

# DETERMINING TEMPERATURE CONTROL OF WASH WATER IN A LAUNDRY ENVIRONMENT

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## Abstract

Water-temperature control during the fill phase of a new kind of washing machine being developed by Fisher and Paykel is considered. The machine and the fill method are described, and the factors which affect temperature control are explored.

A steady-state mass and energy balance of the system, and a linearised analysis of the differential equation governing the temperature of the water sprayed over the clothes are worked through. A linearised analysis of the dynamic situation is also presented, which includes the effects of various dynamic delays.

A numerical approach is coded in MATLAB, which includes a simple model of mass and heat flow to and from the clothes. Representative runs of this program appear to yield results satisfyingly similar to the experimental data provided by Fisher and Paykel, and suggest that the dynamic delays are unimportant, and that temperature control can be achieved to within  $\pm 2$  degC in most cases.

A cascade control process is detailed in which the input temperature is measured simultaneously with the sump temperature. Various “smart” control strategies are also enumerated.

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## 1. Introduction and Outline

### 1.1. Introduction

The question Fisher and Paykel were most interested in having answered at MISG2005 was *What is the best way to regulate water temperature in the new, low-water-using washing machine?*

The Fisher and Paykel representatives helped the team to understand the following.

- 1 It will be harder to regulate the temperature in the new machine since there is less water.
- 2 The disturbing effects are:
  - (a) Cold slugs of water from the hot tap;
  - (b) Wet, cold loads, or more generally, loads containing an unknown amount of water at an unknown temperature.
  - (c) The temperature of the hot water supply is variable or unusually low;
  - (d) Various machine components absorb heat;
  - (e) The lid is raised, allowing heat to escape from the machine;
  - (f) The user adds clothes during the fill;
  - (g) The flow rates of the taps are variable;
  - (h) The ambient temperature is high or low;
  - (i) Varying load masses;
  - (j) Varying absorbency rates of different loads due to different fabric types.

Another question of interest was *Where is the best place to put the temperature sensor and what is the benefit of 2 sensors?* (The preconception was that the cost of a second sensor will far exceed the benefit).

As demonstrated in the remainder of this paper, during the course of MISG2005 the effects (2a) to (2d) were shown to be important, the effects (2e) and (2f) were found to be important only in rather extreme situations, and the effects (2g) to (2j) were found to be much less important.

### 1.2. Outline

Water-temperature control during the fill phase of a new kind of washing machine being developed by Fisher and Paykel is considered. The

machine and the fill method are described (Sections 2 and 3), and the factors which affect temperature control are explored. The process variable is chosen to be the temperature of the water sprayed over the clothes.

The magnitude of all the disturbing effects itemised above can be quantified by using various strategies – both analytical and numerical. These strategies are discussed and some simple examples are worked through in order to guide future exploration.

The analytical approaches include a steady-state mass and energy balance of the system (Sections 4.1.1 and 4.1.2), and, in Section 4.1.3, a linearised analysis of the differential equation governing the temperature of the water sprayed over the clothes (equal to the temperature in the machine's "sump"). A simple model of mass and heat flow to and from the clothes is developed and solved in Section 4.2.

A numerical approach is coded in MATLAB (Section 4.3) which incorporates differential equations for the important factors found in Section 4.1 and the model of Section 4.2. Representative runs of this program appear to yield results satisfyingly similar to the experimental data provided by Fisher and Paykel.

Proportional and proportional-integral controllers are discussed in Section 5.1. An analytic linearised analysis of the dynamic situation is also presented, which includes the effect of delays due to a controller being constrained to operate only at fixed points in time, the finite-time response of the temperature sensor, the time taken for water transport between the input and the sump, and the finite time taken for water to mix in the sump (Section 5.2).

A proportional controller has been included in the MATLAB program in Section 5.3. Two example runs appear to illustrate that the dynamic delays itemised above are unimportant. They also suggest that temperature control may be achieved to within  $\pm 2$  degC unless the clothes contain an inordinate amount of cold water or the input temperatures (including the cold slug) are such that the aim temperature cannot be achieved irrespective of the control strategy.

To counter the effect of time-varying hot and cold input temperatures a cascade control process is detailed in Section 6.1 in which the input temperature is measured simultaneously with the sump temperature. Various "smart" control strategies are also enumerated in Section 6.2: estimating values at the beginning of the fill; installing a memory chip in the machine; initialisation of the machine on installation; placing the temperature sensor on a sill; and measuring input and sump temperatures with one sensor.

## 2. System description.

The system is shown in Figure 1, and is described below.

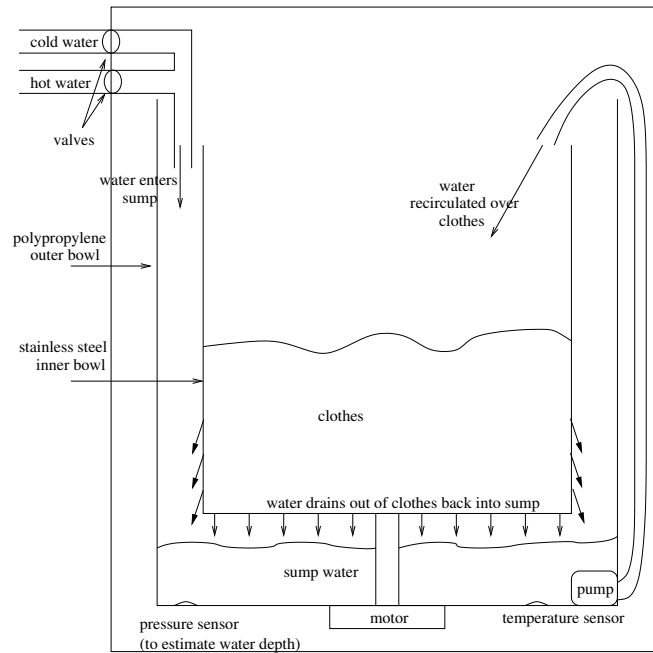


Figure 1. Diagram of washing machine geometry.

The washing machine sits in an environment that is between 0 and 45 degC, with relative humidity between 20 and 95%. The machine has hot and cold water inputs. The hot temperature ranges between 0 and 75 degC, while the cold is between 0 and 45 degC. There are solenoid valves on each supply which switch them on and off; when on, typical flowrates are: hot 10 l/min, and cold, 17 l/min. To isolate the feedrates from supply pressure changes, throttles are used, these are effective at maintaining constant flowrates for any supply pressure above 1 bar and most domestic supplies are above this threshold. The hot and cold inputs pass through a mixing chamber before being discharged into the machine.

The outer surface of the washing machine is cuboid and is constructed from painted sheet metal. Inside this, and insulated from it by a layer of air, sits a cylindrical polypropylene bowl of approximate diameter 550mm, which is 2mm thick and has mass 3.5 kg. The bottom of the bowl is called the sump and it contains water while the machine is wash-

ing. The under-surface of the sump is ribbed for strength. The motor is attached to the bottom of the sump and its shaft passes vertically up through the sump to the wash bowl. The wash bowl is also cylindrical with rough dimensions 500mm. It is coaxial with the outer polypropylene bowl/sump. Its curved surface is constructed of stainless steel of mass 1.5 kg and its circular bottom is polypropylene of mass around 1 kg. All its surfaces are perforated to allow water to move from it to the outer polypropylene bowl/sump. A polypropylene agitator of mass 500 g sits coaxially within the wash bowl.

Clothes are placed within the stainless steel wash bowl. The mass of the dry clothes is up to 8 kg, but occasionally the clothes may also contain some water which will typically be cold. An extreme case would be towels, which weigh 8kg when dry, which are saturated with 40kg of water. The machine is then filled with up to 35 kg of water, as described in Section 3.

A variable-speed pump is attached to the sump, which can pump between 14 l/min and 34 l/min. The pump's output is either directed towards the drain (to empty the machine), or back over the clothes, which is called *recirculating*.

A pressure sensor is attached near the bottom of the sump which allows measurement of the water height in the sump.

In current washing machines, the temperature of the water in the input mixing chamber is measured; however, Fisher and Paykel suggest that the sump is a more appropriate position for the temperature sensor in the prototype machine considered here.

### **3. Fill cycle description.**

The fill cycle begins with clothes being placed into the machine's wash-bowl. The machine then begins to fill with a certain mixture of hot and cold water, the proportions of which are the topic of this report. Currently, Fisher and Paykel use a 30 second periodic cycle time and determine the dwell time of the hot and cold water (0 to 30 seconds) within this period. When this period has finished, the dwell times may be altered for the next period. This cycle time can be reduced if it is found that the response time of the system to disturbances (see sections below) is adversely affected. The water is mixed in the mixing chamber and is discharged down the inside of the outer polypropylene bowl and into the sump.

When the sump is full to a pre-set level (currently around 1.8 litres), the pump starts removing water from the sump, pumping it through the recirculation line and spraying it over the top of the clothes. The

wash-bowl is rotating at 20 r.p.m. so that the spray is directed evenly over the top surface of the clothes. The pump initially removes water at its minimum speed so that the water level in the sump continues to rise. The pump speed is then increased linearly with the sump's water level (but this does not mean that the rate of pumping increases linearly since there is an exponential-type relationship between pump speed and rate of pumping), reaching its maximum when the sump contains around 3.5 l of water. For standard loads this occurs at around 2.5 mins into the fill cycle.

Some of the water that is sprayed over the clothes is absorbed, while the rest finds its way through the perforated wash bowl and back into the sump.

When the sump water level reaches 5 l, which takes about 3 minutes for a standard load of clothes, the input water valves are cut off. The pump is still removing water from the sump at its maximum rate, and the clothes are typically still absorbing water, so the water level in the sump falls. When the water level falls to 4 l, the input valves are toggled on, and remain on until the sump water level reaches 5 l.

This on-off cycle repeats until the sump water level remains more-or-less constant (the input valves are off), at which time the recirculation rate is equal to the rate of drip-back from the clothes, and the clothes are deemed saturated. The fill cycle is then complete. The whole process takes around 10 minutes for a standard load of clothes.

The detergent will be placed somewhere within the system, possibly in the recirculation line.

Summarising:

- Hot/cold inputs are *on* (with some determined ratio) until the sump level is 5 l. Then they are *off* if the sump level is greater than 5 l, but turned *on* if the sump level ever falls below 4 l.
- The pump is only turned *on* (to the recirculation line) if the water level is above 1.8 l. Its speed varies linearly with sump level, from a minimum value at 1.8 l to a maximum value at 3.5 l and above.

The sump water level as a function of time depends on the absorbency of the clothes. Two examples are illustrated below.

#### **4. A suite of simple models without any control strategy.**

During MISG2005 a number of different models of the fill cycle and the washing machine were developed, as team members considered different approaches to and aspects of the problem.

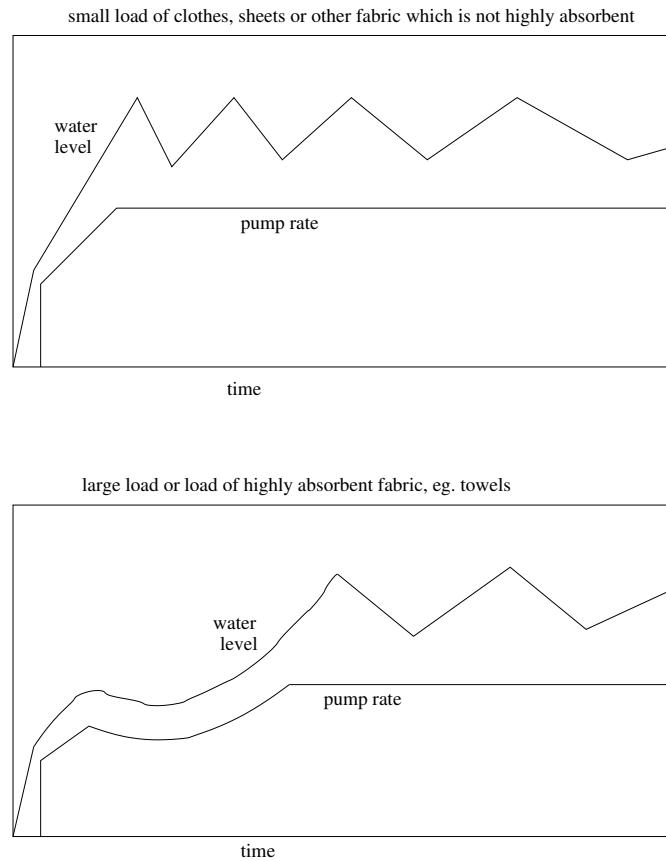


Figure 2. Water level and pump rate for 'typical' fills.

