MODELLING AND ANALYSIS OF WHEEL REPLACEMENT AND RESTORATION

1. Problem description

The problem was presented to the Study Group by Mr Bill Thomas of Queensland Railways.

Queensland Railways have 88 suburban electric trains, hereafter called EMUs, which each consist of three cars permanently coupled together. Each car has four wheelsets, in two bogie pairs, a wheelset being an axle and two wheels. Two of the cars have motored wheelsets, the other has unmotored, or trailer, wheelsets; the two types are not interchangeable.

During running of the EMUs, the wheels gradually wear. The aspect of wear of particular interest is thinning of the wheel flange. A thin flange is more liable to crack or to cause a derailment, particularly when travelling over points. So once the flange wear reaches a critical level the wheel must be returned to its correct flange profile. This can occur in one of two ways:

- by *reprofiling*, that is turning the wheel to the correct profile
- by *replacing* the worn wheel with a new one

Reprofiling involves a loss of metal from the whole wheel rim, so after its return to the correct profile it has a smaller diameter. Thus it is possible to have different sizes (*i.e.* diameters) of wheels on an EMU simultaneously. Because of wheel slippage however, there are limitations on the wheel sizes which can coexist on an EMU - if the size differences are too great the resultant differences in rotational speeds are interpreted by the automatic controller as slippage, and it makes incorrect adjustments to wheel speeds, resulting in loss of traction and excessive wear. A further consequence of differing sizes is that smaller wheels wear faster than big ones.

Note that both wheels on an axle are always dealt with similarly as regards profile assessment and restoration, so that wheelsets are the basic units in this problem.

Flange wear is monitored during the routine three-weekly inspection of an EMU. Once it has reached a critical level (currently defined as 5mm of wear at a fixed distance from the flange top) the wheel profile must be restored, and the EMU is 'flagged' for attention. The restoration process consists of bringing the EMU into the workshop, dropping the worn wheelsets off and replacing them with wheelsets of sizes compatible with those left on the EMU. The replacements come (if available) from a stock of correctly profiled wheelsets. The worn wheelsets are then either reprofiled or they have new wheels pressed onto the axle; once corrected they are returned to stock.

There is another level of flange wear that is also relevant, namely 7mm. Beyond this level the wheelset is considered unserviceable, and the EMU must be dealt with immediately or it can no longer run. There are other occasional emergencies, such as cracking or skids, that also render a wheelset unserviceable.

The question originally asked by Queensland Railways was:

> *Given this system with its inherent constraints, what is the relationship between the stock of spare wheelsets on hand and the probability of not having a wheelset of the correct size on hand to effect a wheel change?*

This eventuality is one to be avoided if at all possible, since it means an EMU will be forced out of service for a period. There are several variants and corollaries of this question, which are also of interest, some of which will be mentioned in this report.

Data

Queensland Railways provided a considerable amount of quantitative information about various aspects of the system. This included data on such matters as:

- lifetimes of wheels
- size distribution of wheels
- service times for wheel replacement, reprofiling *etc.*
- costs

Not all of it will be reported here, but some items are relevant for later discussion.

Reprofiling: This reduces wheel radius by 12.5mm on average, with a standard deviation of 1.58mm. This appears to be independent of the original size of the wheel.

Wheel sizes: For operational convenience, wheel sizes are categorised into 7 size classes each of 10mm diameter range. Size 1 corresponds to a diameter between

830mm and 840mm etc. Clearly the smallest tolerable diameter is 770mm. New wheels are very near to 840mm.

Note that some size classes will be rather uncommon. For instance, one reprofiling gives a mean diameter of 815mm for the new wheel, right in the middle of size class 3, so classes 2 and 4 will be relatively unpopulated. This is unfortunate, as class 4 wheels are in a sense 'universal'; they are compatible with any other class (see below).

Wheel wear: The average life for a wheel is approximately

$$74 - 4.3 \times (\text{wheel size})$$

with a rather large standard deviation (about half the size of the mean).

Compatibility constraint: The size classes of wheelsets on the same EMU must not differ by more than *three* sizes classes *i.e.* a difference of at most 40mm.

