DISCRIMINATION AND IDENTIFICATION OF UNEXPLODED ORDINANCES (UXO) USING AIRBORNE MAGNETIC GRADIENTS

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Abstract

The problem of discriminating the magnetic dipoles of objects on the surface of the earth, possible unexploded ordinances (UXO), from the effect of the earth's magnetic field in airborne magnetic field gradient data was proposed. A model of simultaneous equations was developed which hoped to discriminate between the effect of the earth's magnetic field and the possible UXO by solving for multiple dipoles using multiple data points. The simplifying assumption, that the location of each dipole is known, proved to produce unfavorable results when the flight path has a varying altitude making the model impractical. Current work suggests that a more practical solution to the problem can be achieved with subspace tracking.

1 Introduction

The removal of UXO, such as landmines and mortar shells, is an expensive and dangerous task. An airborne system for locating, and possibly identifying, such objects will greatly assist in the clearing process. The problem proposed was to locate magnetic field dipoles (dipoles), possible UXO, from the measured magnetic gradient field tensors. The earth's magnetic field is three orders of magnitude greater than that of any objects located near the surface and so a major part of the problem is concerned with separating the earth from the dipoles. Data was provided which included the measured magnetic gradient tensors, the total magnetic field, global positioning satellite (GPS) coordinates for the measurements and magnetic gradient tensors which had been compensated for the earth's magnetic field.

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An outline of the paper is as follows. Section 2 briefly describes the magnetic field equations. In section 3 the model of a system of linear equations, based on the assumption that the locations of the dipoles are known, developed at MISGSA 2004 is explained, followed in section 4 by an evaluation of the model explaining why the model assumptions are not practical. Section 5 briefly discusses work that has been done on the problem since the model developed at MISGSA 2004.

2 Magnetic field equations

The magnetic field $\mathbf{H}$ for a magnetic dipole with magnetic moment $\mathbf{m}$ is given by

$$\mathbf{H} = \frac{3(\mathbf{m}\cdot\mathbf{r})\mathbf{r}}{r^5} - \frac{\mathbf{m}}{r^3}, \tag{2.1}$$

where $\mathbf{r}$ is the position vector of the field point relative to the dipole and r is the magnitude of $\mathbf{r}$. Equivalently, in index form, adopting Einstein's summation convention

$$H_j = \frac{3m_i r_i r_j}{r^5} - \frac{m_j}{r^3}. \tag{2.2}$$

The magnetic field gradients must obey Maxwell's equations in a vacuum

$$\partial_j H_j = 0, \tag{2.3}$$
$$\partial_j H_k = \partial_k H_j. \tag{2.4}$$

Thus there are only 5 linearly independent field gradients. The measured magnetic field gradient tensor consists of 6 components, shown in Figure 1. This data, which was used for the results presented in this paper, was measured using cars for possible dipoles. Five of these components can be pre-multiplied by the corresponding rotation matrix to give the 5 linearly independent magnetic gradients, H_{xx}, H_{xy}, H_{xz}, H_{yy} and H_{yz}, in Euclidean space.

3 Model

The Frahm (1972) inversion only considers one field point at a time and assumes there is only one dipole corresponding to that field point. This means that the data must be compensated to remove the effect of the earth's

