 245.0\mathsf{Al}\\ & +1134\mathsf{V} + 2159\mathsf{N} + 0.08709\mathsf{RFT} + 0.0001328\mathsf{FSTime}\\ & +0.03333\mathsf{FTMid} - 0.1717\mathsf{CTMid} + 0.2613\mathsf{ColdRRedPcnt}\\ & +0.07141\mathsf{TempRedPcnt}\end{aligned}$$

$$\begin{aligned}\mathsf{ElongJIS5} \;=\; & 57.03 + 5.958\mathsf{Gauge} - 0.4535\mathsf{ChemGrd}(110) - 0.1174\mathsf{ChemGrd}(117)\\ & +1.690\mathsf{ChemGrd}(810) - 1.931\mathsf{ChemGrd}(901) - 0.0007079\mathsf{Width}\\ & -0.1726\mathsf{Weight} - 0.3783\mathsf{FurnaceID}(2) + 0.09220\mathsf{FurnaceID}(3)\\ & -1.101\mathsf{TrayPos}(2) - 0.4353\mathsf{TrayPos}(3) + 0.3329\mathsf{TrayPos}(4)\\ & -0.6332\mathsf{TrayPos}(5) + 0.1531\mathsf{TrayPos}(6) + 0.4637\mathsf{TrayPos}(7)\\ & -0.7549\mathsf{TrayPos}(8) + 0.3074\mathsf{TrayPos}(9) + 0.002178\mathsf{HeatPeriod}\\ & -18.53\mathsf{C} + 12.20\mathsf{Si} - 86.60\mathsf{P} + 13.08\mathsf{S} - 7.152\mathsf{Mn} + 76.10\mathsf{Al}\\ & -239.2\mathsf{V} - 500.5\mathsf{N} - 0.01491\mathsf{RFT} - 0.00001566\mathsf{FSTime}\\ & -0.008546\mathsf{FTMid} - 0.002082\mathsf{CTMid} + 0.1532\mathsf{ColdRRedPcnt}\\ & +0.6732\mathsf{TempRedPcnt}\end{aligned}$$

$$
\begin{aligned}
\mathsf{HRB} = {} & 83.39 + 1.613\mathsf{Gauge} + 0.5086\mathsf{ChemGrd(110)} - 5.033\mathsf{ChemGrd(117)} \\
& -3.258\mathsf{ChemGrd(810)} - 0.6399\mathsf{ChemGrd(901)} + 0.004454\mathsf{Width} \\
& +0.3239\mathsf{Weight} + 0.8603\mathsf{FurnaceID(2)} - 0.1193\mathsf{FurnaceID(3)} \\
& +1.683\mathsf{TrayPos(2)} + 0.5880\mathsf{TrayPos(3)} - 0.6024\mathsf{TrayPos(4)} \\
& +0.8800\mathsf{TrayPos(5)} - 0.2847\mathsf{TrayPos(6)} - 0.8120\mathsf{TrayPos(7)} \\
& +1.191\mathsf{TrayPos(8)} - 0.5431\mathsf{TrayPos(9)} - 0.006508\mathsf{HeatPeriod} \\
& +39.68\mathsf{C} + 10.42\mathsf{Si} + 186.1\mathsf{P} - 74.49\mathsf{S} + 2.263\mathsf{Mn} - 110.0\mathsf{Al} \\
& +349.3\mathsf{V} + 735.0\mathsf{N} + 0.01748\mathsf{RFT} + 0.00005752\mathsf{FSTime} \\
& -0.02054\mathsf{FTMid} - 0.06078\mathsf{CTMid} - 0.09362\mathsf{ColdRRedPcnt} \\
& -0.7000\mathsf{TempRedPcnt}
\end{aligned}
$$

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