Service and repair times:

• Removal and replacement of wheelset	:	1 day	(motored)
		0.5 day	(trailer)
• Reprofiling a worn wheel	:	1 day	
• Repressing new wheels onto an axle	:	2.5 days	

2. Flow chart for the system

The system described in the previous section is a rather complex one. The group's initial task, then, was to set up a detailed flow chart. This is necessary for several reasons:

- To extract the essential elements of the system and their inter-relations
- To establish the critical points in the system, where congestion or unavailability of stock may occur
- To set down the decision points of the system
- As the basis for a possible simulation study

The chart is given as Figure 1. It includes the important components of the problem, the points at which congestion (queues) can occur (Q1 - Q4) and at which decisions must be made (D1 - D7). Further details of the queues and decisions are given below.

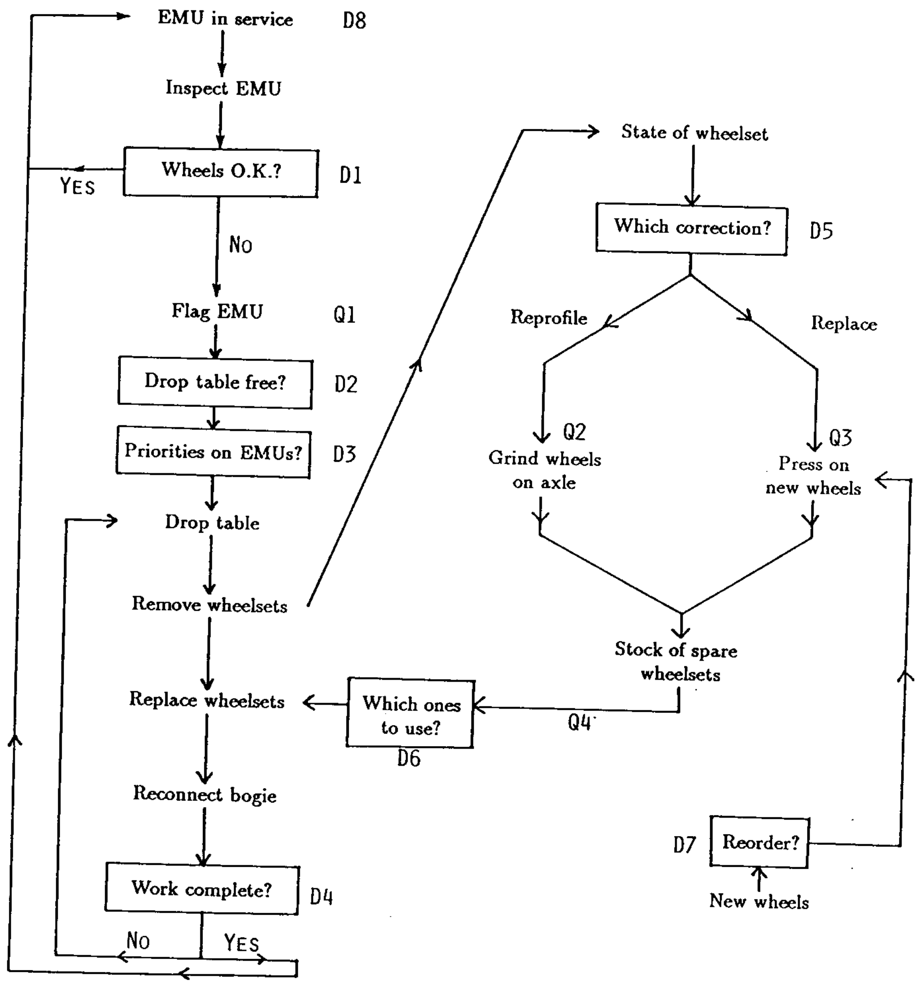

Figure 1: Flow chart for system, with queues and decision points.

There is, of course, more detail which could be given for several areas in the system, and which might be necessary if that area is to be studied more closely. One example is the detailed operation of the workshop when wheelsets are removed, replaced and reprofiled; specific time-scheduling of the connecting parts of the operation is especially important. However it would burden this report unnecessarily to include all such aspects.

