A two-phase flow model for the cooling of steel by water sprays

Abstract. Bao Steel in Shanghai, P.R. China, are one of the country's major producers of steel by the continuous casting process. As the hot steel is drawn from the casting machine, it solidifies from its edges inwards. To complete the process of solidification and cool the hot steel, jets of water are sprayed onto the surface of the steel. In this study we consider a two-phase flow model for the heating and vaporisation of the water drops. We concentrate in particular on the non- monotonicity in the measured heat transfer from the steel as a function of the steel temperature. The result of the model is that the vaporisation of a single drop has first to be considered and the resulting behaviour governs the mass and momentum transfer between the two phases. Some simple solutions are then derived for the resulting system of ordinary differential equations. It appears that, when a vapour layer is present, a Stefan-type problem must be solved to determine the thickness of this layer.

1 Introduction

In this study we wish to consider the cooling of hot steel by water sprays during continuous casting in the Bao Steel plant, Shanghai, P.R. China. For the purposes of the following discussion, we assume that the spray involved takes the form of a large number of isolated droplets of water. The main purpose of this investigation is to explain the qualitative results of figure 1, where the measured heat transfer Q from the steel is plotted against the surface steel temperature. One

might expect that for a constant rate of spraying, as the steel temperature increased, so the heat transfer from the steel would increase. Figure 1 shows that this is not the case. Some discussions with a representative from Bao Steel in Shanghai, during which he revealed that, for strong sprays, droplets of water can clearly be seen "running off" the hot steel, and this has led us to conjecture the following phenomenological explanation for figure1:

- For low spraying rates, the heat transfer from the steel is sufficiently large that all water droplets are vaporised before they reach the surface of the hot steel. This means that the steel is surrounded by a vapour blanket much thicker than a typical droplet size. Obviously this is a non-optimal condition as far as the cooling of the steel is concerned as this blanket acts as an insulator for the hot steel.

- For high spraying rates, many droplets arrive at the surface of the steel without having vaporised. Once such droplets impact the steel, the Leidenfrost effect allows the droplets (a) to remain intact for longer than might be expected, owing to the insulating vapour blanket that is formed at the interface between an individual droplet and the hot steel and (b) to move across the steel in a virtually friction-free manner. The latter manifestation of the Leidenfrost effect is supported by the observations of Dr. Go Zhaohui (Bao Steel) who notes that for high spraying rates many unvaporised droplets can be seen "falling off" the hot steel.

This speculative mechanism for the non-monotonicity in figure 1 leads us to propose that, as far as the cooling of hot steel is concerned,

optimal conditions prevail when the droplets arrive at the surface of the steel just as their radius drops to zero; one of the major outputs of the model that we seek to develop below will be to predict the conditions under which this optimal vapourisation can occur.

2 Mathematical modelling of the water spray process

We now propose a simple two-phase flow model for the flow and vaporisation of water drops in the vicinity of hot steel. The model is based on the two- phase flow methodology outlined in [2], where the usual ensemble averaging takes place. For simplicity we only consider flow in one space dimension, and assume that there are two phases only: the droplets (dispersed phase) and the vapour (continuous

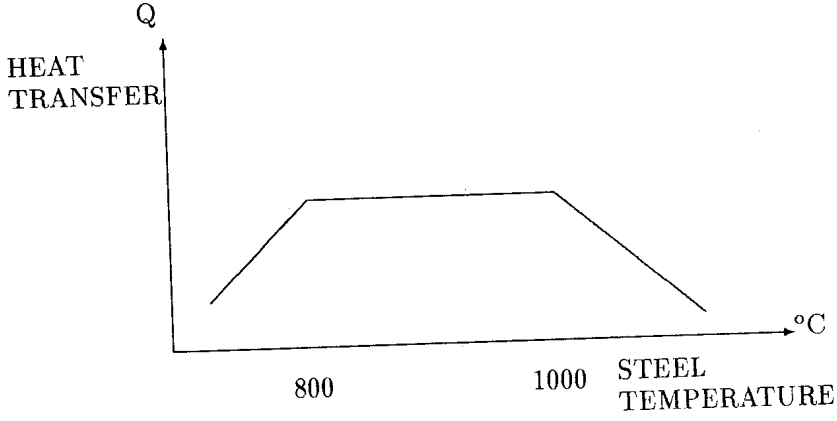


Figure 1: Heat transfer Q vs. steel temperature for steel cooling at a constant spray rate