In Section 4.1 steady state mass and energy balances for the system at the end of the fill are considered. This analysis was used to determine which of the components of the machine absorb significant amounts of heat during the fill in order to decide which components to include in subsequent more detailed models. The model can also be used, as illustrated in Section 4.1.3, to determine the limits on disturbing effects that can be tolerated.

The model of Section 4.2 provides a detailed treatment of the absorption of heat and water into the clothes, but does not include any consideration of the dynamics in the water inputs, the sump or the temperature sensor.

These delays are included in later Sections.

These models are simple, but the results can be obtained analytically. Section 4.3 discusses a MATLAB model which is a numerical implementation of a set of six differential equations describing the important factors found in Section 4.1 and the model of Section 4.2. The MATLAB model has been checked against experimental data and satisfactory agreement has been obtained.

#### 4.1. Overall, steady state mass and energy balances.

During the fill, all components are heated by the hot water supply. The heat required by each component will be determined and compared. This shows which components are most significant in terms of the heat load they represent. A general analysis is presented followed by an illustrative example. This model can then be used to determine where the limits are for the system, i.e. what is the largest heat load that can be tolerated before the target temperature can not be reached.

##### 4.1.1 A general steady-state analysis.

Let us assume that

- the losses to the environment, including the sheet-metal box, are negligible.
- the clothes and the sump are in thermodynamic equilibrium at the end of the fill: that is, the temperature of the clothes, the water in the clothes, the water in the sump, the bowls and the ambient air are all equal (and denoted  $T$ ).

Table 1 describes the components of the system and how much energy is gained by each during the fill. The sum of these terms is zero by conservation of energy.

Energy conservation thus reads:

$$\begin{aligned}
 0 = & M_c c_c (T - T_c(0)) + M_{w.in.c} c_w (T - T_c(0)) + M_b c_b (T - T_b(0)) \\
 & + M_a c_a (T - T_a(0)) + M_{w.in.a}^i c_s (T - T_{w.in.a}(0)) \\
 & + \Delta c_w (100 - T_{w.in.a}(0)) \\
 & + \gamma \Delta + \Delta c_s (T - 100) + M_{hot} c_w (T - T_{hot}) \\
 & + M_{cold} c_w (T - T_{cold}) + M_{slug} c_w (T - T_{slug}) .
 \end{aligned}$$

The total mass of water in the machine at the end of filling is the mass of water in the sump plus the water in the saturated clothes (denoted



Component	$T_0$	$M$	$c$	Energy change
Clothes	$T_c(0)$	$M_c$	$c_c$	$M_c c_c (T - T_c(0))$
Water in clothes	$T_c(0)$	$M_{w.in.c}$	$c_w$	$M_{w.in.c} c_w (T - T_c(0))$
Bowls	$T_b(0)$	$M_b$	$c_b$	$M_b c_b (T - T_b(0))$
Dry air in machine	$T_a(0)$	$M_a$	$c_a$	$M_a c_a (T - T_a(0))$
Water in air	$T_{w.in.a}(0)$	$M_{w.in.a}^i \rightarrow M_{w.in.a}^i + \Delta$	$c_s$	$M_{w.in.a}^i c_s (T - T_{w.in.a}(0)) + \Delta c_w (100 - T_{w.in.a}(0)) + \gamma \Delta + \Delta c_s (T - 100)$
Hot water	$T_{hot}$	$M_{hot}$	$c_w$	$M_{hot} c_w (T - T_{hot})$
Cold water	$T_{cold}$	$M_{cold}$	$c_w$	$M_{cold} c_w (T - T_{cold})$
Cold slug	$T_{slug}$	$M_{slug}$	$c_w$	$M_{slug} c_w (T - T_{slug})$

Table 1. Basic components of the system, their initial temperature  $T_0$ , mass  $M$ , specific heat capacity  $c$ , and their energy changes. The latent heat of vapourisation of water is denoted by  $\gamma$ .

$M_{w.in.s}$  and  $M_{w.in.c}^f$ ) plus the water in the air. Thus the mass conservation reads:

$$M_{w.in.c}(0) + M_{hot} + M_{cold} + M_{slug} + M_{w.in.a}(0) = M_{w.in.s} + M_{w.in.c}^f + M_{w.in.a}^f .$$

**4.1.2 An example and comments on relative magnitudes of different heat sinks.**

To get a feeling for the relative magnitudes of each term, a realistic example can be worked through. Suppose 8kg of dry fabric at 20 degC containing 2 kg of water at 20 degC is placed in the wash bowl. Suppose the ambient temperature is 20 degC and the relative humidity is 20%. Suppose the hot water input has a 5 kg cold slug of temperature 15 degC, the hot-water temperature is 60 degC and the cold water temperature is 15 degC. The aim temperature is  $T = 45$  degC, and the final water mass is 35 kg. Suppose finally that the relative humidity inside the machine at steady-state is 100%. The result is shown in table 2.

This analysis shows the following points.

- 1 The effects of the stainless-steel bowl and the dry air are negligible. The sensible heat changes for the water evaporated and the water

Component	Initial degC	Mass kg	c kJ kg <sup>-1</sup> K <sup>-1</sup>	E kJ	%
clothes	20	8	1.5	300	18
water in clothes	20	2	4.2	210	13
bowls — polypro	20	5	2	250	15
— stainless	20	1.5	0.5	19	1
air — dry	20	0.5	1	13	1
— water	20	1.5 → 32.5 g	4.2	79	5
hot water	60	26.6	4.2	-1675	—
cold water	15	1.4	4.2	174	10
slug	15	5	4.2	630	38

*Table 2.* The components, their initial temperature and mass, their specific heat capacity, and the energy gained  $E$ , for this example. The final column shows the energy gained by each component as a percentage of the energy lost by the hot water. The latent heat of vaporisation is  $\gamma = 2340$  kJ/kg.

initially in the air are likewise small. The energy taken up by the air can be approximated as the latent heat  $\gamma\Delta$  only.

- 2 The effect of the polypropylene could be large, however, because it is fixed (it cannot be altered by the user), it could be accounted for by Fisher and Paykel by performing lab trials without any clothes, as detailed below. Polypropylene's diffusivity is  $20 \text{ s mm}^{-2}$ , so the time taken for heat to travel 2 mm (the thickness of the polypropylene) is roughly 80 seconds. In the 10 minutes of a typical fill, it can therefore be expected that the majority of the sump attains thermal equilibrium with the water, but that the upper regions of the outer bowl are at a lower temperature. This could be taken into account by using a "reduced mass" approximation, which would reduce the energy absorbed in the above table, of course.
- 3 The effect of opening the lid for a long period of time could be significant. For example if the lid were open for a period of time sufficient to allow 10 air changes this is  $\sim 5$ kg of air and so, ignoring any dynamic effects, the energy lost would be  $\sim 920$ kJ (chiefly from the latent heat of vaporisation). In the above example this heat loss is so high that the aim temperature could not actually be achieved. This calculation is taking the model too far, however, since it is unlikely that each air change will have to take the air from 20 degC and 20% RH, to 45 degC and 100% RH (i.e. dynamical effects are important).

- 4 The mass of water evaporated is negligible and could be removed from the mass balance for simplicity.
- 5 Finally, the analysis also demonstrates that the cold slug and the amount of cold water in the clothes can have a potentially large effect.

#### 4.1.3 The disturbance envelopes.

To quantify the limit a particular variable can take, values can be assigned for all other variables and the value of that variable determined from the equations. Since, in most limiting cases the wash water will be cooler than desired, it can be assumed that the control system will not admit any water from the cold tap so  $M_{\text{cold}} = 0$ .

An illustrative example is now worked through.

For  $T_c(0) = 20$  degC,  $M_c = 8$  kg,  $c_c = 1500$  J/kg/K,  $c_w = 4200$  J/kg/K,  $T_{\text{hot}} = 55$  degC,  $T_{\text{slug}} = 15$  degC,  $M_{\text{slug}} = 2$  kg,  $M_{\text{w.in.s}} = 7$  kg and  $M_{\text{w.in.c}}^f = 16$  kg we can determine how much cold water can be present in the clothes before a final temperature of 38 degC can not be attained.

There are two unknowns ( $M_{\text{w.in.c}}(0)$  and  $M_{\text{hot}}$ ) and two equations:

$$\begin{aligned}
 0 &= M_c c_c (T_c(0) - T) + M_{\text{w.in.c}}(0) c_w (T_c(0) - T) + M_{\text{hot}} c_w (T_{\text{hot}} - T) \\
 &\quad + M_{\text{slug}} c_w (T_{\text{slug}} - T) , \\
 0 &= M_{\text{w.in.c}}(0) + M_{\text{hot}} + M_{\text{slug}} - M_{\text{w.in.s}} - M_{\text{w.in.c}}^f .
 \end{aligned}$$

These are solved for  $M_{\text{w.in.c}}(0) = 4.5$  kg with  $M_{\text{hot}} = 14.5$  kg. This gives a measure of how much cold water can be tolerated with the clothes whilst still attaining 38 degC at the end.

Similarly, operating envelopes can also be developed for pairs of variables, for example, the limiting line in  $(T_{\text{hot}}, M_{\text{w.in.c}}(0))$  space.

## 4.2. Model of water and heat absorption into the clothes.

In addition to water supply disturbances (cold slugs etc), the clothes are a significant problem in controlling the temperature. This is for two reasons: firstly, they might contain a large mass of cold water so that the aim temperature might not be achieved even if the machine fills purely with hot water; and, secondly, that there might be significant lag due to the slow percolation of water through the clothes so that dynamic control might not be possible. This means that the magnitude of the disturbing effect of the cold drip-down from the clothes on the sump

temperature may not be observed by the controller until the fill cycle is well advanced.

It is therefore useful to build a model of water and heat absorption into the clothes. This model is based on the following experimental observations obtained during the course of MISG2005.

- 1 As water is sprayed onto the top surface of the clothes some is absorbed and some finds its way through the perforated drum back into the sump. The pure “diffusivity” of water through the clothes appears to be very small, so that on the time scale of the fill (less than 10 minutes) a damp (but not dripping) article of clothing will not significantly wet its neighbours. Clothes thus become saturated in “layers” due to the water recirculation. The “layers” need not be vertically stratified, although this is assumed in the simple model below; rather, they may be like layers in an onion, or have more complicated geometry. In the model developed here, the top layer rapidly becomes saturated and starts dripping onto the next layer, which in turn becomes saturated and drips onto the third layer, and so on. As the top layers drip onto layers below, they also drip from their sides through the perforated drum and back into the sump. The clothes are thus either saturated (top layers) or “dry” (bottom layers), with very few regions being partially saturated.
- 2 The rate of absorption of water into dry clothes varies with the fabric from synthetic/cotton sheets taking around 2 l/min, to a “standard load” absorbing around 4 l/min, to towels which absorb around 9 l/min. These absorbency rates are well below the minimum pump rate of around 15 l/min, so the absorbency is not retarded by the pump rate. (However the model can be modified easily to allow for low pump rates.)
- 3 As the dry clothes become saturated in the way described in (1), the absorbency rate of the *whole* mass of clothes appears to decrease roughly linearly with time. After 12 minutes of pumping the synthetic/cotton sheets (8kg dry weight) were completely saturated. After 1 minute of pumping, the “standard load” (8kg dry weight) was only absorbing roughly 1 l/min. After 5 minutes the towels (8 kg dry weight) were completely saturated.

The model described here does **not** model the following experimental observation:

- 4 The surfaces of fabrics appear to have some initial resistance to wetting, but once they have absorbed some water, their absorbency increases.

The model is constructed as follows. A mass of clothes of height  $h$ , initially containing  $M_{w.in.c}(0)$  kg of water at temperature  $T_{sat.c}(0)$ , is placed into the washing machine. The maximum absorbency rate of the clothes is  $\dot{M}_{max}$  (kg/min). They are sprayed with water from the recirculation pump at rate  $\dot{M}_{re}$  at temperature  $T_{sump}$ . It is assumed that  $\dot{M}_{re} \geq \dot{M}_{max}$ , so that in time  $dt$ , an amount  $(\dot{M}_{re} - \dot{M}_{max})dt$  splashes from the top surface of the clothes, through the perforations and into the polypropylene bowl. This splash has temperature  $T_{sump}$ . For simplicity  $T_{sump}$  is taken to be constant, although as in Section 4.3, dynamic effects can be included in a full dynamical system approach including the sump water, the polypropylene, etc. Essentially then, we are dousing the clothes with water at temperature  $T_{sump}$  and rate  $\dot{M}_{max}$ . This is shown in the figure 3.