The queues

Q1. The queue of EMUs with worn wheelsets. If the wheelset cannot be replaced immediately (as is usually the case), because the workshop is not free or because no compatible wheelset is currently in stock, the EMU is returned to service with a nominal 'flag' to show it needs attention. Because of the 7mm unserviceability level referred to in Section 1 there is likely to be a form of priority in this queue; see D3.

Q2, Q3. The queues of worn wheelsets awaiting appropriate correction. There may be priorities here if imminent need for a particular size of wheelset is foreseen.

Q4. The stock of spare wheels; it is perhaps more accurately described as an inventory. Demands on the inventory come from replacement of worn wheelsets. Because of the size compatibility constraint, it is the types of wheelset and not just the total number that is important.

The decisions

D1. Does a wheelset need replacing at the regular inspection? The criterion for this has already been discussed.

D2. Should the workshop drop table be used for wheelset replacement, or should the equipment and personnel be otherwise employed? This will depend on the queue length and priorities in Q1 and the stock in Q4, among other things.

D3. If a wheelset is to be replaced, which EMU will be serviced? This will depend on the number of wheelsets to be replaced on an EMU, how close its wheelsets are to unserviceability and the stock at Q4.

D4. Is the EMU now fit for service?

D5. Can the worn wheelset be reprofiled, or must the wheels be replaced? In practice, this means deciding whether a twice–reprofiled wheelset will still be suitable after a third reprofiling. There may also be some priorities in which wheelset is next corrected - see Q2,Q3.

D6. If more than one compatible wheelset is available, which one should be used? This will depend on stock levels (for instance, size 4 wheelsets are valuable as they are always compatible), and the states of Q1 - Q3.

D7. Should new wheels for pressing onto axles be ordered? This will depend on existing stock levels and on prediction of future demand.

D8. Can the operating constraints on the system be changed? This is not a tactical decision related to the immediate operation of the system. It includes such matters as a revised size classification for wheelsets (Section 4), the critical levels of flange and diameter wear, the compatibility condition, and so on. These are strategic questions with which the mathematical analyses or simulation study described in this report can help.

3. Simulation

While it is possible to model and analyse subsystems of the whole system in some detail, the only practical method for exploring the whole system seems to be simulation. Such an approach has the usual pros and cons, some of which are listed below.

- It is capable of handling complex, realistic models
- Both stochastic and deterministic models can be treated
- A variety of objectives can be studied without major changes to the *process* of analysis
- The task of setting up, validating and debugging a complex simulation model can be very lengthy
- If changes to input data or system operations are proposed, the whole simulation must usually be rerun

Examples of objectives, other than the one originally proposed, which could be investigated, are:

- Maximise the probability of being able to immediately service an unserviceable wheelset
- Make the Q1 queue as short as possible, in some sense
- Minimise the cost of the operation

The flow chart of Figure 1 is the basis of any simulation model. We need a system state description, and a time period over which it will be augmented.

The elements of the *state vector* may vary depending on the objective, but should include the following information:

- The current diameters of the 8 motor and 4 trailer wheelsets on each of the 88 EMUs
- The levels of flange wear on each wheelset, or some other information to determine when a wheelset will become critical and unserviceable
- The time since last routine inspection

- Whether the EMU is currently flagged
- Size details of any wheelset awaiting correction (Q2,Q3)
- Stock levels of each size class of motor and trailer wheelset

The time period will be the smallest unit over which any changes in the state vector can occur. This appears to be $\frac{1}{2}$ a shift, for removal of a worn trailer wheelset.

Some discussion of the second item in the state vector list is needed. We must be able to decide when an EMU has at least one critical wheelset at its routine inspection. This could be done by including the level of flange wear and augmenting it according to an appropriate hazard function (since it is a random quantity). However, this requires a random experiment for *each* wheelset or an EMU at *each* time period, which does not appeal! An alternative is to generate a lifetime, from an appropriate distribution, for each wheelset when it is first fitted to the EMU, and decrement this; this requires only one random experiment for each lifetime. We shall still need to monitor flange wear once it exceeds criticality (5mm), to track its approach to unserviceability. Since we do not have data on this aspect, we cannot decide yet the best way to incorporate it in the state vector.

