Gas Retention and Release Mechanisms in Tank Waste

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The retention of undissolved gasses (ie hydrogen) in semi-solid tank wastes is complicated and poorly understood. How the gas is retained affects the release mechanisms, rates and release volumes that can be achieved. This in turn affects our estimates of risk and our decisions on how to mitigate risks and how to retrieve the waste for processing for long-term storage.

One type of tank waste is a clay-like material called sludge. This is a fine particulate with a yield stress that may vary from 10 to 10,000 Pa, depending on the composition and history of the waste. Laboratory experiments on simulated wastes suggest that weak sludges retain round bubbles while strong sludges contain noodle- or sheet-shaped gas volumes. We would like to understand the gas-retention process and identify the parameters that control the retained gas bubble shape and size (yield stress, void fraction, ...?).

We expect that the mechanisms controlling the growth of the retained gas volume determine the shape and size of the bubbles. Factors affecting the growth mechanisms might be

- steady growth
- surface instabilities
- agglomeration of small bubbles
- feeder networks
- heterogeneity of the medium
- breathing
- bubble interactions- ie will two nearby bubbles tend to grow towards each other or away from each other?

Saltcake waste has additional complications. This material is made up of larger particles and so acts more like a porous medium, but does not form a rigid matrix. Particles may be displaced, or fractures may form. There may also be "caverns"- larger regions without solids- created when the material is transferred into the tank. We would like to know if the gas retention mechanisms are greatly different from that in sludge, and if so, what additional factors must be considered?

Finally, we would like to know if there are factors that can limit the release from a "dendritic" (noodle or sheet) gas volume to be significantly less than the full connected volume. For example will pores collapse as the pressure is released? This is perhaps the most important question as far as its impact on safety issues. If there is no significant difference between the release fraction of a spherical or a dendritic bubble.

Gas release from sludge

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1 Introduction

Nuclear waste in the form of sludge of rather uncertain composition is held in 177 cylindrical tanks, typically 23m in diameter and 10m high. The radioactivity splits some water molecules into free radicals, which are thought to recombine on suitable catalytic sites to form hydrogen gas. The rate of production of gas (hydrogen and other species) is typically about $4\,\mathrm{m}^3/\mathrm{day}$ in the SY-101 tanks. When the concentration of hydrogen in the roof of the tank exceeds 4%, there is a risk of deflagration. The study group was asked to consider the mode in which the gas is released, whether as dangerous large eruptions or as acceptable continuous background release. We were also asked how much gas was stored in the sludge, where and in what form. The sludge is neither solid nor fluid, but rather a yield material with a yield stress of about $10^3\,\mathrm{Pa}$.

The study group based its approach on a video produced by Pacific Northwest of a laboratory experiment in which gas was generated in a yield material. In these experiments oxygen was produced from hydrogen peroxide, and the material was a clay suspension. The video showed bubbles growing in the material, and the height of the sample rising, rather like baking bread. After some time, some bubbles were large enough to overlap, and they merged. The result of several mergers was to form cracks, fairly horizontal, which grew by being inflated by gas and then breaking sideways into a nearby bubble. A model of this crack growth is given in section 3.1 below. Gas was released to the surface from the network of cracks.

In addition to sludge, some tanks contain salt-cakes. These had been the subject of a previous study group.

Before proceeding to the detailed modelling, two simple observations can be made.

1.1 Largest immobile spherical bubble

Outside a stationary bubble there will be a hydrostatic pressure gradient $\rho_s g$ with $\rho_s = 2 \times 10^3\,\mathrm{kg\,m^{-3}}$ the density of the sludge. Inside the bubble the pressure is constant if we ignore by comparison the density of the gas. There will therefore be a pressure imbalance across a spherical bubble of radius a of $2\rho_s g a$. The yield material can support this without moving by pushing with its yield stress τ_y at the top and pulling similarly at the bottom. Thus the spherical bubble will not move if

$$2\rho_s g a < 2\tau_y.$$

For the given parameters we find the diameter of the largest immobile spherical bubble is 10 cm. This distance will recur frequently in the following. Larger bubbles will move upwards with a only finite region of material around them above the yield stress and so in motion.

If the bubble is not spherical, then its vertical height will give the driving hydrostatic pressure for motion. For an isolated bubble, the resisting yield stress will be exerted over an area approximately equal to the surface area of the bubble, since material from the top of the bubble must be driven down to the bottom as the bubble moves. Hence the critical height will be somewhat larger than 10 cm. However the bubbles are not isolated, but are surrounded by other bubbles which make the medium very compressible. Hence it is necessary only to move sludge from the tip of the mobile bubble to the compression of surrounding bubbles. In this case we retain 10 cm as the estimate of the height of the largest immobile bubble.