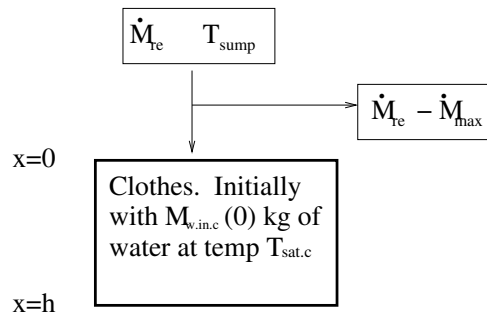


Figure 3. Basic setup in the absorption model.

For ease of exposition, the mass of the bone dry clothes has not been included in this model ( $M_{w.in.c}$  is the *mass of water in the clothes*, **not** the mass of the clothes). This will be of no consequence in the absorbency part of the model, but it will mean that if the model is applied naively to the case of bone-dry clothes ( $M_{w.in.c}(0) = 0$ ), the drip temperature always equals  $T_{sump}$ . In Section 4.3, the model is slightly extended by including the mass of the dry clothes as well as some initial water.

There are two parts to the model – the mass and heat flows.

#### 4.2.1 The mass flow.

Consider a small slice of thickness  $dx$  which is *saturated*. If  $\dot{M}dt$  kg

of water is incident on its top surface then the model assumes that  $r \cdot dx \cdot \dot{M} dt$  kg of water flows to the sump (i.e. gets added to the “drip”), and the rest,  $(1 - r \cdot dx) \dot{M} dt$  flows down to the next layer. This is the key assumption in the model, and  $r$  is the key unknown parameter. Soon we will give  $r$  another physical interpretation.

At time  $t$  (after pumping begins), the material between  $x = 0$  and  $x = x(t)$  will be saturated, while the material between  $x = x(t)$  and  $x = h$  will be still in its initial state. Denote by  $M_{\text{sat}}$  the mass of water in the completely saturated clothes. Then at time  $t$ , the mass of water in the clothes will be

$$M_{\text{sat}} \frac{x(t)}{h} + M_{\text{w.in.c}}(0) \frac{h - x(t)}{h} .$$

In the time interval  $dt$  pump  $\dot{M}_{\text{max}} dt$  kg of water on the top layer. Then the model’s key assumption implies that the amount to the “drip” is  $r \cdot x \cdot \dot{M}_{\text{max}} dt$ , and the amount to the unsaturated layer is  $(1 - r \cdot x) \dot{M}_{\text{max}} dt$ . This is depicted in figure 4.

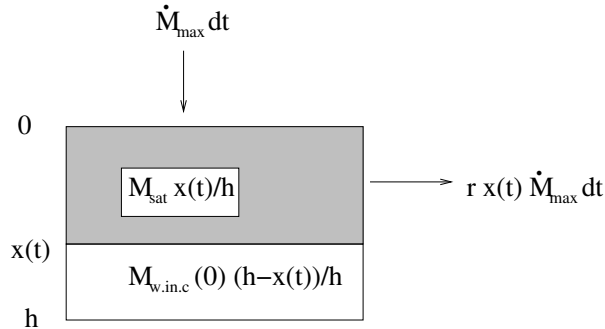


Figure 4. Depiction of the fundamental assumption of this model. Assumed mass flow rates and situation at time  $t$  in the absorption model.

The resulting differential equation can be written in various ways. Expressing in terms of the total mass of water in the clothes,

$$M_{\text{w.in.c}}(t) = \frac{(M_{\text{sat}} - M_{\text{w.in.c}}(0))x(t)}{h} + M_{\text{w.in.c}}(0) ,$$

gives

$$\dot{M}_{\text{w.in.c}} = \dot{M}_{\text{max}}(1 - xr) = \dot{M}_{\text{max}} \left( 1 - \frac{M_{\text{w.in.c}} - M_{\text{w.in.c}}(0)}{M_{\text{sat}} - M_{\text{w.in.c}}(0)} hr \right) .$$

This has solution

$$M_{w.in.c}(t) = M_{w.in.c}(0) + \frac{M_{sat} - M_{w.in.c}(0)}{hr} \left[ 1 - \exp\left(\frac{-\dot{M}_{max}hrt}{M_{sat} - M_{w.in.c}(0)}\right) \right].$$

This model has the property that if  $hr < 1$  then the clothes become completely saturated in time

$$t_{sat} = \frac{M_{w.in.c}(0) - M_{sat}}{\dot{M}_{max}hr} \log(1 - hr),$$

(after which time  $M_{w.in.c}(t) = M_{sat}$ ), but if  $hr > 1$  then the clothes never completely saturate — they reach  $M_{w.in.c}(\infty) = M_{w.in.c}(0) + (M_{sat} - M_{w.in.c}(0))/hr$  — since the drip rate is too large for the water to percolate past  $x(t) = 1/r$ .

This yields a useful interpretation of the parameter  $r$ . If  $h$  is very large (a very high stack of clothes within the washing machine), then the percolation described by this model saturates clothes only to the depth  $1/r$ .

Let us examine two instructive cases.

(a) Towels. The data described above suggests that  $\dot{M}_{max} = 9$  kg/min. Saturated towels weigh roughly 5 times more than bone-dry towels, so for a 8kg bone-dry load,  $M_{sat} = 40$  kg. Load the washing machine to height  $h = 0.4$  m with damp towels with  $M_{w.in.c}(0) = 8$  kg. The only parameter that must be guessed at is  $r$ . Let us choose  $r = 1$  m<sup>-1</sup>, which means that a stack of towels of height greater than  $1/r = 1$  m would not get thoroughly saturated, irrespective of how long we sprayed them (because the loss through the perforated drum will equal  $\dot{M}_{max}$  eventually — recall we're ignoring plain diffusion). These parameters give  $t_{sat} = 4.5$  minutes as the time for complete saturation, which agrees rather well with the experimental observations. (Note that choosing  $r = 0.5$  m<sup>-1</sup> gives  $t_{sat} = 4$  mins, while  $r = 2$  m<sup>-1</sup> (a rather extreme value) gives  $t_{sat} = 7$  minutes, neither of which are too bad. Note also that with  $M_{w.in.c}(0) = 0$  we get  $t_{sat} = 5.6$  minutes.) The total mass of water in the clothes is shown in the figure 5.

Note that the increase is roughly linear with time. This pattern is repeated for many other choices of parameters, except for rather extreme cases (where  $rh \approx 1$ ) in which the exponential form is visible. The other important quantity is the mass flow rate from the clothes as “drip back”. This is given by  $r \cdot x \cdot \dot{M}_{max}$ , and is plotted in figure 5.

Again a roughly linear relation is obtained. Once  $t > t_{sat}$  the clothes start dripping at rate  $\dot{M}_{max}$  (=9 kg/min in this case) since they are not only dripping from their sides, but are dripping from their bottom too. At that point the machine is deemed to have completed the “fill”.

(b) Synthetic/cotton sheets. The data described above suggests that  $\dot{M}_{\max} = 21/\text{min}$ . Let us suppose that saturated sheets weigh roughly 2 times more than bone-dry sheets, so for a 8kg bone-dry load,  $M_{\text{sat}} = 16\text{kg}$ . Load the washing machine to height  $h = 0.4\text{m}$  with slightly damp sheets with  $M_{\text{w.in.c}}(0) = 1\text{kg}$ . Choose  $r = 1.4\text{m}^{-1}$ , which means that a stack of sheets of height greater than  $1/r = 0.7\text{m}$  would not get thoroughly saturated, irrespective of how long we sprayed them. Then  $t_{\text{sat}} = 11\text{mins}$ , again in good agreement with experiment. The graphs are shown in figure 6.

Again, roughly linear relationships are obtained.

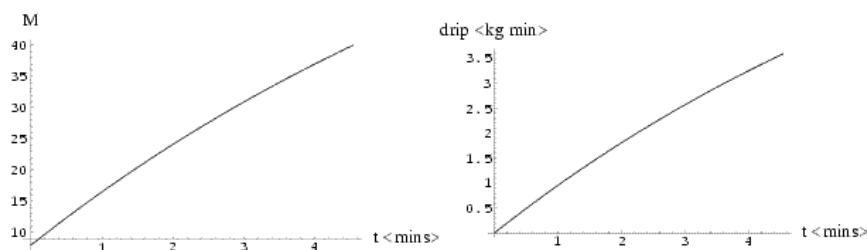


Figure 5. Left: Mass (kg) of water in the towels as a function of time (min). Right: Drip-back rate (kg/min) (excluding the contribution of “splash”) for towels.

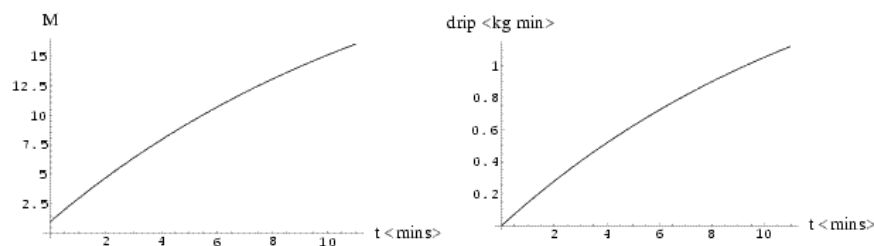


Figure 6. Left: Mass of water in the sheets as a function of time. Right: Drip-back rate (excluding the contribution of “splash”) for sheets.

#### 4.2.2 The heat flow.

Denote the *average* temperature in the *saturated* portion of the clothes



by  $T_{\text{sat.c}}(t)$ . By the key assumption (1), *it is also the temperature of the drip coming out of the clothes* (disregarding the “splash” portion of the drip). The temperature of the water in the unsaturated part of the clothes is  $T_{\text{sat.c}}(0)$ .

In time  $dt$ , the heat energy added to the clothes is  $\dot{M}_{\text{max}}dtT_{\text{sump}}$  (multiplied by the heat capacity of the water which is common to all terms below). The amount removed by the drip is  $\dot{M}_{\text{max}}dt.x.r.T_{\text{sat.c}}(t)$ . The energy balance equation is

$$\begin{aligned} & \dot{M}_{\text{max}}dtT_{\text{sump}} + M_{\text{sat}}x(t)T_{\text{sat.c}}(t)/h + M_{\text{w.in.c}}(0)(h - x(t))T_{\text{sat.c}}(0)/h \\ & = M_{\text{sat}}x(t + dt)T_{\text{sat.c}}(t + dt)/h + M_{\text{w.in.c}}(0)(h - x(t + dt))T_{\text{sat.c}}(0)/h \\ & \quad + \dot{M}_{\text{max}}dt.x(t + dt).r.T_{\text{sat.c}}(t + dt) . \end{aligned}$$

The LHS is the energy before mixing the recirculated water into the water-clothes system, consisting of the energy of the small amount of recirculated water plus the energy of the partially saturated clothes, while the RHS is the energy of the clothes after plus the energy in the small amount of drip water. Expanding to first order in  $dt$  gives the conventional differential expression

$$\dot{M}_{\text{max}}T_{\text{sump}} = \frac{d}{dt} \left( \frac{M_{\text{sat}}xT_{\text{sat.c}} + M_{\text{w.in.c}}(0)(h - x)T_{\text{sat.c}}(0)}{h} \right) + \dot{M}_{\text{max}}xrT_{\text{sat.c}} .$$

This may be solved to give a rather lengthy expression for  $T_{\text{sat.c}}(t)$ .

An important observation is that the initial saturated layer, and consequently the initial drip, is at temperature

$$T_{\text{sat.c}}(0) = ((M_{\text{sat}} - M_{\text{w.in.c}}(0))T_{\text{sump}} + M_{\text{w.in.c}}(0)T_{\text{sat.c}}(0)) / M_{\text{sat}} .$$

As expected, if  $M_{\text{w.in.c}}(0) = 0$ ,  $T_{\text{sat.c}}(0) = T_{\text{sump}}$  (recall that we’re essentially setting the heat capacity of the clothes to zero). Also, if  $M_{\text{w.in.c}}(0) = M_{\text{sat}}$  then the initial drip temperature is  $T_{\text{sat.c}}(0)$ , which is true by definition. (It is clear that the drip comes out at this temperature because the addition of an initial infinitesimal amount of recirculated water is not going to have any effect on the temperature of a large thermal mass at  $T_{\text{sat.c}}(0)$ .)

When the clothes are completely saturated at time  $t_{\text{sat}}$ , the differential equation simplifies to

$$\dot{M}_{\text{max}}(T_{\text{sump}} - T_{\text{sat.c}}) = M_{\text{sat}}\dot{T}_{\text{sat.c}}$$

giving an exponential approach of  $T_{\text{sat.c}}$  to  $T_{\text{sump}}$ .

Again let us explore the two examples.