As mentioned above, we believe that simulation is the only fully satisfactory approach in general. However, we emphasize that detailed studies of subsystems would also be of great practical value. The purpose of such studies is to develop an appreciation of subsystem behaviour and hence to provide guidance on the critical factors affecting them. This is valuable information for any simulation study, since it will highlight the operating rules and regimes it is particularly important to include. A specific instance is the need for relevant decision strategies at D3 and D6 when questions of priority in the order of servicing EMUs arise; such a problem is distinctly nonstandard.

It is possible to use the techniques of operations research, especially queueing theory, to undertake such studies. Some of these are reported in Sections 4 - 7.

4. Rating scales for wheelsizes

The group considered the possibility of simplifying the rating scale for wheelsizes, with consequent simplifications of the information needing to be kept on a wheelset and the rules for compatibility of wheelsets.

Data provided suggested that the reduction in diameter caused by reprofiling is normally distributed with a mean of 25mm, standard deviation of 3.16 (*i.e.*

variance of 10). Assume that successive reprofilings are statistically independent. Then, for $n = 0, \ldots, 3$:

$$\text{Wheel diameter after } n \text{ reprofilings has a } N(840 - 25n, 10n) \text{ distribution.} \quad (1)$$

The independence assumption seems very reasonable.

Five-class rating scale

From (1) the successive means after reprofiling are 815, 790, 765. So a scale more closely aligned to those numbers has appeal, bearing in mind also the compatibility requirement. The following scale is proposed, based on a 12.5mm class interval.

Size Class	Range of Diameter in mm
I	827.5 - 840
II	815 - 827.5
III	802.5 - 815
IV	790 - 802.5
V	777.5 - 790

We further propose that wheels at most two size classes apart are regarded as compatible (I - III, *etc.*)

This scale has the following properties.

- All new wheels are in class I (all $\equiv$ 840). The first reprofiling will distribute wheels equally in classes II and III, the second equally in classes IV and V. In fact, (1) shows that

$$\left.\begin{array}{l} \text{Prob } \{ \text{ 1st reprofiling is in II or III } \} = 2\{1 - \Phi(\frac{827.5-815}{\sqrt{10}})\} = .99997 \\ \text{Prob } \{ \text{ 2nd reprofiling is in IV or V } \} = 2\{1 - \Phi(\frac{802.5-790}{\sqrt{20}})\} = .995 \end{array}\right\} \quad (2)$$

  here, $\Phi(\cdot)$ is the standard normal distribution function

- The maximum tolerable diameter difference here is 37.5mm per EMU. This is close to the 40mm currently used and is in line with Queensland Railways' desire to reduce this tolerance.

- A possible drawback of this scale is that is assigns wheels with diameter between 770 and 777.5 for replacement. Such wheels would virtually all come from a third reprofiling (only about 1% of third reprofilings will have diameter above 777.5), so this scale really discards the possibility of such a procedure. What do we lose ? The chance of a third reprofiling producing an acceptable wheel is $1 - \Phi(\frac{770-765}{\sqrt{30}}) = .18$. It may be that discarding the possibility of a third reprofiling for about 1/6 of all wheelsets, in the long run, will be unacceptably costly.

Any attempt to overcome this difficulty by introducing a class VI leads to the dilemma of either having the class narrower than all others, or lowering the minimum diameter to 765mm. Both seem unappealing.

Number of reprofilings scale

As noted above, the first two reprofilings are very reliably identified with particular sets of size classes (*cf.* equation(2)). This suggests a further simplification of the scale, to classes R0, R1, R2, where:

$$\text{Rn} = \text{class of wheels which have had } n \text{ reprofilings } (n = 0, 1, 2)$$

The proposed tolerance standard for wheelsets on the same EMU is now adjacent size classes.

- What is the chance of this standard producing an incompatibility on an EMU? The potential problem is a mix of R1 and R2 wheelsets. But the chance of such an incompatibility is

  $$p = \text{Prob}(X_1 - X_2 > 40)$$

  where X_i $(i = 1, 2)$ is a typical diameter after i reprofilings. From (1),

  $$p = 1 - \Phi(\frac{40 - 25}{\sqrt{30}}) = .003$$

  So this standard is very unlikely to lead to incompatibility, even though actual wheel diameters are not being recorded.