Some thought was given to the size of the smallest bubbles. In an ordinary fluid, it is necessary for the capillary pressure to be smaller than the pressure necessary to drive gas into solution. In a yield material, there is a problem of capillary pressure overcoming the yield stress in order to expand an isolated bubble – while this can be achieved near to the surface of the bubble, it cannot at some distance. What can be predicted is the minimum size of undulations along the surface of a crack: with the given value for surface tension, $\gamma = 8 \times 10^{-2}\,\mathrm{N\,m^{-1}}$, undulations smaller than $\gamma/\tau_y \sim 80$ microns will smooth out.

1.2 Rate of gas retention

The rate of gas production of $4\,\mathrm{m^3/day}$ within a fairly full tank is equivalent to $1\,\mathrm{m^3}$ of sludge producing $1\,\mathrm{m^3}$ gas at standard pressure and temperature in 3 years. Hence in the top few metres of the tank, bubbles must start overlapping on a time scale of 3 years. At the bottom of the tank where the pressure is 3 atmospheres, it may take 10 years before the bubbles overlap one another. Thus there could be a considerable build up of gas at depth in undisturbed tanks.

Some estimate can be made between the distance between large bubbles after the lapse of 3 years. Gas dissolved in the sludge will prefer to go to the larger bubbles whose penalty from the capillary pressure is smaller. The distance gas can diffuse in the sludge is given by $\delta = \sqrt{Dt}$. Using $D = 10^{-11}\,\mathrm{m^2\,s^{-1}}$ and $t = 10^8\,\mathrm{s}$ (3 years), we have $\delta = 3\,\mathrm{cm}$ as the typical distance between bubbles. At this separation, there should be sufficient compressibility available for critical height of immobile bubbles to be 10 cm.

2 Porous media models

2.1 Overall permeability

The very simplest view of the gas flow through the sludge is that through a porous medium. Now in steady state, gas moves through the height 10 m within the time to generate a volume of gas equal to that of the sludge, say 3 years. Thus there must be an average superficial velocity of $V = 10^{-7}\,\mathrm{m\,s^{-1}}$. We estimate the typical gas viscosity as $\mu_g = 10^{-3}\,\mathrm{Pa\,s}$. Finally the driving pressure gradient is the weight of the sludge $\nabla p = \rho_s g$ with $\rho_s = 2 \times 10^3\,\mathrm{kg\,m^{-3}}$. Darcy's law then gives a permeability of $k = \mu_g V/\nabla p = \frac{1}{2}10^{-14}\,\mathrm{m^2}$. Using the standard correlation $k = (\text{pore size})^2/100$, this corresponds to pores of the size of about $10^{-6}\,\mathrm{m}$. Now all expectations are of bubbles very much larger. Hence we conclude that almost all of the pores are blocked for almost all of the time, in order that the relative permeability is much smaller than that appropriate to the expected pore size.

2.2 Layered model with relative permeabilities

Based on the depth of the tank being 10 m and our assumption that bubbles taller than 10 cm rise, the tank is divided into 100 horizontal layers. Each layer contains the same fixed mass of sludge and a variable amount of gas. The gas pressure in the layers is set equal to the hydrostatic pressure from the weight of sludge above, and so increases with depth but does not vary in time. The same mass of gas is produced per unit time in each layer. Let ϕ_n be the volume fraction of gas in the nth layer. This volume fraction of gas therefore increases according to the uniform mass-production rate and the pressure in the layer. Flow from one layer to another is controlled by a relative permeability which vanishes if $\phi_n < \phi_c$ and varies as $(\phi_n - \phi_c)^2$ above the critical volume fraction ϕ_c which is set equal to 0.1. When the relative permeability exceeds the critical value, a fraction $\alpha = 0.7$ is released into the layer above.

The algorithm proceeds as followed. Time is advanced by 10 days by increasing the volume fraction of the gas by 1% in the top layer and by an amount corresponding to the pressure in the lower layers. The layer with the largest non-zero permeability is found, and a fraction α of its gas transferred to the layer above. This process of transfer from the layer with the largest permeability is repeated until all the permeabilities vanish. Time is then advanced again by 10 days.