phase). We also assume that the vapour is incompressible (though compressibility could be added to the model without too much difficulty). After averaging has taken place, we find that the equations that govern the flow are

$$(\rho_d\alpha)_t + (\rho_d\alpha u)_x = -f \tag{1}$$

$$(\rho_g(1-\alpha))_t + (\rho_g(1-\alpha)v)_x = f \tag{2}$$

$$(\rho_d\alpha u)_t + (\rho_d\alpha u^2)_x = -\Phi - fu \tag{3}$$

$$(\rho_g(1-\alpha)v)_t + (\rho_g(1-\alpha)v^2)_x + p_{gx} = \Phi + fu \tag{4}$$

$$\rho_d c_p((\alpha T)_t + (\alpha u T)_x) = k\nabla^2 T - E - f c_p T_v \tag{5}$$

$$\rho_g c_{pg}(((1-\alpha)T_g)_t + ((1-\alpha)vT_g)_x) = k_g\nabla^2 T_g - E \tag{6}$$

In (1)–(6) ρ_d and ρ_g denote the density of the droplets and the vapour respectively, α denotes the void fraction of droplets ($0 \leq \alpha \leq 1$), u and v denote the respective speeds of the droplets and the vapour, p_g denotes the pressure in the vapour phase and the droplet and gas temperatures are given by T and T_g. T_v denotes the temperature of vaporisation of the fluid in the droplets. Thermal conductivity and specific heat are denoted by k and c_p respectively, a subscript g denoting properties in the gas. All such thermal properties are assumed to be constant (see appendix for typical values). The two-phase flow problem is driven by the sourrce terms f, Φ and E. The term f characterises the mass that is transferred from the droplets to the vapour as a droplet vaporises. Specifically, f denotes the mass per second per unit volume that is transferred. At present, f is unknown and needs to be specified using a submodel. The term Φ accounts for the viscous drag experienced by the droplets as they move through the gas. E denotes the distributed energy sources and sinks that

affect the gas and the droplet flow. We also note that, if the number density (number per unit volume) of droplets in the flow is denoted by N, then

$$\frac{4}{3}\pi s^3 N = \alpha. \tag{7}$$

It should be noted that a number of common two-phhase flow assumptions have been made in deriving the govening equations (1)–(6). In particular

- All "profile coefficients" arising from the fact that during the aveaging process the product of the aveage is not equal to the average of the products have been sot to unity;
- All sources of drag in the momentum equations have been lumped together via a single drag term Φ;
- The standard "dusty gas" two-phase flow hypothesis has been invoked in order to set the pressure in the drops equal to a constant.

We now consider the submodels that must be specified to close the equations (1)–(6). First, we note that the mass transfer term is given by

$$f = -4\pi\rho_d s^2 s\dot{s}N, \tag{8}$$

because there are N droplets each of density ρ_d, the surface are of each is $4\pi s^2$, and the surface recedes at a rate $\dot{s}$; of course, a submodel for the vaporisation process is still required if (8) is to be of any use.

Dotermining an accurate form for the drag Φ may not be a simple matter as the averaging involved has to be carried out rather carefully in the full study of the influence of Φ on void fraction carried

out in [3], however, for disperse low Reynolds number flows the law it was found that

$$\Phi = \frac{9\alpha\mu_g}{2s^2}(u - v),$$

where μ_g denotes the dynamic viscosity of the gas.

One of the govening equations (1)–(6) may be dealt with at once. We observe that the conduction time scale τ_c in a drop of radius s_0 is given (according to the values in the appendix) by

$$\tau_c = \frac{s_0^2\rho_d c_p}{k} \sim 6 \times 10^{-4}\text{sec}.$$

This suggests that a drop travelling a typical distance of 1m at a speed of 5m/s takes only about three thousandths of its total travel time to heat up by conduction in the drop. We conclude that sensible heating of a drop is thus virtually instantaneous and the vaporisation process is controlled completely by latent heat. The temperature in a droplet is thus constant and so from (5) we have $E = -fc_pTv$.