- 1 Towels. Choose the parameters as above in addition to  $T_{\text{sat.c}}(0) = 20 \text{ degC}$  and  $T_{\text{sump}} = 50 \text{ degC}$ . Then the *average* temperature of

the *saturated* part of the clothes, and consequently of the drip is shown in figure 7.

- 2 Sheets. Choose the parameters as above in addition to  $T_{\text{sat.c}}(0) = 20, \text{degC}$  and  $T_{\text{sump}} = 50 \text{degC}$ . The temperature is shown in figure 7.

The important point here is that  $T_{\text{sat.c}}(t)$  does not vary significantly with time.

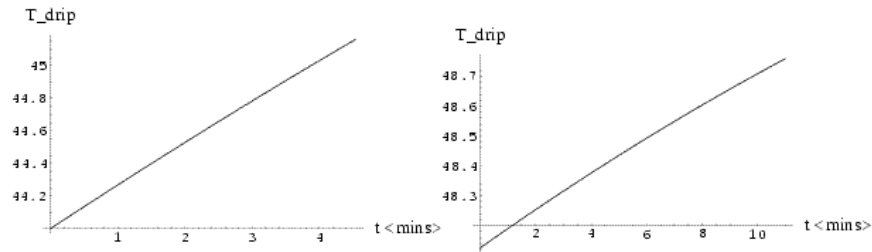


Figure 7. Temperature of drip (excluding any “splash” contribution) as a function of time. Left: Towels. Right: Sheets.

### 4.2.3 Conclusions from absorption model.

There are a number of important conclusions to be drawn from the study of this model. Assume that the parameters (in particular “ $r$ ”) do not attain extreme unphysical values. Then

- 1 The time taken to saturate the clothes is

$$t_{\text{sat}} = \frac{M_{\text{w.in.c}}(0) - M_{\text{sat}}}{M_{\text{max}}hr} \log(1 - h.r)$$

- 2 The mass of water in clothes increases roughly linearly with time from  $M_{\text{w.in.c}}(0)$  at time  $t = 0$  to  $M_{\text{sat}}$  at time  $t = t_{\text{sat}}$ . This is satisfying since it agrees with experimental observations. At time  $t = 0$  the absorbency rate is defined by  $\dot{M}_{\text{max}}$ .
- 3 The drip mass rate increases roughly linearly with time from 0 to  $\dot{M}_{\text{max}}hr < \dot{M}_{\text{max}}$  at  $t_{\text{sat}}$ . After  $t_{\text{sat}}$ , the drip rate will suddenly rise to  $\dot{M}_{\text{max}}$ , due to dripping from the bottom of the clothes as well as from their sides, but this does not concern us as the “fill” will be complete.

- 4 The temperature of the drip and of the saturated portion of the clothes stays more-or-less constant during the fill at the value

$$T_{\text{sat.c}}(0) = \frac{(M_{\text{sat}} - M_{\text{w.in.c}}(0))T_{\text{sump}} + M_{\text{w.in.c}}(0)T_{\text{sat.c}}(0)}{M_{\text{sat}}} .$$

After the fill the temperatures converges exponentially to  $T_{\text{sump}}$ .

The pump can recirculate water at any rate greater than  $\dot{M}_{\text{max}}$ , but any excess is removed virtually instantaneously as “splash” back into the sump with temperature  $T_{\text{sump}}$ , and so would be unobservable by a temperature sensor in the sump.

The more complete model presented below includes the sump water as well as the hot/cold inputs, and thus  $T_{\text{sump}}$  varies with time. What is clear from this model is that in the initial stage of the fill, reasonably dry clothes have very little effect since the drip-back rate to the sump is low. During this stage, then, the controller will be battling the effect of the polypropylene bowl. The key assumption of “layers” of clothes becoming progressively saturated means that the drip temperature is simply the average temperature of the saturated portion of the clothes which remains quite constant. During the final stages of the fill the clothes have more of an effect because of the high drip rate, and this may make temperature control hard to achieve; although the main effect of the clothes actually occurs when they are completely saturated and the “fill” is complete.

### 4.3. A dynamic heat and mass transfer model in MATLAB.

The team proposed a series of first-order differential equations as a model of the washing machine. Many of these equations have been discussed above, but are re-written here for ease of reading. A MATLAB program has been written and produced the representative output which is given graphically, below. This program also contains a controller and has been given to Fisher and Paykel in order that they can carry out numerical experiments in conjunction with the analytical approaches presented in other sections. Although good agreement with experiment is apparently obtained, further work is required to determine if this model effectively captures all the mechanisms that need to be captured.

Given the aforementioned experimental observations and order-of-magnitude calculations, it appears appropriate to model the washing machine using the following components.

Component	Associated variables	Notation	Units
Hot/cold inputs	mass flow rate	$\dot{M}_{in}(t)$	kg/min
	temperature	$T_{in}(t)$	K
Polypropylene	mass	$M_{poly}$	kg
	average temperature	$T_{poly}(t)$	K
Clothes	mass of dry fabric	$M_c$	kg
	mass of water in clothes	$M_{w.in.c}(t)$	kg
	average temperature of saturated portion of clothes	$T_{sat.c}(t)$	K
Water in sump	mass	$M_{sump}(t)$	kg
	temperature	$T_{sump}(t)$	K
Recirculation	mass flow rate	$\dot{M}_{re}(t)$	kg/min
	temperature	$T_{sump}(t)$	K

The dry parts of the clothes are assumed to be in thermal equilibrium with the surrounding water at all times, and it is sufficient to consider just the average temperature of the saturated portion, rather than consider the spatial dependence.

Let us examine the mass and heat flow rates to and from each component separately.

#### 4.3.1 The hot/cold inputs.

$\dot{M}_{in}$  can be pre-set: for instance, Fisher and Paykel's current scheme is to set  $\dot{M}_{in}(t) = \text{constant}$  until  $M_{sump} = 5 \text{ kg}$  and then  $\dot{M}_{in}(t) = 0$  for  $M_{sump} > 5 \text{ kg}$  and  $\dot{M}_{in}(t) = \text{constant}$  when  $M_{sump}$  returns to  $4 \text{ kg}$ . Alternatively,  $\dot{M}_{in}$  and  $T_{in}$  could be dynamically controlled by a P/PI controller in order to investigate the performance of the controller.

#### 4.3.2 The polypropylene.

It has already been argued that heat transfer to the surrounding air within the sheet-metal box can be neglected. Moreover, because the polypropylene is only  $2 \text{ mm}$  thick and the diffusivity is  $20 \text{ s mm}^{-2}$ , given good thermal contact between the sump water and the polypropylene, it will attain a uniform distribution of heat within around a minute of contact. Therefore, it is appropriate to use a "lumped approximation" where the temperature in the polypropylene satisfies

$$c_{poly} M_{poly} \frac{dT_{poly}}{dt} = -h_{p.to.w} A_{poly} (T_{poly}(t) - T_{sump}(t)) . \quad (1)$$

The constants are

Notation	constant	suggested value
$c_{\text{poly}}$	heat capacity	$2 \text{ kJ kg}^{-1} \text{ K}^{-1}$
$M_{\text{poly}}$	mass	5 kg
$h_{\text{p.to.w}}$	heat transfer coefficient from water to poly	$1 \text{ kW m}^{-2} \text{ K}^{-1}$
$A_{\text{poly}}$	area of contact	$1 \text{ m}^2$

The heat transfer coefficient (or, more particularly  $h_{\text{p.to.w}}A_{\text{poly}}/M_{\text{poly}}c_{\text{poly}}$ ) could be estimated by performing an experiment with an empty machine. A suitable initial condition is  $T_{\text{poly}}(0) = T_{\text{sat.c}}(0)$ .

### 4.3.3 The clothes.

The model used here follows closely the model of Section 4.2. Introduce the following constants.

Notation	constant	suggested value
$\dot{M}_{\text{max}}$	maximum absorbency rate	9 kg/min (towels) to 2 kg/min (sheets)
$M_{\text{sat}}$	maximum mass of water that can be absorbed by clothes	5 times dry weight (towels) 2 times dry weight (sheets)
$c_{\text{w}}$	heat capacity of water	$4.2 \text{ kJ kg}^{-1} \text{ K}^{-1}$
$c_{\text{c}}$	heat capacity of dry clothes	$1.5 \text{ kJ kg}^{-1} \text{ K}^{-1}$
$h$	height of clothes in machine	0.4 m
$1/r$	maximum height before complete saturation is impossible via dripping	1 m (towels) 0.7 m (sheets)

Also introduce the convenient variable

$$x(t) = h \frac{M_{\text{w.in.c}}(t) - M_{\text{w.in.c}}(0)}{M_{\text{sat}} - M_{\text{w.in.c}}(0)}, \quad (2)$$

which measures the total “height” of the saturated layer.

The drip-back rate to the sump is  $(\dot{M}_{\text{re}} - \dot{M}_{\text{max}}) + \dot{M}_{\text{max}}rx$ , the term in parentheses being the “splash” contribution, while the final term is from dripping through the clothes. The differential equations of mass and energy conservation are

$$\frac{dM_{\text{w.in.c}}}{dt} = \dot{M}_{\text{max}} - \dot{M}_{\text{max}}rx(t), \quad (3)$$

$$\begin{aligned} & \frac{d}{dt} \left( (c_{\text{w}}M_{\text{sat}} + c_{\text{c}}M_{\text{c}}) \frac{x(t)}{h} T_{\text{sat.c}}(t) + (c_{\text{w}}M_{\text{w.in.c}}(0) + c_{\text{c}}M_{\text{c}}) \frac{h - x(t)}{h} T_{\text{sat.c}}(0) \right) \\ & = c_{\text{w}}\dot{M}_{\text{max}}T_{\text{sump}}(t) - c_{\text{w}}\dot{M}_{\text{max}}rx(t)T_{\text{sat.c}}(t). \end{aligned} \quad (4)$$

When the clothes are completely saturated ( $M_{\text{w.in.c}} = M_{\text{sat}}$ ) then the drip-back rate to the sump becomes  $\dot{M}_{\text{re}}$  and the differential equations

simplify to

$$\begin{aligned}\frac{dM_{w.in.c}}{dt} &= 0, \\ (c_w M_{sat} + c_c M_c) \frac{dT_{sat.c}}{dt} &= c_w \dot{M}_{max}(T_{sump}(t) - T_{sat.c}(t)).\end{aligned}$$

#### 4.3.4 The sump.

The mass and energy balance equations read

$$\begin{aligned}\frac{dM_{sump}}{dt} &= \dot{M}_{in}(t) - \dot{M}_{max} + \dot{M}_{max}rx(t), \quad (5) \\ c_w \frac{d}{dt}(M_{sump}(t)T_{sump}(t)) &= c_w \dot{M}_{in}(t)T_{in}(t) - h_{p.to.w}A_{poly}(T_{sump}(t) - T_{poly}(t)) \\ &\quad - c_w \dot{M}_{max}T_{sump}(t) + c_w \dot{M}_{max}rx(t)T_{sat.c}(t). \quad (6)\end{aligned}$$

After the clothes become saturated the equations simplify to

$$\begin{aligned}\frac{dM_{sump}}{dt} &= \dot{M}_{in}(t), \\ c_w \frac{d}{dt}(M_{sump}(t)T_{sump}(t)) &= c_w \dot{M}_{in}(t)T_{in}(t) - h_{p.to.w}A_{poly}(T_{sump}(t) - T_{poly}(t)) \\ &\quad - c_w \dot{M}_{max}T_{sump}(t) + c_w \dot{M}_{max}T_{sat.c}(t).\end{aligned}$$

#### 4.3.5 The recirculation line.

The pump moves water at the arbitrary rate  $\dot{M}_{re}(t)$ . Currently, Fisher and Paykel use a linear function:

$$\dot{M}_{re}(t) = \begin{cases} 0 & \text{if } M_{sump} < 1.8 \text{ kg} \\ 14 + \frac{34-14}{3.5-1.8}(M_{sump} - 1.8) & \text{if } 1.8 < M_{sump} < 3.5 \text{ kg} \\ 34 & \text{if } M_{sump} > 3.5 \text{ kg} \end{cases}$$

#### 4.3.6 Matlab model (without control).

The differential equations (1), (3), (4), (5) and (6) (with the definition of  $x$  in equation (2)) and a simplified delay equation comprise the “virtual machine” model. The delay equation is discussed in more detail in Section 5.3 — as no control strategy is used here, the delay is not important for these results.