This layered model allows one (i) to follow the evolution of the total height of sludge, (ii) to determine the statistics of the volumes of gas released and see how frequent catastrophic releases are, and (iii) to see how gas is distributed throughout the depth.

Computer simulations found that the height of the sludge increases by about 10% or 1 m, corresponding to the volume fraction ϕ_n in all the layers hovering around the critical value $\phi_c = 0.1$. The height oscillates randomly with a period of about 15 years and an amplitude of about 5%. The time-scale is controlled by the rate of production of gas, and the amplitude by α, the size of the fraction of gas released from one layer to another. As one should probably view each layer as having a cross-section of size 10 cm, the 23 m wide tanks should average out these variations in the height of the sludge.

The simulations showed that very often as one layer releases gas into the layer above, a cascade to the top is triggered. The next layer above often contains gas, and the addition from below pushes its volume fraction over the critical value. In addition the higher layers are at lower pressures and so a critical mass from below has a larger than critical volume higher up. Thus the volume of gas released varies between that from a single top layer, say a 10 cm bubble, to that from about 50 layers (volume $2.5 \times 10^{-2}\,\mathrm{m}^3$). All sizes in between these limits seem to be equally likely. The larger releases from deep down cannot be very frequent because one must wait longer there for the greater mass required to increase the volume fraction to the critical value.

We therefore conclude from this model that gas is released fairly constantly. It is however worrying that every layer contains about 10% gas by volume. Depressurizing this gas at the bottom would produce 30% volume of gas.

2.3 Continuum version

While there are good reasons to study a model with discrete layers 10 cm thick, a continuum version was considered. The gas production is described by

$$\frac{\partial}{\partial t}(\rho_g \phi) = \nabla \cdot \mathbf{q} + Q,$$

where Q is the mass production rate per unit volume, and $\mathbf{q}$ is the flux. This flux is given by Darcy's law

$$\mathbf{q} = -\frac{k(\phi)}{\mu_g} \nabla p_g,$$

where

$$k(\phi) = \begin{cases} k_0(\phi - \phi_c)^2 & \text{if } \phi > \phi_c \\ 0 & \text{if } \phi < \phi_c \end{cases} .$$

is the relative permeability. The pressure and density in the gas are related by the gas law $p_g = \rho_g RT$. Finally the expansion of the sludge is controlled by the excess of the gas pressure above the hydrostatic sludge pressure

$$\frac{\partial \phi}{\partial t} = A(p_g - p_s).$$

Numerical solution of these partial differential equations found that the sludge goes to a steady state almost monotonically (a phenomenon observed in most single shell tanks), in which gas is released steadily and the volume fraction goes to just above the critical value throughout the depth. In the future it might be more interesting to incorporate the effect of a yield stress in the last equation above, although this might necessitate going back to a discrete layered system. This modification might produce a chaotic release.

3 Crack models

There is a large literature on crack propagation in different materials, e.g. cracks in purely elastic materials and cracks in Newtonian viscous fluids. A major difficulty is to decide on the mechanism by which the cracks become longer.

3.1 Inflating crack

The video showed individual bubbles growing until some merged, which grew further in the form of a crack. The crack expanded in width through the addition of more gas from the sludge. Occasionally the length of the crack increased by jumping to the next bubble adjacent to the tip. We suppose that this jump occurred when the stress at the tip exceeded the yield stress.

Let the half-length of the crack be l and its half-width w. Let the gas pressure inside the bubble be p_g, and the hydrostatic pressure in the sludge far outside the crack be p_s. Now because the crack grows from a slit and because the pressures inside and far outside are constant, the bubble will take an ellipsoidal form

$$\frac{x^2+y^2}{l^2} + \frac{z^2}{w^2} = 1.$$

Its volume is therefore $\frac{4}{3}\pi l^2 w$. Standard elasticity theory relates the width to the pressure excess

$$w = \frac{2l}{\pi} \frac{p_g - p_s}{\mu_s(1-\nu)}.$$

where μ_s is the elastic shear modulus and ν is the Poisson ratio. Pure sludge will be effectively incompressible, $\nu = 0.5$, but once there are sufficient bubbles in the sludge, one should include an effect from the compressibility of the gas $\partial(1/\rho_g)/\partial p$ multiplied by the volume fraction of the bubbles and cracks in the sludge.