3 Drop vaporisation: local problem

The two-phase flow model presented in section 2 cannot be analysed until a sultable submodel has been proposed for the vaporisation of an individual droplet so that the mass source term f can be determined. We therefore consider the local problem close to a single droplet. If we denote the vaporisation temperature of the fluid in a droplet by T_v, then since, as we have already seen, the temperature in a droplet is effectively constant, the details of the drop vaporisation are determined entirely by a local Stefan problem in the gas. We assume that the velocity of the gas produced by the vapor- ising droplet liquid is given by w, and that the local problem is spherically

symmeric. We denote by τ the distance from the centre of a droplet and assume that the droplet (which has initial radius s_0) has radius $s(t)$ at time t. Conservation of mass for the droplet then yields

$$\frac{1}{r^2}(r^2 w)_r = 0. \tag{9}$$

The local gas temperature $T(r,t)$ satisfies

$$\frac{k_g}{r^2}(r^2 T_r)_r = \rho_g c_{pg}(T_t + wT_r) \tag{10}$$

with

$$T \to T_g(x,t) \text{ as } r \to \infty, \quad T = T_v \text{ at } r = s(t). \tag{11}$$

The specification of the local problem is completed by supplying a boundary condition for w at the vaporisation front $s(t)$ and a Stefan condition to allow the free boundary $s(t)$ to be determined. To derive the former, suppose that

the free boundary r = s(t) moves in a time dt to $r = s(t+dt)$. Then the amount of droplet liquid that is gasified is $\rho_d[s(t+dt)-s(t)]$. Since at the free boundary the gas moves away with a radial speed $w + \dot{s}$, mass conservation dictates that

$$w = \left(\frac{\rho_g - \rho_d}{\rho_g}\right)\dot{s} \text{ at } r = s(t). \tag{12}$$

Since in all the current cases of interest $\rho_d \gg \rho_g$, we immediately approxi- mate (12) by

$$w = -\frac{\rho_d}{\rho_g}\dot{s} \text{ at } r = s(t). \tag{13}$$

The Stefan condition is, as usual, given by

$$-\rho_d L\dot{s} = k_g T_r \tag{14}$$

where L denotes the latent heat of vaporisation of the fluid in a droplet, 8 denotes the total (Lagrangian) derivative of s and use has been made of the fact that, by our earlier arguments, the temperature gradient within the drop is zero.

We now nondimensionalise the equations (9) and (1O) and the boundary conditions (11)-(14). We set $s = s_0\bar{s}$, $t = \tau\bar{t}$, $r = s_0\bar{r}$, $w = W\bar{w}$ and $T = T_v + \Delta T\bar{T}$ where $\Delta T = (T_g - T_v)$. The time scale τ is given

$$\tau = \frac{\rho_d L s_0^2}{k_g \Delta T},$$

this scaling having been determined by the Stefan condition (14). We note that, using the values in the appendix, we find that

$$\tau \sim \frac{10}{\Delta T} \gg \tau_c$$

thus confirming our earlier assertion that the role played by the latent heat in the melting dominates that of the sensible heat. Now that T has been determined, the gas velocity scale W follows from (13). We find that

$$W = \frac{k_g \Delta T}{\rho_d L s_0} \sim 2 \times 10^{-3}/K\Delta T,$$

so that when the steel is hot relatively high gas velocities may be produced. When these scalings are used, we find that, upon dropping the bars for convenience, and assuming that $T_t = 0$, the problem becomes

$$\frac{1}{r^2}(r^2 T')' = \gamma w T' \tag{15}$$

where a dash denotes differentiation with respect to r and

$$T \to 1 \text{ as } r \to \infty, \quad T = 0 \text{ at } r = s(t) \text{ and } -\dot{s} = T_r \text{ at } r = s(t).$$

The non-dimensional parameter γ is given by

$$\gamma = \frac{c_{pg}\Delta T}{L}.$$

We may estimate γ using the values found in the appendix: we find that

$$\gamma \sim (10^{-3}/K)\Delta T$$

and so, depending on the temperature of the steel that is being cooled, we conclude that $\bar{\gamma}$ may be small, but is at most $O(1)$.