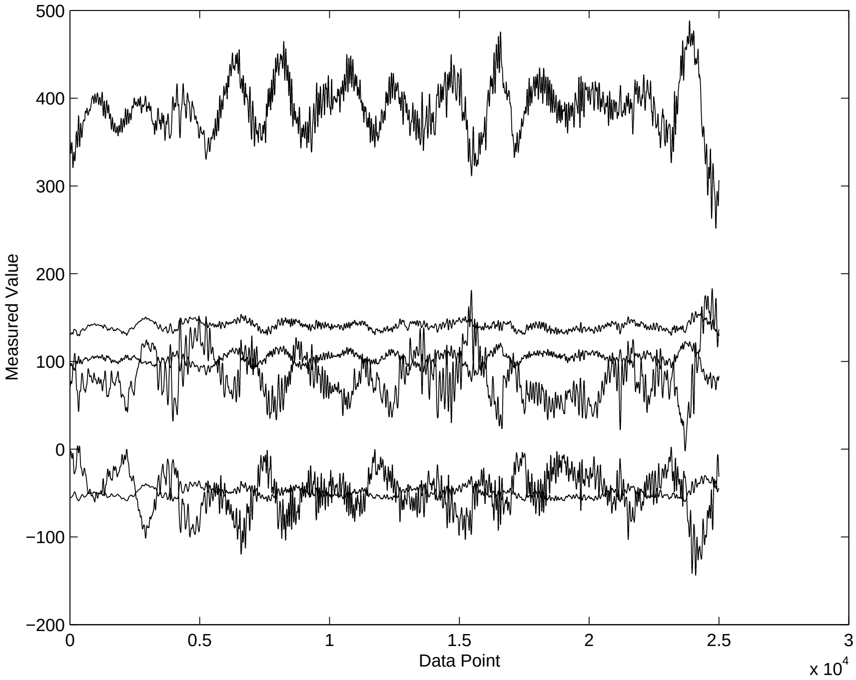

Figure 1: Magnetic gradient field tensor data.

magnetic field, before the Frahm inversion algorithm can be applied. The earth's magnetic field is far greater than that of the expected targets and the model presented here hoped to find the targets in the uncompensated data by solving for multiple dipoles (earth and target), using multiple field points. In order to simplify the multiple points multiple dipoles (MPMD) model the locations of the dipoles are fixed. Thus $\mathbf{r}$, in equation (2.1), is known and the problem is now linear in $\mathbf{m}$. Now let

$$\mathbf{n} = \frac{\mathbf{r}}{r}. \tag{3.1}$$

Then from equation (2.1)

$$\mathbf{H} = \frac{1}{r^3}[3(\mathbf{m} \cdot \mathbf{n})\mathbf{n} - \mathbf{m}]. \tag{3.2}$$

Rewriting in index form, equation (3.2) becomes

$$H_j = \frac{1}{r^3}(3m_i n_i n_j - m_j). \tag{3.3}$$

For the magnetic gradient field

$$\begin{aligned} \partial_k H_j &= \frac{1}{r^3}(3n_i n_j \partial_k m_i - \partial_k m_j) \\ &= \frac{1}{r^3}(3n_i n_j - \delta_{ij})\partial_k m_i. \end{aligned} \tag{3.4}$$

Now for P dipoles the magnetic field at point q is given by

$$\mathbf{G}^q = \sum_{p=1}^{P} \mathbf{H}^{pq}, \tag{3.5}$$

where $\mathbf{H}^{pq}$ is the magnetic field at field point q corresponding to the magnetic moment $\mathbf{m}^p$. The gradient tensors are given by

$$\partial_k G_j^q = \sum_{p=1}^{P} \partial_k H_j^{pq} \tag{3.6}$$

$$= \sum_{p=1}^{P} \frac{1}{(r^{pq})^3}(3n_i^{pq} n_j^{pq} - \delta_{ij})\partial_k m_i^p, \tag{3.7}$$

where $\mathbf{m}^p$ is the moment of dipole p, $\mathbf{r}^{pq}$ is the position vector of field point q relative to dipole p, r^{pq} is the magnitude of $\mathbf{r}^{pq}$ and $\mathbf{n}^{pq} = \mathbf{r}^{pq}/r^{pq}$. For Q field points, there is the system of $9 \times Q$ equations in $9 \times P$ unknowns, which can be represented in matrix form by

$$\hat{\mathbf{G}} = A\hat{\mathbf{m}}, \tag{3.8}$$

where

$$\hat{G}_f = \partial_k G_j^q, \qquad f = (3k + j - 3)q, \tag{3.9}$$

$$\hat{m}_g = \partial_k m_i^p, \qquad g = (3k + i - 3)p, \tag{3.10}$$

$$A_{fg} = \frac{3n_i^{pq} n_j^{pq} - \delta_{ij}}{(r^{pq})^3}. \tag{3.11}$$

Then

$$\hat{\mathbf{m}} = A^{-g}\hat{\mathbf{G}}, \tag{3.12}$$

where A^{-g} is the generalised pseudo-inverse of A:

$$A^{-g} = VS^{-1}U^T, \tag{3.13}$$

where $A = USV^T$ is the singular value decomposition (SVD) of A, U and V are orthonormal matrices and S is a diagonal matrix of the singular values of A such that $S_{11} \geq S_{22} \geq \cdots \geq 0$.