Two simulation runs of the model, with no control strategy, were conducted to compare with experimental results from the two hot fills of the machine performed at MISC2005. In these experimental fills,

the machine was filled with hot water only and the slug of cold water from the hot tap had previously been cleared. Two different loads were used, a load of sheets and a load of towels, each of dry weight 8kg. The initial conditions used in the simulation runs were that all temperatures initially were room temperature (25 degrees C), the mass of water in the load was zero and the mass of water in the machine was 0.1kg. The water was assumed to be input at a constant flow rate of 15l/min (when the tap was on) and a constant temperature of 50 degC. The values used for absorbency, heat capacity, etc are as suggested above. The results are shown in figure 8.

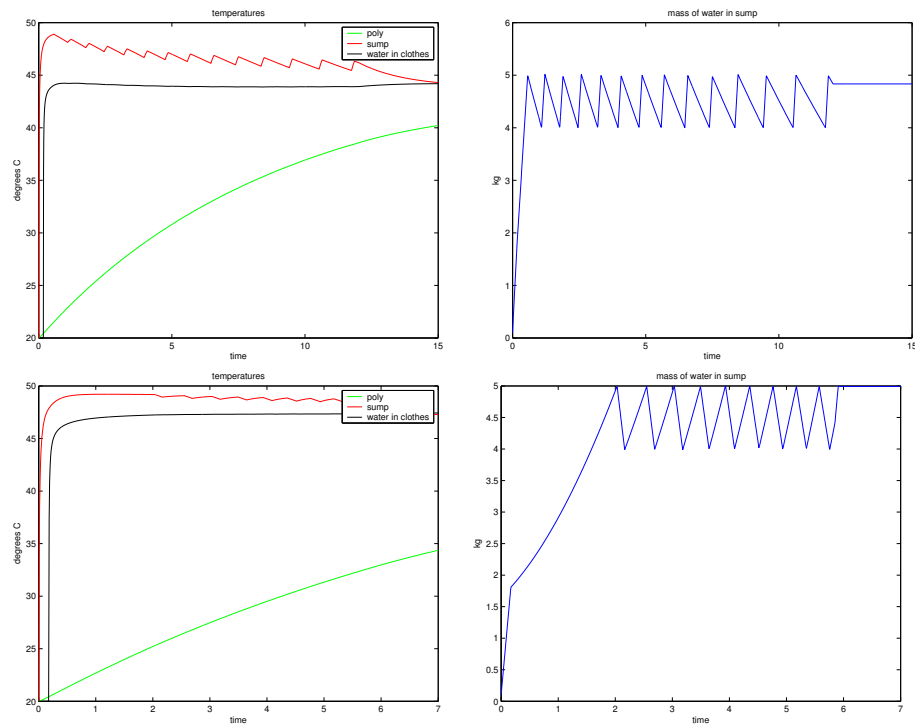


Figure 8. Output from MATLAB model. Upper graphs: towels. Lower graphs: sheets. Left graphs: Temperatures,  $T_{\text{sump}}$  in red (upper line),  $T_{\text{sat.c}}$  in black (middle line),  $T_{\text{poly}}$  in green (lower line). Right graphs: mass of water in the sump as a function of time (in minutes).

The time taken in the model for both sheets and towels to reach saturation is in good agreement with the times observed from the ex-

periments performed at MISG2005: around 12 minutes and 6 minutes respectively. The temperature graphs are not unrealistic considering the initial conditions used, and the mass of water in the clothes increases roughly linearly as predicted by the analytic solution of the absorbency model.

The graphs for the mass of water in the sump (RH graphs in figure 8) are the furthest from the observed values as there is a larger number of “sawteeth” and the towel graph doesn’t show the expected dip in the water level (see the sketches in Section 3). This is possibly because the model assumes that once the pump is on the water is always pumped over the clothes at a higher rate than can be absorbed, so that pump speed has no impact on the absorbency rate. However, if the input rate is less than the recirculation rate is less than the maximum absorbency rate, a flattening of the curve for the mass of water in the sump can be achieved (see figure 9). Still, the water mass does not drop significantly as the recirculation pump switches off. (Note that while the maximum absorbency rate is greater than the pump rate, the actual rate of water absorption into the clothes must always be less than or equal to the pump rate. )

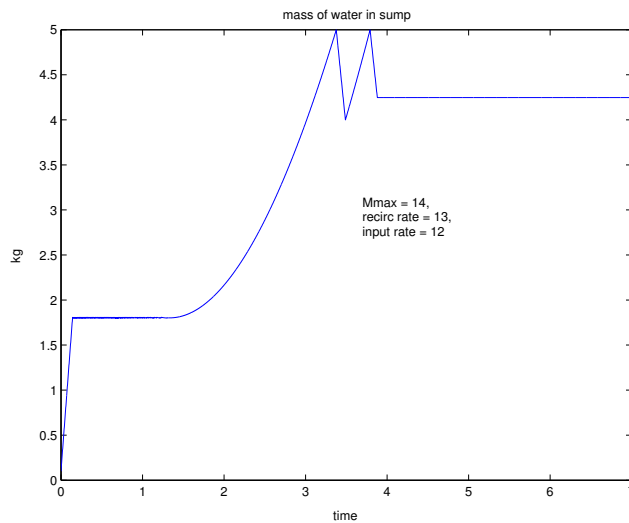


Figure 9. Mass of water in sump when hot/cold input rate < pump rate < maximum absorbency rate.

To see the “dip” in the water mass that was observed experimentally, changes would need to be made. Either the pump rate needs to be



directly included and the absorbency related to the pump rate, or the absorbency model needs to be changed. The current absorbency model has maximum absorbency at the beginning of the fill cycle and the total absorbency of the mass of clothes decreases as the clothes become wetter (due to water escaping through the perforated drum), but, as mentioned in Section 4.2, clothes appear to have some initial resistance to wetting, meaning their raw absorbency can change with time. It is hoped that this effect is not large.

## 5. Simple control and models with delays

### 5.1. Simple feedback.

In control theory, the process variable(s) is the thing that is being regulated. In this case it is the sump temperature. This is an appropriate choice for the process variable since the aim of the regulation is to ensure that the clothes are not thermally damaged and that the detergent is completely dissolved. The sump temperature is indicative of the temperature at which the wash water is sprayed onto the clothes and also the temperature for dissolving the detergent.

This is regulated by manipulating the dwell times for the hot and cold water inlet valves, denoted  $U_{\text{hot}}(t)$  and  $U_{\text{cold}}(t)$  respectively. These are the fraction of time for which the valve is open, so range between 0 and 1. The controller chooses  $U_{\text{hot}}$  and  $U_{\text{cold}}$  when it is given an input, which in this case is denoted  $T_{\text{sump}}^{\text{input.to.controller}}(t)$ . This is not necessarily equal to  $T_{\text{sump}}(t)$  because of various delays discussed in Section 5.2. The aim temperature for the water in the sump is denoted  $T_{\text{sump}}^{\text{set.point}}$ .

It is suggested that for simplicity, the choice for the value of the manipulated variables could be linked by defining  $U_{\text{hot}}(t)$  from the controller equation and then setting  $U_{\text{cold}}(t) = 1 - U_{\text{hot}}(t)$ . If fill time is important then an alternative is to use the following to maximise fill rate:

If  $T_{\text{sump}}^{\text{input.to.controller}}(t) < T_{\text{sump}}^{\text{set.point}}$  then set  $U_{\text{hot}}(t) = 1$  and let  $U_{\text{cold}}(t)$  be set less than 1 as dictated by the controller equation, else if  $T_{\text{sump}}^{\text{input.to.controller}}(t) > T_{\text{sump}}^{\text{set.point}}$  then set  $U_{\text{cold}}(t) = 1$  and let  $U_{\text{hot}}(t)$  be set less than 1 as dictated by the controller equation.

Feedback control strategies first calculate the “error” signal, denoted  $e(t)$  and defined as:

$$e(t) = T_{\text{sump}}^{\text{set.point}} - T_{\text{sump}}^{\text{input.to.controller}}(t) .$$

A proportional controller will then relate the manipulated variable (assume this is  $U_{\text{hot}}(t)$  for illustration) to the error signal with the equation:

$$U_{\text{hot}}(t) = k_p e(t) + \text{offset} ,$$

where  $k_p$  is the controller's proportional gain and "offset" is the controller's offset.

Such a controller is very simple with just 2 parameters to choose. The main drawback is that there is usually a steady state error, that is we do not have  $T_{\text{sump}}^{\text{input.to.controller}}(t)$  exactly equal to  $T_{\text{sump}}^{\text{set.point}}$ . To see this, for the prevailing values of the disturbances there will be a value of  $U_{\text{hot}}(t)$  that is required to make  $e(t) = 0$  and this value will generally not be equal to the offset chosen.

The proportional gain can be chosen using a modelling or experimental approach. Various values can be tried in the MATLAB model of Section 5.3.

To remove this steady state error, a Proportional + Integral controller can be used, this has the form:

$$U_{\text{hot}}(t) = k_p \left( e(t) + \frac{1}{T_i} \int e(t) dt \right)$$

where  $T_i$  (s) is the controller's integral time. This effectively automatically adjusts the controller's offset to remove any steady state error. This benefit comes at a cost, though, as the controller's integral action adds 90 degrees of phase lag and it is now effectively the frequency at which the System Transfer Function has 90 degrees of phase lag (not 180 degrees as for proportional control) that is the limit of the disturbance rejection bandwidth. This might not be a problem in the washing machine where the MATLAB model of Section 5.3 suggests that delays are unimportant.

The controller's integral time should be "tuned" first the proportional gain to follow. A good rule of thumb for choosing the integral time is to set  $T_i = 1/\omega_{90}$  where  $\omega_{90}$  (rad/sec) is the frequency by which the phase lag of the System Transfer Function reaches 90 degrees (or approximately twice the sum of the longest pure delay time and the largest time constant).

## 5.2. A dynamic, equilibrium perturbation model of the sump.

A dynamic, equilibrium perturbation model will be constructed to assist with the analysis and design of a control strategy for the wash water temperature. The generic architecture used for such analysis is shown in figure 10.

The disturbance signals are anything that impacts the process variable excluding the manipulated variables. There are essentially 2 categories of disturbance variable: water supply and clothes. The water supply disturbances are represented by the signals for the hot and cold wa-

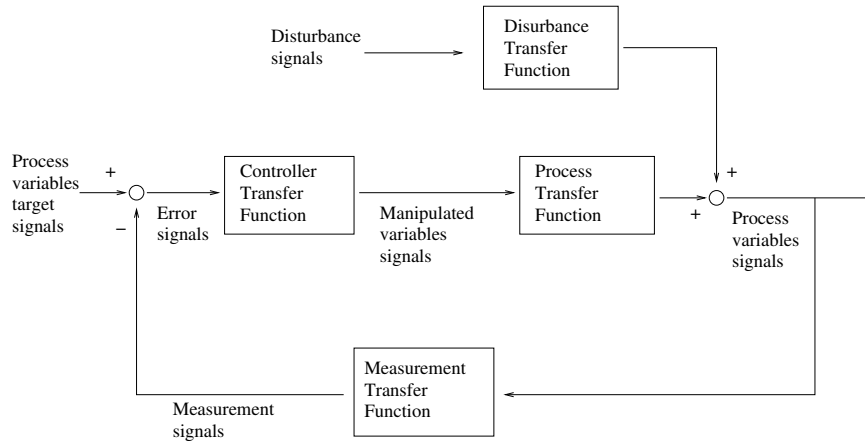


Figure 10. Generic architecture for control system analysis.

ter supply temperatures ( $T_{hot}(t)$  and  $T_{cold}(t)$ ), it is assumed that the flowrates of hot and cold water will depend only on the dwells of the valves (not dependent on line pressure since the supplies are throttled to produce a fixed flow for any line pressure  $> 1$  bar). The flowrates of hot and cold supply will be given by  $\dot{M}_{hot}U_{hot}(t)$  and  $\dot{M}_{cold}U_{cold}(t)$  where  $\dot{M}_{hot}$  and  $\dot{M}_{cold}$  are the mass flowrates (kg/s) that prevail when the valves are fully open.

There are 2 parts of the machine to model to produce the Process and Disturbance Transfer Functions: the mixing chamber for the supply water and the sump.

The dynamic energy balance for the mixing chamber is given by:

$$\frac{d(M_{mix}T_{mix}(t))}{dt} = \dot{M}_{hot}U_{hot}(t)T_{hot}(t) + \dot{M}_{cold}U_{cold}(t)T_{cold}(t) - \dot{M}_{mix}(t)T_{mix}(t) , \quad (7)$$

where  $M_{mix}$  (kg) is the (fixed) mass of water in the mixing chamber,  $T_{mix}(t)$  is the temperature of the water in the mixing chamber and  $\dot{M}_{mix}(t)$  is the mass flowrate of mixed water to the sump.