The gas velocity w is easily determined using the non-dimensional forms of (9) and (13). We find that

$$w = -\frac{\dot{s}s^2}{r2}.$$

This allows (15) to be solved completely When the boundary conditions are applied we find that

$$T = 1 - \left(\frac{1 - \exp(\gamma \dot{s} s^2/r)}{1 - \exp(\gamma \dot{s} s)}\right)$$

When we further set $-\dot{s} = T_r$ at $r = s(t)$, we find that

$$-\dot{s} = -\frac{\gamma \dot{s} \exp(\gamma \dot{s} s)}{1 - \exp(\gamma \dot{s} s)}$$

and thus finally

$$\dot{s} = -\frac{\log(1+\gamma)}{\gamma s}. \tag{16}$$

For $\gamma \ll 1$, the obvious limit may be taken to yield $\dot{s} = -1/s$. Recast in dimensional form, (16) becomes

$$\dot{s} = -\left(\frac{k_g}{s\rho_d c_{pg}}\right) \log\left(1 + \frac{c_{pg}}{L}(T_g(x,t) - T_v)\right)$$

or, when $\gamma \ll 1$,

$$\dot{s} = -\frac{k_g(T_g - T_v)}{\rho_d s L}.$$

4 Analysis of the equations

Now that a submodel has been proposed for $\dot{s}$, we are in a position to analyse the equations of motion (1)-(6). We concentrate only on steady flows and set all time derivatives in (1)-(6) to zero; the start-up or wind-down processes could of course be considered if the time derivatives were included, but this would lead to a much more involved problem for which a numerical approach would be required.

In addition we ignore equations (4) and (5) on the grounds that the former serves only to provide an equation for the gas pressure (which could be calculated if desired once all the other variables have been determined) and, as already noted, the latter simply states that the temperature in a droplet is constant, the sensible heat having no role to play in the vapourising of a droplet. We therefore study the ordinary differential equations

$$(\rho_d \alpha u)_x = -f \tag{17}$$

$$(\rho_g (1-\alpha) v)_x = f \tag{18}$$

$$(\rho_d \alpha u^2)_x = -\frac{9\alpha\mu_g(u-v)}{2s^2} - fu \tag{19}$$

$$us_x = -\left(\frac{k_g}{s\rho_d c_{pg}}\right) \log\left(1 + \frac{c_{pg}}{L}(T_g(x,t) - T_v)\right) \tag{20}$$

$$\rho_g c_{pg}((1-\alpha)vT_g)_x = k_g T_{gxx} + f c_{pg} T_v \tag{21}$$

where

$$f = \frac{3\alpha k_g}{s^2 c_{pg}} \log\left(1 + \frac{c_{pg}}{L}(T_g - T_v)\right).$$

The equations are to be solved in the region $0 \le x \le D$ where $x = 0$ denotes the spray jet orifices and $x = D$ the surface of the steel. A number of different specifications of boundary conditions are possible, but for the present we assume that the properties of the water spray

are known at $x = 0$, the temperature gradient is zero there, and at $x = D$ the gas has the same temperature as the steel and no gas can pass through the steel. Thus

$$T_{gx}(0) = 0, \quad s(0) = s_0, \quad \alpha(0) = \alpha_0, \quad u(0) = U$$

and

$$v(D) = 0, \quad T_g(D) = T_s$$

where T_s denotes the steel temperature, α_0 and s_0 denote respectively the initial void fraction and drop radius, and U denotes the drop speed at thespray nozzle exit. Obviously in general numerical calculations will be required. There are a few cases though where some analytical progress may be made. We begin by non-dimensionalising (17)-(21) according to

$$T_g = T_v + \bar{\theta}(T_s - T_v), \qquad \alpha = \alpha_0\bar{\alpha}, \qquad s = s_0\bar{s},$$
$$u = U\bar{u}, \qquad v = \frac{U\rho_d\alpha_0}{\rho_g}\bar{v}, \qquad x = D\bar{x}.$$

This choice of scalings reflects the fact that the scale for the gas velocity v is effectively determined by (18), and that the gas temperature must at all times be between T_v and T_s. We shall also assume for simplicity that $\gamma \ll 1$ so that we may approximate the source term by using

$$f = \frac{3\alpha k_g(T_g - T_v)}{s^2 L}$$

and that $\alpha_0 \ll 1$ so that we may approximate $1 - \alpha$ by 1.