The cracks grow by the diffusion of gas from the sludge. If Q is the mass of gas produced per unit time per unit volume of the sludge, and if δ is the distance between cracks (estimated earlier to about 3 cm) then there will be a mass flux of gas into the cracks of $Q\delta$ per unit surface area. With a surface area of $2\pi l^2$, the growth of the mass of gas in the cracks is given by

$$\frac{d}{dt}\left(\rho_g \frac{4}{3}\pi l^2 w\right) = 2\pi l^2 Q\delta.$$

As the width of the crack grows, stress will build up at the tip of the crack. If we assume that the tip of the crack is a bubble of radius a, the tensile stress at the tip is

$$2(p_g - p_s)\sqrt{(l/a)}.$$

When this exceeds the yield stress τ_y, the tip of the crack will jump forward to the next bubble. This will increase the volume of the crack and so decrease the gas pressure inside it. The crack will therefore pause at the new size, waiting for gas to inflate it further.

We can smooth out the jumping to find the approximate evolution of the size of the crack with time. We first assume that the difference in pressures remain just below critical value for the tip to yield. Substituting this value into the expression for the width of the crack, we find

$$w = \sqrt{la}\frac{\tau_y}{\pi\mu_s(1-\nu)}.$$

While the difference in pressures is important for the width of the crack, it is likely to be small absolutely, except near the top of the tank. Thus to calculate the density of the gas ρ_g from the gas law, we can take the gas pressure p_g to be constant in time, equal to the hydrostatic pressure p_s in the sludge. Using this, together with the above expression for the width of the crack, the equation for the mass increase of the crack can be integrated to give

$$\sqrt{l} = \frac{3\pi}{10}\frac{\delta}{\sqrt{a}}\frac{\mu_s(1-\nu)}{\tau_y}\frac{Qt}{\rho_g}.$$

Thus the length of the crack grows quadratically in time. Taking $\rho_g/Q = 3\,\text{years}$, $\delta = 10\,\text{cm}$, $a = 1\,\text{mm}$, and guessing $\tau_y = \mu_s(1-\nu)$, it will take 1 year for the crack to grow to $2l = 2\,\text{m}$ long, at which time it will be about $2w = 6\,\text{cm}$ wide and have a volume of about $0.1\,\text{m}^3$.

3.2 Application

In the video the cracks grew fairly horizontally. This will have been due to the side walls exerting a confining pressure, forcing the clay suspension to expand only in the vertical direction. The stress in the bubbly clay suspension will thus be anisotropic, the (negative) horizontal normal stress exceeding the (negative) vertical normal stress. In such an anisotropic background stress, it is easier to open up the material with horizontal cracks rather the vertical cracks. This argument may not be appropriate to the sludge tanks which are much wider. The time-scales are also much longer, so giving time for creep to relax the anisotropy (stress would be isotropic in a fluid). One further observation is that the process of crack growth in the bubbly sludge is by the tip jumping to a nearby bubble, which will not be strictly horizontal. The cracks will therefore not remain strictly horizontal. The study group were unable to determine whether the deviations from horizontal of the cracks were likely to be significant in determining when the cracks were sufficiently large to start to rise.

Once the cracks reach the critical height to be mobile (see section 1.1), they will migrate to the top of the tank, collecting on their way any gas in cracks they pass through (see section 2.2). The study group thought that after the passage of a crack, the sides would seal together, although they might remain a weak point ready to open up for the passage of subsequent crack/bubbles. It is possible that the suface of a crack might not join perfectly everywhere, and this would leave a bubble to grow in the future.

If a crack were to meet another crack before either were mobile, then one would expect the smaller to empty into the larger, because it is at a higher pressure.

4 Conclusions

There are two simple conclusions from the porous medium and crack models. Firstly, we expect gas to released from the sludge safely at a constant rate in small bubbles, typically 10 cm in size. Equivalently, the height of the sludge should remain fairly constant. This conclusion assumes that there is no seal or crust which stops gas passing through any level (although PNNL point out that a seal could occur if, for example, a rigid salt cake had its pores plugged by fines). One would worry about the top layer drying out creating such a seal. Second and less happily, quite a large mass of gas is probably stored under pressure in the lower levels of the sludge. It would therefore be very unwise to stir up the sludge, releasing this mass.

Turning to the laboratory experiments in the video, some suggestions can be made for future investigations. To test the idea that the hydrostatic pressure at depth in the tanks means that there is a large mass of gas stored there, the experiments could be run under pressure, and changes in gas released measured. A complication might be that the rate of gas produced in the hydrogen peroxide reaction might depend on pressure. Of course it is an open question how far this reaction does model the production of gas by the nuclear decay. One would also be interested in the effect of the aspect ratio of the container and the ratio of the yield stress to the elastic shear modulus.