- The drawback mentioned above for the five-class scheme applies to this scale also. It is a balance between the greatly increased simplicity of this scale and the loss of third reprofiling possibility, and perhaps also the small risk (3 in 1000) of incompatibility between R1 and R2 wheelsets.

A different reprofiling procedure

The final possible change considered was to take off more metal at each reprofiling. The natural choice is to aim for a mean reduction of 30mm in diameter, since for larger reductions a significant proportion of second reprofilings will be below 770mm diameter.

Assume that the variance of each reduction is still 10 (this is almost certainly a lower bound). The diameter distribution after n reprofilings is

$$N(840 - 30n, 10n) \tag{3}$$

On the original rating scale, some 99.9% of first reprofilings now occur in classes 3 and 4. However it seems more desirable to again use the R0, R1, R2 scale, with the same tolerance standard as before. For this scale:

- From (3), the chance of incompatibility between R0 and R1 is $1-\Phi(\frac{10}{\sqrt{10}}) = .001$ and between $R1$ and $R2$ is $1 - \Phi(\frac{10}{\sqrt{30}}) = .035$
- The chance of an R2 wheel being less than 770mm in diameter is $\Phi(\frac{770-780}{\sqrt{20}}) = .013$

If either of the figures .035 or .013 is unacceptably large, a compromise choice of a reduction between 25mm and 30mm is easily assessed by the same method.

5. Analysis of Q1

A critical queue in the system is Q1, the flagged EMUs waiting for a wheelset change. It is unsatisfactory if this queue becomes too large. The group considered an approximate analysis of the average length of the queue.

The Khintchine-Pollaczek formula for mean queue size just after a departure from the queue is (Cox & Smith, 1961),

$$E(Q) = \rho + \frac{\rho^2}{2(1-\rho)}\{1 + \mathrm{var}(S)/E(S)^2\} \tag{4}$$

Here, λ is the arrival rate of customers *i.e.* the rate of flagging EMUs
S denotes a typical service time, mean $E(S)$, variance $\mathrm{var}(S)$
$\rho = \lambda E(S)$ is the *traffic intensity*

For (4) to make sense we must have $\rho < 1$; that is, customers are served on average faster than they arrive. Strictly, (4) requires a Poisson process of arrivals. Since the arrival process is the sum of the arrival processes for each EMU, the classical limit theorem for such sums (Khintchine, 1960, Ch 5) suggests that the Poisson assumption should be approximately true. In any case, (4) offers valuable insight into the queue's behaviour.

We calculate the parameters in (4) using a shift as the basic time unit. An EMU needs a new wheelset 1.61 times/year, so the total rate for all 88 EMU's per 3 week inspection cycle is

$$88 \times 1.61 \times 3/52 = 8.17$$

If x shifts are worked in 3 weeks, the arrival rate $\lambda = 8.17/x$ per shift. The total time to change the worn wheelsets on an EMU is 2.2 shifts, on average, so

$$\rho = 17.98/x \simeq 18/x \tag{5}$$

Clearly the system is unstable if $x \leq 18$. For larger x we get, from (4) and (5),

$$E(Q) \simeq \frac{18}{x(x-18)}\{x - 19 + 1.86 \text{ var}(S)\} \tag{6}$$

No variances were available, but the range of values of S is 0.25 – 5 shifts, so a value for var(S) in the range 1-3 is plausible. If in fact S is uniform on $(0, 5)$, $\text{var}(S) \simeq 2$.

Table 1: Values of $E(Q)$ from (6), for various var(S)

x	ρ	$E(Q)$		
		var(S) = 1	var(S) = 2	var(S) = 3
19	0.95	11.2	13	14.8
20	0.9	5.8	6.6	7.5
21	0.86	4	4.5	5

The values $x = 20$ and 8 flagged EMUs are fairly typical, so (4) - (6) do appear to be relevant. The results can therefore be used with some confidence to assess the quantitative effects of any changes.