In non-dimensional form, the equations become

$$(\bar{\alpha}\bar{u})_{\bar{x}} = -M\frac{\bar{\theta}\bar{\alpha}}{\bar{s}^2} \tag{22}$$

$$(\bar{v})_{\bar{x}} = M\frac{\bar{\theta}\bar{\alpha}}{\bar{s}^2} \tag{23}$$

$$(\bar{\alpha}\bar{u}^2)_{\bar{x}} = -\frac{9N\bar{\alpha}}{2\bar{s}^2}\left(\frac{\rho_g\bar{u}}{\rho_d\alpha_0} - \bar{v}\right) - M\frac{\bar{u}\bar{\alpha}\bar{\theta}}{\bar{s}^2} \tag{24}$$

$$(\bar{u}\bar{s})_{\bar{x}} = -M\frac{\bar{\theta}}{3\bar{s}} \tag{25}$$

$$(\bar{v}\bar{\theta})_{\bar{x}} = \Gamma\bar{\theta}_{\bar{x}\bar{x}} + RM\frac{\bar{\alpha}\bar{\theta}}{\bar{s}^2} \tag{26}$$

where the key nondimensional parameters are given by

$$M = \frac{3Dk_g(T_s - T_v)}{U\rho_d L s_0^2},\ \Gamma = \frac{k_g}{c_{pg}\rho_d\alpha_0 UD},\ N = \frac{\mu_g D\alpha_0}{U\rho_g s_0^2},\ R = \frac{T_v}{T_s - T_v}.$$

We note first that, according to the values in the appendix,

$$M \sim 0.07(T_s - T_v)$$

and since the values of T_s that are of interest range between about 1000°C and 2000°C it therefore seems reasonable to assume that $M \gg 1$. We alsonote tha

$$\Gamma \sim \frac{0.25 \times 10^{-8}}{\alpha_0},$$

so that diffusive effects in the gas are negligible unless α_0 is very small or a boundary layer is present. Finally we observe that R is likely to range between about 0.1 and 0.05, but may evidently be considered as being small.

Some analysis may now take place. Assuming for simplicity that the viscous drag may be ignored (i.e. taking the limit $N = 0$) we find

that $\bar{u} = 1$, and equation (24) may be dispensed with. Addition of (22) and (23) gives, after an integration,

$$\bar{\alpha} + \bar{v} = \bar{\alpha}(1)$$

(since we are assuming that $\bar{v} = 0$ at $\bar{x} = 1$). Dividing (22) by (24) and integrating now gives that

$$\bar{\alpha} = \bar{s}^3.$$

The whole problem may thus be reduced to the two ordinary differential equations

$$\bar{\alpha}' = -M\bar{\theta}\bar{\alpha}^{1/3} \tag{27}$$

$$((\bar{\alpha}(1) - \bar{\alpha})\bar{\theta})' = \Gamma\bar{\theta}'' + RM\bar{\theta}\alpha^{1/3} \tag{28}$$

where a prime denotes $d/d\bar{x}$.

We now consider the orders of magnitude of the various parameters in the problem. From the practical data we have observed that M is large, and so we will first examine the equations in the formal limit $M \to \infty$. By examining (27), we expect that near to the steel (where $\theta \sim 1$) a thin layer of width $1/M$ will be present and we therefore scale using

$$1 - \bar{x} = \tilde{x}/M.$$

We find that (with dash now denoting $d/d\tilde{x}$) the governing equations are

$$\bar{\alpha}' = \bar{\theta}\bar{\alpha}^{1/3} \tag{29}$$

$$((\bar{\alpha} - \bar{\alpha}_s)\bar{\theta})' = M\Gamma\bar{\theta}'' + R\bar{\theta}\alpha^{1/3} \tag{30}$$

where $\bar{\alpha}_s$ now denotes the (yet to be determined) value that $\bar{\alpha}$ takes at thes teel. Note that though we have taken the limit$M \to \infty$ we

have retained terms of order $M\Gamma$ and R in (30); this is because these terms relate to the thermal problem rather than to the evaporation problem, and will give rise to boundary layers within the thin layer that we are currently analysing. Assuming that the nondimensional gas temperature tends to $\bar{\theta}_\infty$ and $\alpha \to 1$ as $\tilde{x} \to \infty$, the boundary conditions for (29) and (30) are

$$\bar{\theta}(0) = 1,\ \bar{\alpha}(0) = \bar{\alpha}_s,\ \theta_\infty(0) = \bar{\theta}_\infty,\ \bar{\alpha}(\infty) = 1. \tag{31}$$