While the study group was considering how gas might percolate to the top surface through a network of disk-shaped cracks, it received two visits by the creator of Percolation Theory, J.M. Hammersley. Rather disarmingly, he said that we knew nothing about percolation (by which he meant that very little has been rigorously proven) and that it would take 6 months for a computer to produce an estimate (a somewhat longer time-scale than that the study groups permit themselves). Perhaps in this topic of utmost danger, we should keep in mind that we have established nothing with any certainty. It would therefore be wise to monitor very regularly the rate of release of gas, there being serious danger when it fluctuates (either up or down!).

ADDENDUM

Crack bubble dynamics

When the yield stress of the sludge is high, the material resembles and behaves like a wet clay soil, and its possible behaviour can be understood on this basis. In particular, accumulation of H_2 and formation of voids is similar to fracture initiation and propagation in elastic materials.

Failure in soils is associated with a transition from elastic to plastic behaviour on a critical surface called the yield surface, consisting of three parts: the Hvorslev surface, the Roscoe surface, and the tension failure surface. It is the last which concerns us here, and a simple expression describing failure on this surface is

$$\tau = c + p_e \tan \psi, \tag{1}$$

where τ is the shear stress, c is the cohesion, p_e is the effective pressure (or effective normal stress) and ψ is the angle of solid grain to grain friction. The idea behind (i) is that c represents the grain to grain adhesion (e.g. due to adsorption forces in clays), while p_e is essentially the pressure transmitted through the soil grains, and thus $p_e \tan\psi$ represents solid friction. If the shear stress (a measure of which may be the second invariant of the deviatoric stress tensor) reaches the yield surface, then plastic deformation occurs, so as to maintain the soil on the yield surface. Plastic flow is thus not normally associated with viscous behaviour, which is more relevant beyond the 'liquid limit' (when soils behave viscously). In order to be specific, we focus on the concept of fracture in an elastic-plastic sludge, while realising that, particularly at lower yield stresses, a viscoelastic law may be more appropriate.

Crack model

Consider a two-dimensional crack of width h and length $2l$ in an elastic medium. If p is the excess pore pressure over the far field normal stress, then (if p is spatially uniform) h is given by

$$h \sim \frac{p}{\mu}(l^2 - x^2)^{1/2}, \tag{2}$$

where x is distance along the crack, and μ is the shear modulus. The volume (per unit width) of the crack is

$$V \sim hl, \tag{3}$$

and conservation of mass takes the form (if we adopt the perfect gas law $p = \rho RT/M_{H_2}$ for H_2, where M_{H_2} is its molecular weight)

$$\frac{d}{dt}\left[\frac{M_{H_2}}{RT}pV\right] = 4Jl, \tag{4}$$

where J is the mass flux of H_2 diffusing to each side of the crack.

4.1 Crack tip propagation

The stress field at the tip of a crack is characterised by a square root singularity,

$$\sigma \sim \frac{K}{\sqrt{2\pi r}}, \tag{5}$$

where r is polar distance from the crack tip, and K ($\sim pl^{1/2}$) is the stress intensity factor. This local stress field is associated with stored elastic energy, which is released as the crack propagates. The energy release rate (per unit length of crack) is G, and is given by

$$G \sim \frac{K^2}{\mu}. \tag{6}$$

The Griffith criterion asserts that crack extension occurs if G is equal to the energy R required for growth. For example, for a brittle material, this will be surface energy γ, thus growth will occur if $G = 2\gamma$. In metals, however, the relevant energy is plastic energy.

In order to specify the crack tip speed, we need to know the rate-limiting mechanism for energy transfer. For example, *dynamic* fracture propagation occurs if $G > R$, when the excess energy generates kinetic energy in the medium. The resultant crack speeds are on the order of the elastic wave speed. Subcritical crack propagation (when $G < R$) can also occur, providing another energy transfer mechanism is available to make up the deficit. For example, stress corrosion in rocks and ceramics is mediated by the diffusion of active chemical species.