Since there is no change of mass of water in the mixing chamber, we have

$$\dot{M}_{mix}(t) = \dot{M}_{hot}U_{hot}(t) + \dot{M}_{cold}U_{cold}(t) . \quad (8)$$

To model the transport delay caused by the water flowing from the mixing chamber to the sump we define the variable  $T_{m.to.s}(t)$  which is the temperature of the mixed water entering the sump at time  $t$ . Assuming

no energy loss during transport this is equal to the temperature for the water leaving the mixing chamber  $\delta_{mix}$  seconds ago, where  $\delta_{mix}$  is the transport time. Thus we have the delay equation:

$$T_{m.to.s}(t) = T_{mix}(t - \delta_{mix}). \quad (9)$$

Note, the sump model will assume instantaneous mixing for simplicity, this will not be strictly correct and some of the “lost” dynamics can be effectively “captured” in the model by adding an appropriate amount of delay time to  $\delta_{mix}$ .

The dynamic energy (assuming instantaneous mixing) and mass balances for sump are given by:

$$\frac{d(M_{w.in.s}(t)T_{sump}(t))}{dt} = \dot{M}_{mix}(t)T_{m.to.s}(t) - \dot{M}_{re}(t)T_{sump}(t) + \dot{M}_{drip}(t)T_{drip}(t), \quad (10)$$

$$\frac{d(M_{w.in.s}(t))}{dt} = \dot{M}_{mix}(t) - \dot{M}_{re}(t) + \dot{M}_{drip}(t) \quad (11)$$

where,  $M_{w.in.s}(t)$  (kg) is the mass of water in the sump,  $T_{sump}(t)$  is the temperature of the water in the sump,  $\dot{M}_{re}(t)$  (kg/s) is the mass flowrate of recirculation,  $\dot{M}_{drip}(t)$  (kg/s) is the mass flowrate of the drip back from the clothes (*including* recirculated water that does not penetrate the clothes — this portion is called the “*splash*” in other sections) and  $T_{drip}(t)$  is the temperature of the water dripping back.

Thus the disturbances are parameterised here by the disturbance signals:  $\dot{M}_{re}(t)$ ,  $\dot{M}_{drip}(t)$  and  $T_{drip}(t)$ . Alternative parameterisations are possible, e.g. in terms of the absorbance rate of the clothes and the temperature difference between the sump and the clothes.

The measurement system (the temperature probe) can be assumed to have a first order response, thus

$$\tau_s \frac{dT_{sump}^{measured}(t)}{dt} + T_{sump}^{measured}(t) = T_{sump}(t), \quad (12)$$

where  $T_{sump}^{measured}(t)$  is the measured sump temperature and  $\tau_s$  (s) is the time constant of the temperature sensor.

Finally, the pulse width modulation of the valves effectively produces an additional delay of approximately half the modulation period. This can be accommodated in the measurement dynamics with the delay equation:

$$T_{sump}^{input.to.controller}(t) = T_{sump}^{measured}(t) \left( t - \frac{1}{2}\delta_{PWM} \right), \quad (13)$$

where  $T_{\text{sump}}^{\text{input.to.controller}}$  is the delayed, measured sump temperature as received by the controller and  $\delta_{PWM}$  (s) is the modulation period.

So, to summarise so far, the measured signal  $T_{\text{sump}}^{\text{input.to.controller}}(t)$  and process variable signal  $T_{\text{sump}}(t)$  are related to the manipulated variable signals ( $U_{\text{hot}}(t)$  and  $U_{\text{cold}}(t)$ ) and disturbance variables signals (water supply  $T_{\text{hot}}(t)$  and  $T_{\text{cold}}(t)$ , and clothes  $\dot{M}_{\text{re}}(t)$ ,  $\dot{M}_{\text{drip}}(t)$  and  $T_{\text{drip}}(t)$ ) by equations (6) through (13).

To yield a linear set of ODEs and delay equations, the nonlinear equations will have to be approximated by linearisation about a chosen, equilibrium operating point. For example, equation (6) can be approximated by the linear ODE:

$$\begin{aligned} M_{\text{mix}} \frac{dT_{\text{mix}}}{dt} = & \dot{M}_{\text{hot}} (U_{\text{hot}}(t)T_{\text{hot}}^o + T_{\text{hot}}(t)U_{\text{hot}}^o) \\ & + \dot{M}_{\text{cold}} (U_{\text{cold}}(t)T_{\text{cold}}^o + T_{\text{cold}}(t)U_{\text{cold}}^o) \\ & - \dot{M}_{\text{mix}}(t)T_{\text{mix}}^o - T_{\text{mix}}(t)\dot{M}_{\text{mix}}^o, \end{aligned}$$

where the superscript “o” means the chosen equilibrium operating point for the variable. To ensure that the operating point is an equilibrium we require the operating point to be chosen such that:  $\dot{M}_{\text{mix}}^o = \dot{M}_{\text{hot}}U_{\text{hot}}^o + \dot{M}_{\text{cold}}U_{\text{cold}}^o$  and  $\dot{M}_{\text{hot}}U_{\text{hot}}^oT_{\text{hot}}^o + \dot{M}_{\text{cold}}U_{\text{cold}}^oT_{\text{cold}}^o - \dot{M}_{\text{mix}}^oT_{\text{mix}}^o = 0$ .

Equations (10) and (11) are coupled, the product differential on the LHS of (10) should be expanded and the  $dM_{\text{w.in.s}}(t)/dt$  term replaced by substitution with the expression on the RHS of equation (11). The resultant decoupled ODEs will still be nonlinear and an approximation (similar to that given for equation (6)) will have to be made to linearise them about a chosen equilibrium operating point.

For the purposes of controller design and system assessment, the System Transfer function should be analysed, this is essentially the concatenation of the Process Transfer Function and the Measurement Transfer Function and describes the map from the signals representing the manipulated variables ( $U_{\text{hot}}(t)$  and  $U_{\text{cold}}(t)$ ) to the signal representing the measured variable  $T_{\text{sump}}^{\text{input.to.controller}}(t)$ .

It is the low frequency phase lag of this System Transfer Function that is of primary importance and the phase crossover frequency (The frequency at which the phase lag reaches 180 degrees.) is a good measure of this. If the system has a low phase crossover frequency then it is only possible to engineer a controller which can reject the low frequency components of the disturbance signals (i.e. slow changes in water supply temperatures or absorption behaviour of the clothes can be accommodated but more abrupt changes can not). Conversely, if the system has a high phase crossover frequency then a broader spectrum (bandwidth) of the disturbance signals can be rejected.

The phase lag of the System Transfer function is the sum of the phase lags of the component parts. The most significant parts are likely to be the pure delays (these are the transport and mixing delay ( $\delta_{mix}$ ) and the PWM delay ( $\delta_{PWM}$ )). If the time constant for the temperature sensor ( $\tau_s$ ) is large (say  $> 10$  s) then this may have a significant impact. The phase lag contributed by mixing is likely to be small since the residence time of the mixing chamber ( $M_{mix}/\dot{M}_{mix}$ ) is likely to be small ( $< 5$  s). The phase lag contributed by the sump will increase as  $M_{w.in.s}(t)$  increases during the fill cycle.

Thus, it is possible to analyse the impact which  $\delta_{PWM}$  and  $\tau_s$  have on the controllability of the system, these are both able to be influenced by sensor choice and programming of the controller, noting shorter PWM periods may have a negative impact on valve mean-time-between-failure. Further, it is evident that  $\delta_{mix}$  may have a significant impact on the performance, measures should be taken to minimise the transport delay and to make the sump mixing as ‘instantaneous’ as possible.

If any of the dynamic effects mentioned are much faster than the others they will contribute negligible phase lag at low frequencies and thus can be neglected from the model. The MATLAB simulation of Section 5.3 suggests that none of these delays is particularly important.

### 5.3. A MATLAB model of proportional control.

A controller has been added to the MATLAB model of Section 4.3. The model also incorporates the cold slug of water from the hot tap. To model the delays, the team decided to calculate the dwell times only every 30 seconds, and to introduce another variable,  $T_{measured}$ , which is the temperature of the water in the sump as measured by the temperature sensor. This lags behind the temperature of the water in the sump,  $T_{sump}$  with time constant  $\tau$ :

$$\tau \frac{dT_{measured}}{dt} = T_{sump} - T_{measured}. \quad (14)$$

This is designed to model the finite-time response of the temperature sensor. It is also designed to model the slow mixing of water at different temperatures in the sump: we can assume instantaneous perfect mixing (and denote the temperature by  $T_{sump}$ ), but the sensor takes some time to “see” this mixed water.

Two representative cases were tried. Both concerned 8kg of towels. Both had an aim temperature of 45 degC. All other parameters are as given in Section 4.3, and the input flow-rate was assumed to be constant at 10 l/min (when the valves are open).

In the first case the towels were initially bone dry and at the ambient temperature of 20degC. The hot input was at 60 degC and the cold at 15 degC. The cold slug was of 30 seconds duration. The offset in the proportional controller was taken to be  $(45-15)/(60-15)$ , and the gain  $k_p = 0.02$  degC. The time constant for the sensor was chosen to be  $\tau = 0.2$  min. Control was achieved, with the final sump temperature being around 43 degC. Choosing the gain more carefully, or using a PI controller, would counter the effects of the polypropylene, etc.

The second case was rather extreme. The towels contained 20 kg of cold water at 10 degC. The ambient, and initial conditions were 10 degC. The hot water input was 55 degC and the cold at 15 degC, and the cold slug was of 45 seconds duration. The controller's offset was chosen to be  $(45-15)/(55-15)$  with gain  $k_p = 0.02$  degC. The time constant for the sensor was chosen to be 0.4 min. Control was not achieved, with sump temperature being around 40 degC at the end of the fill.

Both simulations were continued after the fill had completed until the sump-clothes temperatures had equilibrated. For the latter simulation, this was at the low temperature of around 24 degC. This illustrates that post-fill effects can be quite important.

The results are shown in figures 11 and 12.

## 6. Sophisticated control strategies.

### 6.1. Cascade control.

A cascade control strategy utilises information of an additional measurement to improve the performance of the system. In this case, the additional measurement would be the temperature in the mixing chamber. The generic architecture of such a system is shown in figure 13. In this case the 'Inner process variable' is the mix temperature and the water supply disturbance signals will act on this inner loop. The inner controller can be tuned to have a relatively high bandwidth since the phase lag of the inner transfer function is relatively low ( the phase lag contributed by the transport lag to the sump and mixing dynamics in the sump are 'outside' the inner loop). Thus the impact of the water supply disturbances will be significantly reduced if cascade control is used.

The phase lags in the outer loop are similar to those for the single loop control and thus the ability to reject disturbances from the clothes is not significantly changed. To tune the controllers in the cascade architecture, the inner loop is tuned first then the outer loop tuned once the inner loop is fixed.

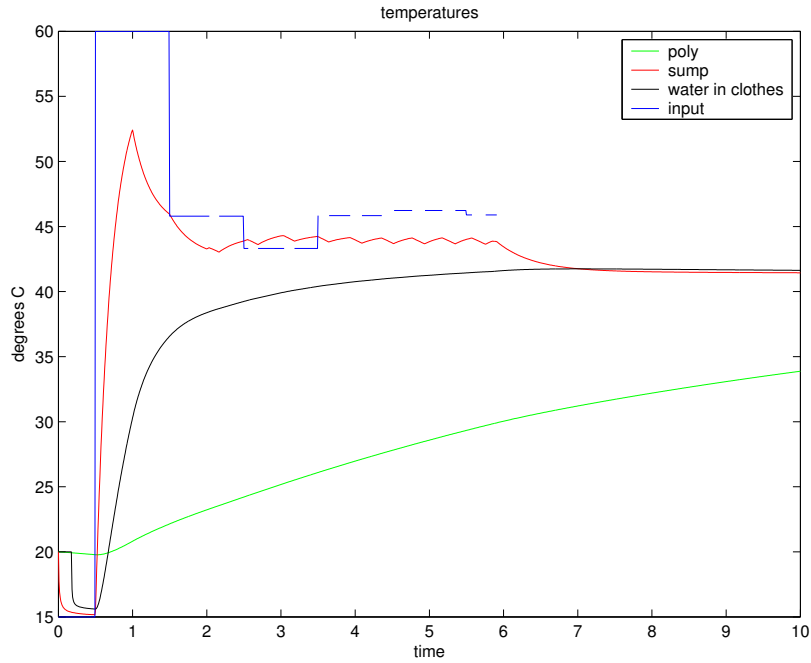


Figure 11. Output from MATLAB model command for “normal” control conditions. Temperature of polypropylene (green, lower at  $t = 5$ ), sump water (red), saturated portion of clothes (black, second lowest at  $t = 5$ ) and water input (blue, highest at  $t = 5$ ) are shown plotted against time. Note that the “fill” ends around 6 minutes when the “input” is shut off. Produced with MATLAB command `>>[t,y] = washp(45,1,10,@normal)`.