Integrating (30) using (29) and assuming that $\bar{\theta}'(\infty) = 0$, we find that the equations may be written in the form

$$\bar{\alpha}' = \bar{\theta}\bar{\alpha}^{1/3} \tag{32}$$

$$\bar{\theta}' = \frac{\bar{\theta}(\bar{\alpha} - \bar{\alpha}_s) + R(1 - \bar{\alpha}) - \bar{\theta}_\infty(1 - \bar{\alpha}_s)}{M\Gamma} \tag{33}$$

so that

$$\frac{d\bar{\theta}}{d\bar{\alpha}} = \frac{\bar{\theta}(\bar{\alpha} - \bar{\alpha}_s) + R(1 - \bar{\alpha}) - \bar{\theta}_\infty(1 - \bar{\alpha}_s)}{M\Gamma\bar{\theta}\bar{\alpha}^{1/3}} \tag{34}$$

and a phase plane analysis may now take place. The task is to find a phase trajectory joining the points $(\bar{\alpha}_s, 1)$ and $(1, \bar{\theta}_\infty)$ in the first quadrant of the$(\bar{\alpha}, \bar{\theta})$-plane; various different cases must be considered.

5 No vapour layer

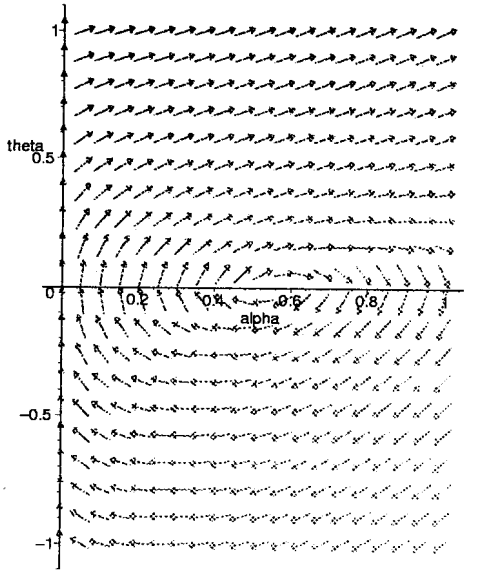
We first analyse the case where there is no vapour layer and the droplets havea radius greater than zero when they reach the surface of the steel. Since the one third power of α in equation (32) means that near $\tilde{x} = 0$ (where $\bar{\theta} \sim 1$) the right-hand side of the equation is

not Lipschitz continuous, we discount for the moment the case where $\bar{\alpha}_s = 0$, preferring to regard this as a vapour layer of width zero. Accordingly we assume for the present that $\bar{\alpha}_s \geq \delta > 0$ where δ is some positive constant.

We now consider the phase plane analysis of (34) above. The phase trajectories in the $(\bar{\alpha}, \bar{\theta})$-plane are vertical when either $\bar{\alpha} = 0$ or $\bar{\theta} = 0$ and are horizontal when

$$\bar{\theta} = \frac{\bar{\theta}_\infty(1 - \bar{\alpha}_s) + R(\bar{\alpha} - 1)}{\bar{\alpha} - \bar{\alpha}_s}.$$

The slope of this nullcline curve is given by $(1-\bar{\alpha}_s)(R-\bar{\theta}_\infty)/(\bar{\alpha}-\bar{\alpha}_s)^2$, and is therefore positive when $R > \bar{\theta}_\infty$. We also note that when $R > \bar{\theta}_\infty$ (i) the phase paths have positive slope in parts of phase space above the the nullcline and (ii) the equations have singular points at $(\bar{\alpha}, \bar{\theta}) = ((R - \bar{\theta}_\infty + \bar{\theta}_\infty\bar{\alpha}_s)/R, 0)$ and $(0, \bar{\theta}_\infty + (R - \bar{\theta}_\infty)/\bar{\alpha}_s)$. The qualitative details of the phase plane are shown in figure 2 (for the particular values $\bar{\alpha}_s = 1/10$, $R = 1/2$ and $\bar{\theta}_\infty = 1/4$- which are not realistic but have been chosen to make the qualitative details of the figure as clear as possible), and we see immediately from the slopes of the phase paths that in this case there can be no connection between $(\bar{\alpha}_s, 1)$ and $(1, \bar{\theta}_\infty)$. We conclude that when $R > \bar{\theta}_\infty$ the problem has no solution.