Viscous separation

We postulate the following mechanism for determining crack tip propagation in sludge. We conceptualise the material as consisting of solid grains separated by thin viscous fluid films. The time required to separate two circular plates of radius $d_p/2$, initially a distance h_0 apart, is

$$t_{\text{sep}} \approx \frac{3\pi\eta d_p^4}{64h_0^2F}, \tag{7}$$

where F is the net separation force and η is the viscosity. Identifying this with the net separation stress σ_{sep} via $F = \pi\sigma_{\text{sep}}d_p^2/4$, we thus have

$$t_{\text{sep}} \approx \frac{3\eta d_p^2}{16h_0^2\sigma_{\text{sep}}}. \tag{8}$$

We also interpret h_0^2 as a measure of porosity via $h_0^2 \approx d_p^2\phi/3$, where ϕ is the liquid saturation, and the factor 3 represents a grain surrounded by six comparable films. Finally, we associate the crack tip velocity with d_p/t_{sep}, where d_p represents grain diameter. Thus

$$\dot{l} \approx \frac{16\phi d_p\sigma_{\text{sep}}}{9\eta} \tag{9}$$

and more generally (analogously to the general form for Darcy's law), we would propose a crack tip velocity of the form

$$\dot{l} = \frac{d_p}{\eta}C(\phi)\sigma_{\text{sep}}, \tag{10}$$

where $C(\phi)$ is an $O(1)$ function of ϕ. It may be possible to test this hypothesis experimentally.

In the present instance, we suggest that the separation stress is $p - \tau$, where τ is the yield stress given by (1). The cohesion acts directly on the separating film, while the frictional resistance $p_e \tan\psi$ acts on the separating grains through grain contacts.

Propagation rates

Measured speeds from the video were on the order of 10^{-5} m s^{-1} (centimetres per hour). If we suppose $\sigma_{sep} \sim \tau \sim 10^3$ Pa, $d_p \sim 10^{-5}$ m (10 microns), $\eta = 10^{-3}$ Pa s, then we have from (10), $\dot{l} \sim 10$ m s^{-1}. It seems that the separation rate predicted by this 'law' is much too rapid.

However, let us examine the model in greater detail. Equations (2), (3), (4) and (10) are essentially

$$h \sim \frac{pl}{\mu},$$

$$V \sim hl,$$

$$(p\dot{V}) = \frac{4RT}{M_{H_2}} Jl,$$

$$\dot{l} = \frac{d_p}{\eta} C(\phi)(p-\tau), \tag{11}$$

giving four relations for the variables h, p, l, V, J, which thus means a further relationship for J is necessary. We consider this below. We have

$$p \sim \frac{\mu h}{l} \sim \frac{\mu V}{l^2}, \tag{12}$$

thus

$$\dot{l} = \frac{d_p}{\eta} C \left[\frac{\mu V}{l^2} - \tau\right]. \tag{13}$$

Now the implication of the large tip propagation rate is that l relaxes rapidly to a stable quasi-equilibrium

$$l \sim \left(\frac{\mu V}{\tau}\right)^{1/2}, \tag{14}$$

and thus the crack tip propagation is in fact controlled by the slower diffusion of H_2 to the crack. Specifically, $(11)_3$ is now (with $p \approx \tau$)

$$\tau \dot{V} \approx \frac{4RT}{M_{H_2}} Jl, \tag{15}$$

and, following the 1995 report (cf. equations (1.30), (1.31)), we estimate J from

$$V_c R^* \approx 4Jl, \tag{16}$$

where R^* is the volumetric production rate, and also

$$Jl \approx \rho_s D \Delta c, \tag{17}$$

where Δc is the critical supersaturation. In fact, (16) determines the catchment volume V_c, and (17) implies

$$\dot{V} \approx \frac{4RT}{M_{H_2}} \frac{\rho_s D \Delta c}{\tau}. \tag{18}$$

These various simple relations, and their three-dimensional equivalents, are presumably capable of fairly straightforward rejection (or not) on the basis of experiments, *e.g.* (18) is a prediction for V, bubble volume, versus time, depending on yield stress which may be used in (14) to give a prediction for crack length, l, versus time. They also form the basis for a more elaborate model involving crack populations and drainage, *i.e.* a more elaborate model could have a population n, where $n(V,t)$ is the number of cracks per unit volume V. The model would then develop along the lines of age-dependent population models so that $\dot{V}$ is given by (18), $\dot{n}$ is some nucleation rate and they are linked by

$$\frac{\partial n}{\partial t} + [\dot{V}] \frac{\partial n}{\partial r} = [\dot{n}] \,, \tag{19}$$

where $[\dot{V}]$ and $[\dot{n}]$ are unknown functions of these variables.

5 Contributors

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