4 Evaluation of the model

A constructed test case was used initially to evaluate the MPMD model, with promising results. However, the results were not favourable on the measured data. The main reason for this was the assumption that $\mathbf{r}$ is a constant vector. In the test case the flight path was at a fixed altitude and thus there was far less variation in the values of r. In the measured data the altitude changes and this affects the solution. This can be seen in Figure 2, which shows the comparison of the measured data's altitude, the Frahm solution and the result of the MPMD model with dipoles placed at ground level directly below the field points, using a window size of 2 ($P = Q = 2$), and taking the smallest of the solutions for each dipole as the target value.

5 Subspace tracking

The MPMD model proposed at MISGSA 2004 proved to be impractical. The assumption that $\mathbf{r}$ is a constant in the magnetic field equation (2.1) does not work with measured data, where variations in r are common. Subsequent to MISGSA 2004, work has continued on the problem, using subspace tracking. The measured data is a mixture of data from different sources as well as noise. This can be written as the equation

$$Y = DS + \mathcal{N}, \tag{5.1}$$

where Y is the measured values for the n sensors, S is the values for the m sources ($m \leq n$), D is a $m \times n$ mixing matrix and $\mathcal{N}$ is the noise matrix. The noise matrix $\mathcal{N}$ can be dropped by assuming that the noise is an additional source. The sources (subspaces) can now be determined by decomposing the matrix Y, into the matrices D and S. The singular value decomposition (SVD) can be used to do this decomposition. Using the SVD, the three most

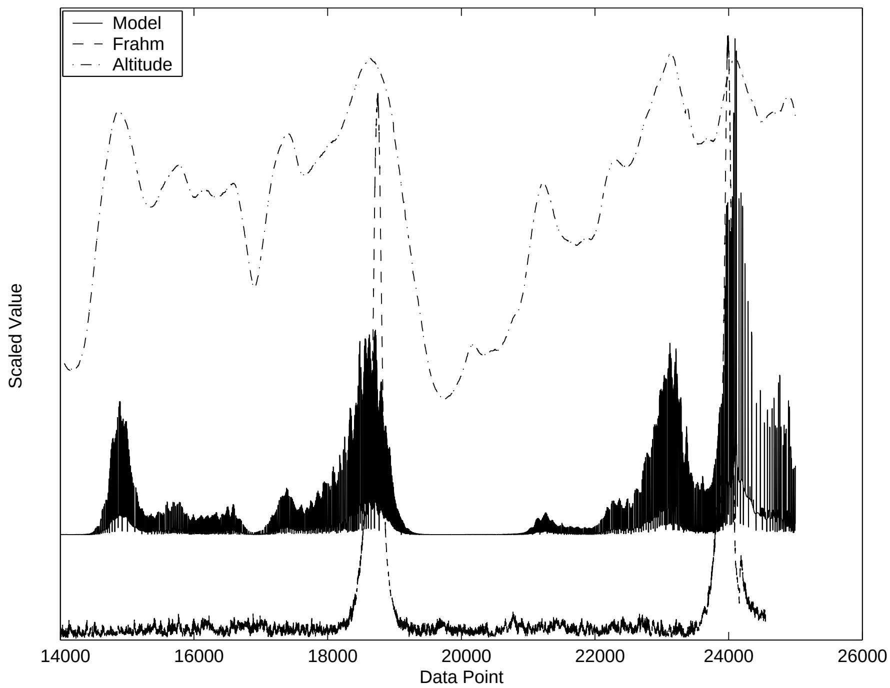

Figure 2: Comparison of MPMD model, Frahm solution and altitude of the flight path.

significant subspaces had a high correlation with the three components of the measured magnetic field and so can be assumed to be the earth's magnetic field. Figure 3 shows the fourth subspace of the data which has a high correlation with the components of the Frahm solution, shown in Figure 4. The remaining two subspaces have significantly small singular values and are assumed to be noise. Having only one subspace for the desired dipoles can be expected since the subspaces are orthonormal, by definition of the SVD (given in section 3), and the measure of three components of the dipoles are not independent. Thus the orientation of the magnetic moment as well as the direction to the dipole are properties of the mixing matrix D and logically if the subspace is a scaled magnitude of the dipoles then the structure of D corresponding to the subspace is the appropriately scaled magnetic field gradient equations. However, since D is constant, any movement in the directions of and to the dipole over the measuring period of Y will be averaged in some way. In order to track the movement of the subspace small windows of the data must be used. The singular value decomposition does