Phase plane for $\bar{\alpha}_s = 1/10$, $R = 1/2$ and $\bar{\theta}_\infty = 1/4$

When$R = \bar{\theta}_\infty$ a special case occurs. Equation (34) becomes

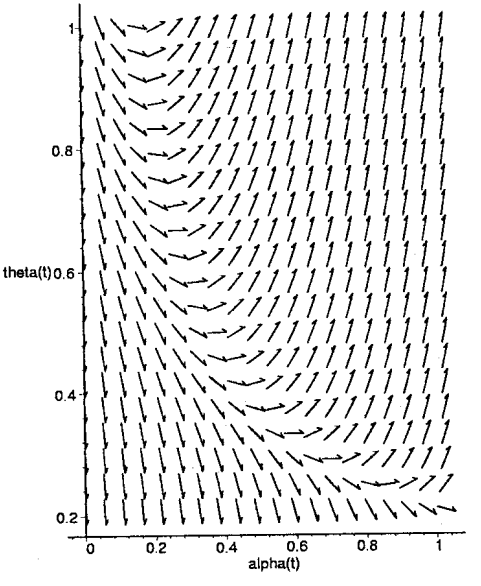
$$\frac{d\bar{\theta}}{d\bar{\alpha}} = \frac{(\bar{\theta} - R)(\bar{\alpha} - \bar{\alpha}_s)}{M\Gamma\bar{\theta}\bar{\alpha}^{1/3}}$$

and the nullclines are thus the straight lines $\bar{\theta} = R$ and $\bar{\alpha} = \bar{\alpha}_s$. A consideration of the signs of the slopes in the phase planes quickly shows again that there can be no solution.

It has therefore been established that we require $\bar{\theta}_\infty > R$ f there is to be a phase path connecting $(\bar{\alpha}_s, 1)$ to $(1, \bar{\theta}_\infty)$. In this case however the slope of the nullcline is negative and the signs of the gradients of the phase paths indicate that a solution will always be possible. We conjecture that for each given value of $\bar{\alpha}_s$ with $0 < \bar{\alpha}_s < 1$ a unique value of $\bar{\theta}_s$ exists which allows a connection in phase space between $(\bar{\alpha}_s, 1)$ and $(1, \bar{\theta}_\infty)$. Though we have not yet been able

to prove this, numerical experiments indicate that this is likely to be the case: a typical connection trajectory is shown in figure 3 for the parameter values $\bar{\alpha}_s = 0.0001$, $R = 1/20$ and $M\Gamma = 0.2$: the computed $\bar{\theta}_s$ in this case was 0.21771385.

Some other typical numerical results are shown in table 1. These resultswere obtained in an extremely simplistic fashion, using the initial value problem numerical integrator in MAPLE to shoot for given values of $\bar{\theta}_s$. In this form, the problem is extremely sensitive and very accurate values of $\bar{\theta}_s$



Phase plane and solution trajectory for $\bar{\alpha}_s = 0.0001$, $R = 1/20$, $M\Gamma = 0.2$. Computed $\bar{\theta}_s = 0.21771385$.

are required to prevent the solution from blowing up. Undoubtedly there are much better ways of attacking the problem numerically; these are being investigated at present.

5.1 Boundary layer behaviour

We have noted above that the problem in a boundary layer of width $1/M$ (given by (29), (30) and (31)) contains two parameters, namely R and $M\Gamma$. Since both of these parameters are small, we can examine and identify further boundary layers within this boundary layer. We now briefly discuss

R	$M\Gamma$	θ_∞
0.05	0.3	0.26576230
0.05	0.2	0.21771385
0.05	0.1	0.158797013
0.1	0.1	0.20425412
0.1	0.01	0.1241753309

Table 1: Numerical results for $\bar{\alpha}_s = 10^{-4}$: computed values of $\bar{\theta}_\infty$ for various R and $M\Gamma$. some of this internal boundary layer structure.

Some study reveals that, away from $\bar{x} = 0$ there is an outer solution to the problem (where diffusion plays no role and convection is balanced by vaporisation) which is obviously given for $\bar{\theta}_\infty > R$ by the nullcline, so that

$$\bar{\theta} \sim \frac{R(1-\bar{\alpha}) - \bar{\theta}_\infty(1-\bar{\alpha}_s)}{\bar{\alpha} - \bar{\alpha}_s}.$$

A boundary layer is also present near to the steel where diffusive effects are important. The behaviour here appears to be much more complicated. Work is continuing at present to completely determine the structure.