6. Analysis of Q4

The group considered a simplified model of Q4 by relating it to the so-called repairman model (Morse, 1958, Ch 11). In this model, K machines are served by M repairman ($K \geq M$). When a machine fails it is serviced immediately if a repairman is available; otherwise it waits in a nominal queue until the repairman is free. Identifying machines with wheelsets on EMUs and repairmen with wheelsets in stock, the formal analogy is clear.

We want the probability that an EMU cannot have its requirements met from current stock. This is the probability of at least M + 1 failed machines. Note that this has the usual alternative interpretation as a long-run proportion of time, which is perhaps more meaningful in the EMU context. There is a formula for this probability, under the typical queuing assumptions of exponentially distributed times between breakdowns and service times. It is (Morse, 1958, eq 11.12)

$$\sum_{n=M+1}^{\infty} u^{K-n}e^{-u}/\{(K-n)!\, Q_{K,M}(u)\} \tag{7}$$

where $Q_{K,M}$ is a rather complicated function involving cumulative Poisson probabilities. Here,

$$u = \frac{\text{mean time between failures}}{\text{mean service time}}$$

"service time" being the time it takes a worn wheelset removed from an EMU to be repaired and replaced in stock.

The formula (7) could be used to explore the effect on this critical unavailability probability of changing M (the size of the wheelset stock) or the mean time between failures (*i.e.* changing the criterion for critical flange wear), among other factors. However, (7) does assume only one wheelset arrives at a time, which is not true, so it would be desirable to generalize (7) to the case of batch arrivals. The relevant equations can be written down in principle but there does not seem to be an explicit solution. The formula also ignores the two types of wheelsets and the size constraints. The latter will be important, and could be incorporated in some fashion.

7. A simple stochastic model

To gain some insight into the behaviour of the system, the group considered several very simple stochastic models. One is described in this section.

Suppose there are only two wheel sizes, and any EMU must have all its wheels of the same size (imagine a massive reprofiling that trimmed 50mm of diameter). So there are really two types of EMU, 1 and 2, say; an EMU retains its type for all time. The correction process causes a wheelset to alternate between types 1 and 2. The two types of wheelsets wear at different rates, so need replacement at different frequencies.

Let N_W be the number of wheelsets in stock. If $N_i(t)$ $(i = 1, 2)$ is the number of type i wheelsets in stock at time $t \geq 0$, then

$$N_1(t) + N_2(t) \equiv N_W \qquad \text{for all } t$$

Assume an EMU only has one wheelset replaced at a time. When a type 1 EMU arrives for wheelset replacement, it takes a type 1 wheelset from stock and its worn type 1 wheelset is converted (instantaneously !) to a type 2. So a type 1 arrival at t means that $N_2(t) \to N_2(t)+1$. A typical path of $N_2(t)$, for instance, is shown in Figure 2.

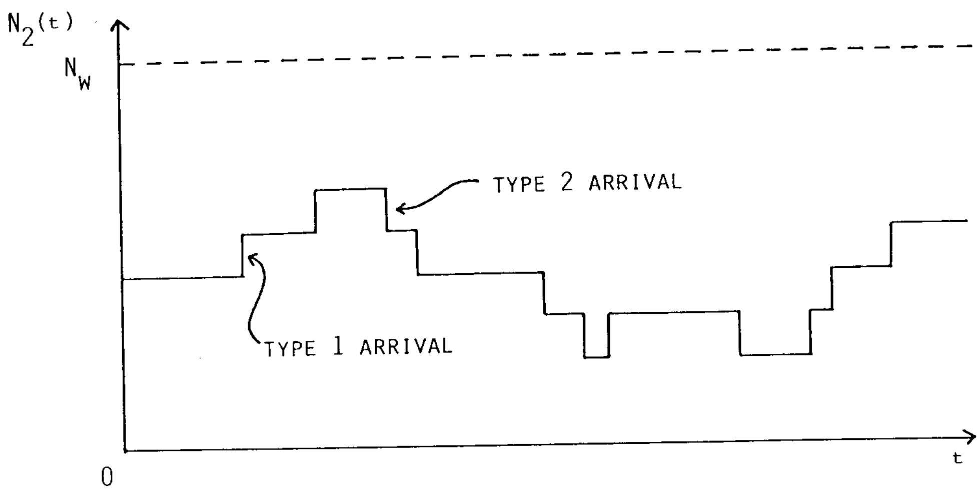

Figure 2: Typical level of type 2 wheelsets.