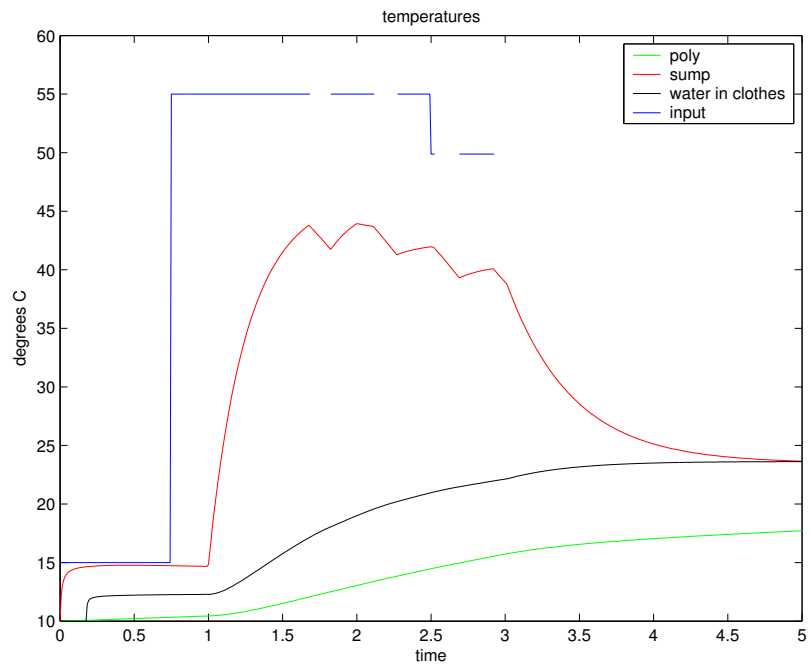


Figure 12. Output from MATLAB model command for “harsh” control conditions. MATLAB command `>>[t,y] = washp(45,1,10,@harsh)`.



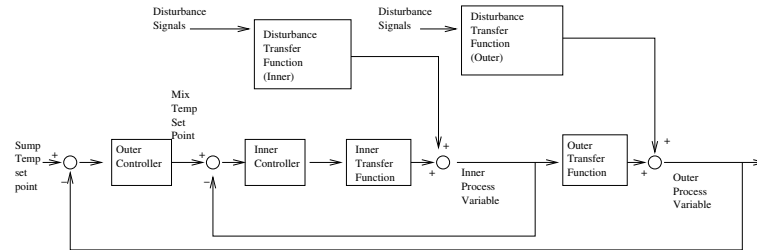


Figure 13. Generic architecture for cascade control system. Note, measurement transfer functions have been omitted for brevity.

## 6.2. “Smart” control systems.

In addition to the models and control strategies discussed above the team that worked on the problem at MISG2005 came up with a number of innovative suggestions to “redesign” the washing machine or control strategy. These suggestions are detailed below.

### 6.2.1 Estimating values from beginning of fill.

In order to obtain accurate values for site specific parameters such as the flow rate and temperature of the hot and cold water supply it was suggested that these be measured in the initial part of the fill cycle. Before the pump is switched on, the hot and cold taps could be turned on separately to measure flow rate (using the data from the pressure sensor) and temperature. However, dealing with the cold slug is a complicating factor and measuring the temperature of the water supplies would require the temperature sensor to be placed in or near the input. Further, adding cold water to the machine in order to measure the cold temperature will worsen the performance of the control strategy in the situation where no cold water should be added in order to reach the aim temperature.

### 6.2.2 Memory chip.

This suggestion built on the previous suggestion and overcomes some of the problems with it, by eliminating the need to measure values each time a load of washing is done. As a Fisher and Paykel washing machine already contains significant computing power it would be feasible to add a memory chip to the machine. This could store information about the hot and cold water temperatures, flow rates, and other site specific in-

formation like the length of the cold slug. Remembering how well the control strategy did last time a hot wash was performed would also be useful — the controller would then know whether it should add more or less hot water. The memory chip could also store information about the user's washing patterns which would aid in data collection and act as a diagnostic tool if the machine required repair.

### **6.2.3 Initialise machine on installation.**

Further building on the previous suggestion, the machine when installed could perform an initial measurement test to obtain the data required by the memory chip for future washes.

### **6.2.4 Placement of temperature sensor on a “sill”.**

Following the realisation that the first suggestion of measuring the hot and cold water temperature in the initial part of the fill would require the temperature sensor to be placed near the input and not in the sump, it was suggested that the temperature sensor instead be located on a ledge in the lower part of the machine (see figure 14). This would allow the temperature of the incoming water to be measured initially, before the water level in the machine reached the sill. Once the water was higher than the sill the temperature sensor would measure the sump temperature. This is a potential compromise between the conflicting needs to rapidly observe temperature disturbances from the water supplies and have an accurate, local measure of the temperature of the water at the suction side of the recirculating pump, i.e. the temperature of the water that will soon be sprayed on the clothes.

### **6.2.5 Placement of temperature sensor in the mixing chamber and diverting the recirculating water.**

An alternative location for the temperature sensor would be in the mixing chamber (see figure 15). This would allow the measurement of the incoming water temperature. The temperature of the recirculation water could also be measured by the same temperature sensor if this water was passed through the mixing chamber. The drawback of this geometry is that another valve would be required so that the water could be directed either into the sump or onto the clothes, to prevent water which is too hot being sprayed onto the clothes.

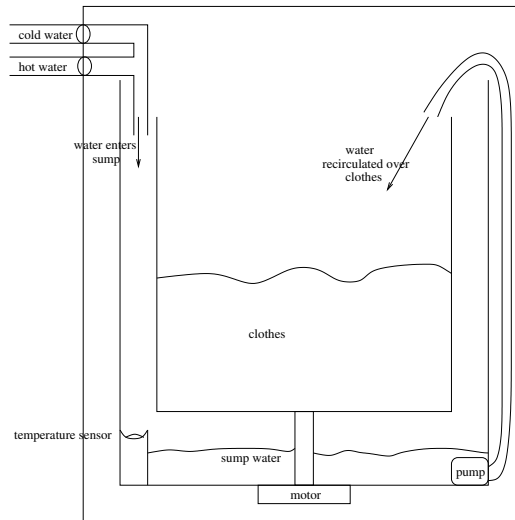


Figure 14. Placement of temperature sensor on a “sill”.

## 7. Conclusions and recommendations

The steady state energy balance illustrates which effects are significant and which are not. Cold slugs in the hot water supply, large amounts of water in the clothes and excessive opening of the lid are all significant. Machine components also absorb heat, although since this effect does not change considerably between loads, Fisher and Paykel can easily compensate for the effect, especially if using a PI controller.

A number of analytic models have been presented which allow an analytical exploration of the disturbance envelopes including dynamic delay effects. The analysis and modelling of the clothes suggests that cold, saturated clothes may continue to drip cold water into the sump for most of the fill and this is a challenging disturbance for the temperature regulation system.

MATLAB simulations suggest that dynamic delays are unimportant and also allow the effects of the following variables to be easily investigated numerically: temperature of the hot water supply, amount and temperature of initial water in the clothes, clothes masses, and absorbency rates of clothes. In all but the extreme cases, a few trial runs of the MATLAB proportional controller routine have suggested that temperature control can be achieved to within  $\pm 2$  degC, and this could be improved by using a PI controller.

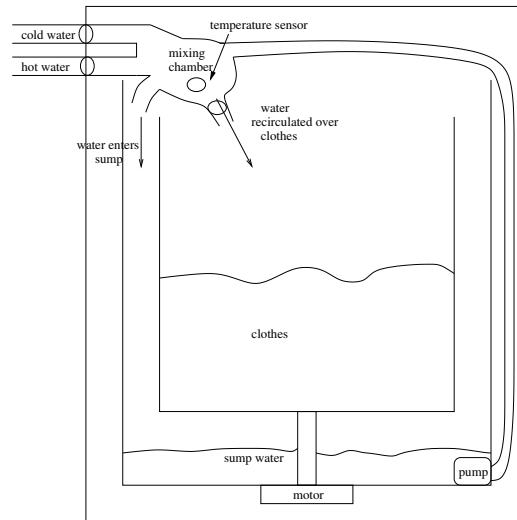


Figure 15. Placement of temperature sensor in the mixing chamber and modifying the geometry so that the recirculating water also flows through the mixing chamber.

If the water supply disturbances are much greater than the clothes disturbances then the best place to locate the sensor is in the mixing chamber. High bandwidth disturbance rejection of the water supply disturbances can then be achieved but the system will be highly exposed to any significant disturbances from the clothes. Conversely, if the clothes disturbances are much greater than the water supply disturbances then the best place to locate the sensor is in the sump. All disturbances can be rejected to an extent but the high frequency disturbances from the water supply will not be able to be rejected. If both water supply and clothes disturbances are significant then the best performance will be achieved by measuring both the mix and sump temperatures and utilising this measured information in a cascade control strategy.

Further, a series of ideas have been presented for “smart” approaches where key external parameters may be identified, memorised and utilised by the regulation system.

Finally, it may be useful to reiterate that the temperature of the sump water may change radically after the fill phase due to equilibration between the sump water and the water in the clothes.

## Acknowledgments

The project moderators, Clive Marsh, Jane Thredgold and Andy Wilkins, thank the representatives from Fisher and Paykel; Steven Mansell, Kerry Newnham and Josh Cox. The MISG team which worked on the problem included Maria Goodiev, Diane Hoffoss, Young Hong, Sam Howison, Don Kulasiri, Chang-Ock Lee, Jongwoo Lee, Gerard Liddell, Mark Nelson, Eun-Jae Park, and Wynand Verwoerd.

## Appendice

### A The MATLAB model

The top level function washp.m calls all the other functions. To run the code with the aim temperature set to 45 deg C, for a load of sheets, with the simulation going up to the 10 minute mark of the fill cycle for a fill under “normal” control conditions, at the command prompt in Matlab, enter:

```
>> [t,y] = washp(45,1,10,@normal);
```

For the same aim temperature, with a load of towels, simulated up to the 5 minute mark of the fill cycle under “harsh” control conditions, enter :

```
>> [t,y] = washp(45,2,5,@harsh);
```

Three load “types” have been defined - sheets (1), towels (2) and empty (3). The empty load is useful for checking the behaviour of the model and the accuracy of parameters such as the heat transfer coefficient from the water to the polypropylene by comparing with experimental fills of an “empty” machine (when the disturbances associated with the load are removed).

To enable some parameters to be changed easily they have been saved in separate files. “Normal” control conditions are defined as ambient temperature 20 deg C, hot and cold water supply temperatures 60 and 15 deg C respectively, a cold slug which takes 30seconds to clear, no water in the load initially and a delay constant for the temperature sensor of 0.2 min. “Harsh” control conditions (an example of a case where we expected the controller to have difficulty reaching the desired temperature) are defined as ambient temperature 10 deg C, hot and cold water 60 and 15 deg C, a cold slug which takes 45seconds to clear, 20 kg of water at 10 degC in the load initially and a delay constant of 0.4 min. The temperature of the cold slug is assumed to be the temperature of the cold water. Note that “harsh” control conditions may only be used

with the towel load type as the sheet load type will not absorb 20kg of water. Files containing parameters for any other operating conditions may be created. It should also be reasonably straightforward to add other parameters to the parameter files (removing the definition of these parameters from within the model proper) if required.

Note that the controller, which updates the temperature of the input water every 30 seconds, requires that the time step is chosen so that there is a time step exactly at 30, 60, 90, etc. seconds. The time step is set within the code so that there is a time point every 0.5seconds, but if the time step is changed this requirement must be taken into account.

The function washp.m returns a vector of the time values  $t$ , and a matrix  $y$  which contains the values of the 6 variables - the temperatures of the polypropylene ( $y(:,1)$ ), the sump water ( $y(:,2)$ ), the water in the clothes ( $y(:,3)$ ); the mass of water in the sump ( $y(:,4)$ ) and in the clothes ( $y(:,5)$ ) and the “measured” temperature of the sump water ( $y(:,6)$ ). This last variable is used to incorporate delays.

The code also produces a plot of each of the mass variables against time in separate figures and a combined plot of the temperatures of the polypropylene, sump water, water in clothes and the input water (the “measured” sump temperature is not included in the plot).

Most of the parameters for the model (aside from those in the parameter files) are set within the code and need to be altered manually to experiment with different situations. Some “typical” parameters are included within the code and occasionally alternative values are given in comments (any line in Matlab code which begins with % is a comment). The gain parameter for the controller is set in temp\_control.m and all other parameters are defined within washmodel.m.