6 Vapour layer

When conditions are such that a droplet completely vaporises before it reaches the wall, account must be taken of the vapour layer that

adjoins the steel. Though it is fairly clear how to proceed under these circumstances, we have not yet pursued the analysis. The fact that a region now exists where $\alpha = 0$ means that the lack of Lipschitz continuity of $\bar{\alpha}^{1/3}$ near $\tilde{x} = 0$ where $\bar{\theta} = 1$ allows the solution to have the required behaviour, as it can take the value zero over a finite region and then become nonzero. The thickness of the vapour layer under these circumstances will be determined by a Stefan type problem. Further analysis of this aspect of the problem is taking place at present.

7 Conclusions and suggestions for further work

A simple two-phase flow model, coupled to a local Stefan problem has been presented for the vaporisation of drops of a liquid spray in the heated layer produced by hot steel. Obviously there are many possibilities for further work. These include:

- carrying out a full literature survey. This has not been done in any detail yet, but it seems very likely that other attempts have been made to model evaporating water sprays;
- carrying out a much fuller analysis of the case when the droplets vaporise before reaching the steel, so that a vapour layer is formed. The problem then becomes one of free boundary type;
- completing a more careful analysis of the various connection problems that arise for different droplet radii;
- incorporation of third phase (air) into the model. Although we do not expect this to make much qualitative difference to

the predictions of the current model, there are significant differences between air and steam and we might expect that the quantitative predictions of the model may change;

- taking account of the fact that the thermal properties (thermal conductivity specific heat, etc.) of both the droplet and the gas phase are likely to vary with temperature. Once again, although the problem will be greatly complicated it is unlikely that the qualitative results will change;

- taking into account the compressibility of the vapour phase. Although we do not expect to encounter pressures higher than a few bar in this process, the variations in temperature will mean that compressibility may play a significant role;

- adding to the model some consideration of the fact that, in the steel plant, the sprays are likely to be discrete. The effect of this may be that the air near to the hot steel moves in a non-uniform manner between two sprays;

- allowing for the fact that, at a given x-station, it is likely that drops of a range of sizes will be present. Inclusion of drop size distribution effects may be expected to complicate the modelling considerably as the number of independent variables will be increased by one.

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Appendix: typical values of constants

There is by no means unanimity concerning the values of density thermal conductivity and specific heat at high pressures and temperatures. In the current study though the temperatures may be as high as 1500°C, we do not expect pressures higher than a few bar as the system is not confined in any way. We have therefore used values from standard steam tables. Although these may not be completely accurate, they are unlikely to be in error by orders of magnitude. Data items marked with a dagger (†) are informed guesses on our part and need to be confirmed with Bao steel.

$c_p = 4.214 \times 10^3$J/kg/K (water, 10 bar, 100°C)([1], p.116)

$c_{pg} = 2.032 \times 10^3$J/kg/K (steam, 1 bar, 100°C)([1], p.116)

$k = 0.670$W/m/K(water, 10 bar, 80°C)([1], p.124)

$k_g = 2.48 \times 10^{-2}$W/m/K (steam, 1 bar, 100°C)([1], p.124)

$k_g = 1.35 \times 10^{-1}$W/m/K (steam, 1 bar, 1000°C)([1], p.124)

$L = 2.257 \times 10^6$J/kg (water, 100°C)([4],p.174)

$\mu_g = 1.81 \times 10^{-5}$kg/m/s (air, 1 bar, 20°C†)

$\rho_d = 958.5$kg/m^3(water, 1 bar, 80°C)([1], p. 132)

$\rho_g = 0.590$kg/m^3(steam, 1 bar, 100°C)([1], p. 132)

$\rho_g = 0.202$kg/m^3(steam, 1 bar, 800°C)([1], p. 132)

$s_0 = 10^{-5}$m(drop radiust)

$T_s = 500 - 2000$°C(varies along the steel†)

$T_v = 100$°C(assuming pressure conditions close to atmospheric†)

$U = 5$m/s (typical drop speed†)

D = 1m (typical spray nozzle to steel distance†)

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