If we assume that times between wheelset replacements are all exponentially distributed, $N_2(t)$ is a *continuous time random walk* with *absorbing barriers* at 0 and N_W; these correspond to having only one type of wheelset in stock.

A sensible operating strategy is to keep the process away from the barriers for as long as possible, *i.e.* to maximise some function of the time to absorption. This requires:

1. that the process has no mean drift in time
2. that it starts as far from each barrier as possible

Requirement (1) means that, if λ_i $(i = 1, 2)$ is the mean rate of replacement for type i wheelsets, and ν_i $(i = 1, 2)$ is their number, we should choose the ν_i such that

$$\nu_1\lambda_1 = \nu_2\lambda_2$$

This ensures the overall arrival rates for the two wheelset types are equal. Requirement (2) means that $N_2(0) = \frac{1}{2}N_W$.

If these conditions are imposed, the process has steps which are ± 1 with probability $\frac{1}{2}$ and which occur at the points of a Poisson process of rate $(\lambda_1\nu_1 + \lambda_2\nu_2)$. If N_T is the total number of wheelsets on trains, then $\nu_1 + \nu_2 = N_T$, so the rate becomes $2\lambda_1\lambda_2 N_W/(\lambda_1 + \lambda_2)$ here. To get the mean time until absorption we can use Wald's identity (Cox and Miller, 1965, p244); we find that

$$\text{mean time to absorption} = \frac{\lambda_1 + \lambda_2}{8\lambda_1\lambda_2}\frac{N_W^2}{N_T}$$

So the mean duration of the process until the first time when one wheelset type is out of stock can be controlled by choice of N_W.

Example: If the type 1 wheels are size class 1, type 2 are size class 4 the empirical formula for lifetime gives $\lambda_1^{-1} = 70$, $\lambda_2^{-1} = 57$ and $N_T = 88 \times 12 = 1056$. Then $(\lambda_1 + \lambda_2)/8\lambda_1\lambda_2 \simeq 16$, so with a stock of 36 wheels the mean time before an out-of-stock is about 19 weeks.

The conclusions generalize to more than two types of EMU. Other generalisations, such as noninstantaneous repair and multiple wheel changes, can also be incorporated, although the answers are unlikely to be as transparent.

8. Some other subsystem models

Several other subsystems were considered by group members during one week. A brief description follows.

Wheelsets on EMUs. The state of the wheelsets on an EMU can be modelled as a semi-Markov process. This can give information about when an EMU is likely to require attention. It may also give important results about a long-run distribution of wheelset sizes on the EMU fleet. Such a conclusion could have important consequences for long-term optimization and for assessing the consequences of modifications in operations. One obvious benefit is to provide a basis for D7.

Costs of different policies. If the transition process for wheelsets is Markovian (*i.e.* exponential lifetimes), it may be possible to set up a Markov programming model to examine possible courses of action and their related costs, and perhaps to optimise costs. However, the assumption of exponential lifetimes does not appear to be even approximately true here.

Acknowledgements

Particular thanks must go to Bill Thomas, who was constantly available to provide information and advice, and who answered all questions with competence and enthusiam. Mary Southwell was the principal developer of the crucial flow chart in Figure 1. Other contributors to the work on this problem included Neville Fowkes, David Gates, Huu-Nhon Huynh, Graham Mills, Bernhard Neumann, Peter Trudinger and Kevin Wilkins.

References

D.R. Cox & H.D. Miller, *The Theory of Stochastic Processes* (Methuen, London, 1965).

D.R. Cox & W.L. Smith, *Queues* (Methuen, London, 1961).

A.Y. Khintchine, *Mathematical Methods in the Theory of Queueing* (Griffin, London, 1960).

P.M. Morse, *Queues, Inventories and Maintenance* (Wiley, New York, 1958).