#### A1 wash.m

```
function [t,y] = washp(Taim,load,tfinal,parafile);

% Taim is aim temperature for sump at end of fill (in Celsius)
% load = 1 for sheets, 2 for towels, 3 for empty
% tfinal is time in MINUTES to run simulation for
% parafile is file containing parameters
% (have collected in file to make it easier to change the
% values)

% n is number of points used for numerical ode solver
% n MUST be chosen so that there is a time point EXACTLY
% at t = 0.5, 1, 1.5, 2, etc. minutes or input temp
% will not be updated
```

```
n = tfinal * 120; % time step every 1/2 second

% load parameters and initial conditions from file
[ambient,Mwinc,Twinc,Thot,Tcold,slugtime,tau]
%= feval(parafile);

% +273 to temperatures for Kelvin scale
Taim = Taim + 273;
ambient = ambient + 273;
Twinc = Twinc + 273;

% set up initial conditions
% temperatures initially all ambient temperature
% unless there is water in clothes, then this is at temp
% Twinc
% mass of water in clothes given by parameter Mwinc
% mass of water in machine initially assumed to be 100mL
% or 0.1kg

if Mwinc == 0,          % make sure if no water in clothes
    Twinc = ambient;   % that this is ambient temp
end
y0 = [ambient, ambient, Twinc, 0.1, Mwinc, ambient];

global inlet;
% used to remember whether taps were previously on or off
% for each call to washmodel
inlet = 1; % set taps on initially

% variables are
% y1 == Tpoly
% y2 == Tsump
% y3 == Tsatc
% y4 == Msump
% y5 == Mwinc
% y6 == Tsumpmeasured

% cannot use Matlab solver as adaptive step size messes up
%tap control,
% have used a fourth order Runge-Kutta method with n
%evenly spaced points
[t,y] = rko4vpara(@washmodelp,0,tfinal,n,y0,parafile,
```

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```
[Taim,load,y0]);

% -273 from temperature values to get Celsius scale
y(:,1) = y(:,1) - 273*ones(length(t),1);
y(:,2) = y(:,2) - 273*ones(length(t),1);
y(:,3) = y(:,3) - 273*ones(length(t),1);
y(:,6) = y(:,6) - 273*ones(length(t),1);

% plot masses
figure;
plot(t,y(:,4)); xlabel('time'); ylabel('kg');
title('mass of water in sump');
figure; plot(t,y(:,5)); xlabel('time'); ylabel('kg');
title('mass of water in clothes');

% combined plot of temperatures
global inputtemps; % used to plot input temperatures
inputT(1) = inputtemps(1)-273; k = 1;
for i = 2:length(t),
    if rem(t(i),1/2) == 0,
        k = k+1;
    end;
    inputT(i) = inputtemps(k)-273;
    if t(i) < slugtime,
        inputT(i) = Tcold;
    end
end

figure;
plot(t,y(:,1),'g',t,y(:,2),'r',t,y(:,3),'k',t,inputT,'b');
xlabel('time'); ylabel('degrees C');
legend('poly','sump','water in clothes','input');
```

## A2 washmodelp.m

This function contains the differential equations of the model. The equations used are equations (1), (3), (4), (5) and (6) from the absorption model of Section 4.2 and the simple delay equation (14) discussed in Section 5.3.

```
function y = washmodelp(t,w,parafile,para);

Taim = para(1);
```



```

load = para(2); % load = 1 for sheets, 2 for towels, 3 for empty
y0 = para(3:8); % initial conditions

% load parameters from file, only use Tcold and tau in
this file
[ambient,Mwinc,Twinc,Thot,Tcold,slugtime,tau]
= feval(parafile);

global inlet          % taps on/off
global inputtemps    % used to plot input temperatures
global Tin
% not used outside this file but need to remember
% this value as it's only updated every 30 seconds

% initialise Tin if t is zero,
start cold water temperature (cold slug)
% model INCLUDES COLD SLUG
if t == 0,
    Tin = Tcold+273;
    inputtemps = Tin;
end
if t < slugtime,
    Tin = Tcold+273;
    inputtemps = Tin;
end

% if 30 second mark, find input temperature,
% given DELAYED MEASUREMENT of sump temperature
if (rem(t,1/2) == 0 && t >= slugtime),
    Tin = feval(@temp_controlp,w(6),Taim,parafile);
    inputtemps = [inputtemps, Tin];
end

% constants which are independent of load
Cpoly = 2;          % heat capacity of polypropylene
Mpoly = 5;          % mass of polypropylene
Hw2p = 1;          % heat transfer coeff from water to poly
Apoly = 1;          % area of contact between water and poly
Cw = 4.2;          % heat capacity of water
Cc = 1.5;          % heat capacity of dry clothes

% constants which depend on load, but which are not changed

```

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```
h = 0.4;          % height of clothes in machine
Mc = 8;          % mass of dry clothes

if load == 1,    % constants for sheets
    Mmax = 2;    % maximum absorbency rate
    Msat = 2*Mc; % max mass of water absorbed by clothes,
                % 2 times mass of clothes
    r = 10/7;   % max height before complete saturation
                % impossible
elseif load == 2, % constants for towels
    Mmax = 9;
    Msat = 5*Mc;
    r = 1;
elseif load == 3, % no clothes!!
    h = 0;
    Mc = 0;
    Mmax = 0;
    Msat = 0;
    r = 1; % r is meaningless but must be defined
else
    fprintf(1,'need to set load parameter to 1, 2 or 3\n');
end

inputrate = 10;
% find Min at current time
if w(4) < 4 % on if water level below 4
    Min = inputrate;
    inlet = 1;
elseif w(4) > 5 % off if water level above 5
    Min = 0;
    inlet = 0;
elseif inlet == 1 % if between 4 and 5 on if previously on
    Min = inputrate;
else % off if previously off
    Min = 0;
end

% calculate x (height of saturated clothes)
if load == 3, % no clothes
    x = 0;
else
    x = h*(w(5) - y0(5))/(Msat - y0(5));
```

```

end

% Three situations
% (1) pump off (beginning) water level isn't high enough
% for water recirc pump to be on, set Mmax to zero,
% no change in mass or temp of water in clothes
% (2) pump on, clothes absorbing water
% (3) pump on, clothes saturated, equations simplify,
% no change in mass of water in clothes

% system of odes
if load == 3, % no clothes
    y(5) = 0; % never any water in clothes
    y(3) = 0; % no clothes so satc is meaningless
    y(1) = -Hw2p*Apoly/(Cpoly*Mpoly)*( w(1) - w(2) ); %Tpoly
    y(6) = (w(2) - w(6))/tau;
    if w(4) < 1.8 % Msump < 1.8, situation (1)
        Mmax = 0;
        y(4) = Min; %Msump
        y(2) = 1/w(4)*(-w(2)*y(4) + 1/Cw*( Cw*Min*Tin -
        Hw2p*Apoly*(w(2) - w(1)) - Cw*Mmax*w(2)
        + Cw*Mmax*r*x*w(3) ) ); % Tsump
    elseif w(5) > Msat % Mwinc > saturation mass, situation (3)
        y(4) = Min; %Msump
        y(2) = 1/w(4)*(-w(2)*y(4) + 1/Cw*( Cw*Min*Tin -
        Hw2p*Apoly*(w(2) - w(1)) - Cw*Mmax*w(2)
        + Cw*Mmax*w(3) ) ); % Tsump
    else % situation (2)
        y(4) = Min - Mmax + Mmax*r*x; %Msump
        y(2) = 1/w(4)*(-w(2)*y(4) + 1/Cw*( Cw*Min*Tin -
        Hw2p*Apoly*(w(2) - w(1)) - Cw*Mmax*w(2)
        + Cw*Mmax*r*x*w(3) ) ); % Tsump
    end
else % clothes
    y(1) = -Hw2p*Apoly/(Cpoly*Mpoly)*( w(1) - w(2) ); %Tpoly
    y(6) = (w(2) - w(6))/tau;
    if w(4) < 1.8 % Msump < 1.8, situation (1)
        Mmax = 0;
        y(4) = Min; %Msump
        y(5) = 0; %Mwinc
        y(2) = 1/w(4)*(-w(2)*y(4) + 1/Cw*( Cw*Min*Tin -
        Hw2p*Apoly*(w(2) - w(1)) - Cw*Mmax*w(2)

```

```

        + Cw*Mmax*r*x*w(3) ) ); % Tsump
    y(3) = 0; % Tsatc
elseif w(5) > Msat % Mwinc > saturation mass, situation (3)
    y(4) = Min; %Msump
    y(5) = 0; %Mwinc
    y(2) = 1/w(4)*( -w(2)*y(4) + 1/Cw*( Cw*Min*Tin -
    Hw2p*Apoly*(w(2) - w(1)) - Cw*Mmax*w(2)
    + Cw*Mmax*w(3) ) ); % Tsump
    y(3) = 1/(Cw*Msat + Cc*Mc)*( Cw*Mmax*( w(2) - w(3) ) );
    % Tsatc
else % situation (2)
    y(4) = Min - Mmax + Mmax*r*x; %Msump
    y(5) = Mmax - Mmax*r*x; %Mwinc
    y(2) = 1/w(4)*( -w(2)*y(4) + 1/Cw*( Cw*Min*Tin -
    Hw2p*Apoly*(w(2) - w(1)) - Cw*Mmax*w(2)
    + Cw*Mmax*r*x*w(3) ) ); % Tsump
    % calculate derivative of x
    dx = h/(Msat - y0(5))*y(5);
    if x == 0,
        y(3) = 0; % no water in clothes,
            % assume no change in temperature
    else
        y(3) = 1/x*( -dx*w(3) + h/(Cw*Msat
        + Cc*Mc)*( Cw*Mmax*w(2) - Cw*Mmax*r*x*w(3) -
        (Cw*y0(5) + Cc*Mc)*y0(3)/h*(-1*dx) ) ); % Tsatc
    end
end
end
end
y = y(:);

```

### A3 temp\_controlp.m

This function is the “controller”. Different control strategies can be used with the model, by varying the way uhot and ucold (and hence inputtemp) are calculated in this file. No other changes need to be made to any other file, unless the flow rate needs to be altered. Note that the model currently has a constant flowrate of 10 L/min when the taps are on. This is defined by the parameter inputrate, set within washmodelp.m.

```
function inputtemp = temp_controlp(sumptemp,aintemp,parafile);
```

```

% given current sump temperature and aim temperature,
% set input temperature

% load parameters from file, only use Thot, Tcold in
this file
[ambient,Mwinc,Twinc,Thot,Tcold,slugtime,tau]
= feval(parafilename);

Tmax = Thot+273;    % max achievable temp (all hot water)
Tmin = Tcold+273;  % min achievable temp (all cold water)

offset = (aimtemp - Tmin)/(Tmax - Tmin);
k = 0.02;

error = aimtemp - sumptemp;
uhot = k*error + offset; % dwell time for u hot
% uhot must be between 0 and 1
if uhot > 1,
    uhot = 1; % all hot water
end
if uhot < 0,
    uhot = 0; % all cold water
end
ucold = 1 - uhot;

inputtemp = uhot*Tmax + ucold*Tmin;

```

#### A4 normal.m

As explained in section 7, this file defines some parameters for “normal” control conditions.

```

function [ambient,Mwinc,Twinc,Thot,Tcold,slugtime,tau] = normal;

% parameters for 'normal' control conditions

ambient = 20;    % ambient temperature
Thot = 60;      % hot water supply temp
Tcold = 15;     % cold water supply temp
slugtime = 0.5; % time for cold slug to clear -
                % based on 5L cold slug, 10L/min flowrate

Mwinc = 0;      % amount of water in clothes initially

```

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```
Twinc = ambient; % temperature of water in clothes initially  
tau = 0.2;      % delay constant
```

#### A5 harsh.m

As explained in section 7, this file defines some parameters for “harsh” control conditions.

```
function [ambient,Mwinc,Twinc,Thot,Tcold,slugtime,tau]=harsh;  
  
% parameters for 'harsh' control conditions  
% - towels with 20kg cold water at 10degC  
% - ambient (initial conditions) is 10degC  
% - T_hot = 55degC, T_cold = 15degC  
% - 30second period, with tau_sensor = 0.4 minutes.  
  
ambient = 10;    % ambient temperature  
Thot = 55;      % hot water supply temp  
Tcold = 15;     % cold water supply temp  
slugtime = 0.75; % time for cold slug to clear  
  
Mwinc = 20;     % amount of water in clothes initially  
Twinc = 10;    % temperature of water in clothes initially  
  
tau = 0.4;     % delay constant
```