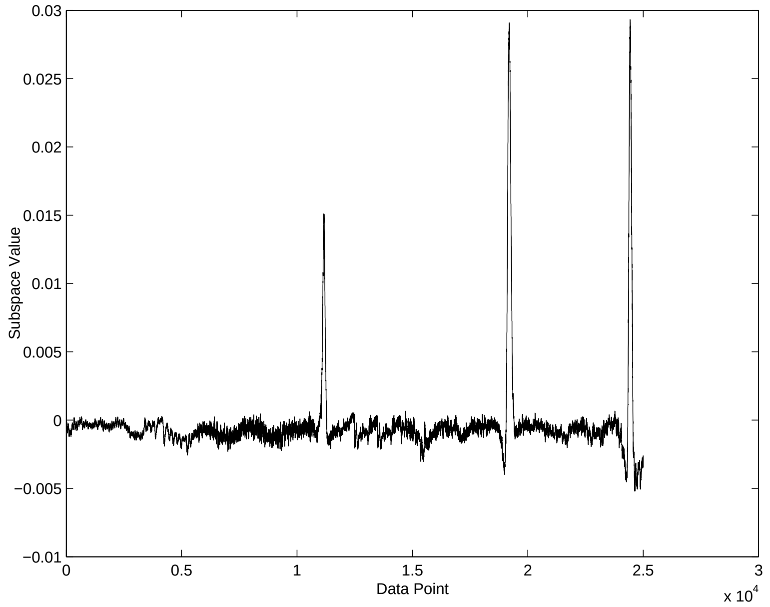

Figure 3: Subspace solution.

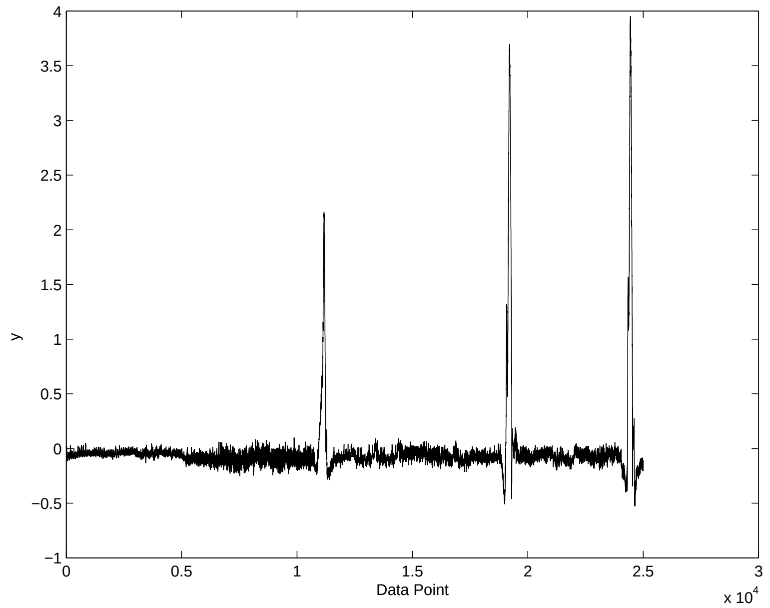

Figure 4: The y-component of Frahm solution.

not produce good results with the small windows. Present work is looking at using statistical and geometric subspace tracking methods, such as those by Miller et. al. (1995) and Srivasta (2000).

6 Conclusion

The problem posed at MISGSA 2004 was that of discriminating the magnetic dipoles of objects on the surface of the earth, possible UXO, from the effect of the earth's magnetic field in airborne magnetic field gradient data. A model of simultaneous equations using multiple data points was developed which hoped to discriminate between the effect of the earth's magnetic field and the possible UXO by solving for multiple dipoles. However the assumption that $\mathbf{r}$ is a constant vector, in order to simplify the model, meant that the change in the magnetic field gradients due to the change in the altitude of the sensor, results in an undesirable solution. Work, subsequent to MISGSA 2004, using subspace tracking has shown far more desirable results.

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