Jet Break-up (Elkem)

Elkem propose to produce drops of liquid metal from the break-up of a cylindrical jet by capillary instabilities (see figure 1). They require the drops to be large (1 to 5 cm in diameter) with a narrow size distribution. They also want the jets to break-up in a reasonably short distance and to have some means of controlling the drop size. The study group was asked to determine which dimensionless groups govern jet break-up, and what effect the design of the outlet nozzle has on the break-up of the jet. The problem of jet break-up has been studied intensively and the main activity of the study group was to survey the literature.

Dimensionless Groups

There are two dimensionless groups which control jet break-up. These are the fluid Reynolds number (Re) and the Weber number (We). For a jet of diameter d falling at velocity V these are given by

$$Re = \frac{Vd}{\nu}, \qquad We = \frac{\rho V^2 d}{\sigma}$$

where ν is the kinematic viscosity, ρ is the density, and σ the surface tension (actual values are confidential). For a jet of the metal considered, $Re \simeq 2000$ and $We \simeq 40$. In order to keep these two numbers constant, an equivalent flow with water would have a jet of diameter approximately 0.15 cm with a velocity of approximately 1.3 m s^{-1}.

Experimental Observations

The many experimental observations of jet break-up have been reviewed by McCarthy and Molloy (1974). The variation in the break-up length with the jet velocity is sketched in figure 2. For jet velocities below a critical velocity V_{crit} the jet develops a varicose instability (see figure 3) and breaks up into drops of uniform size (with some small satellite drops). The break-up length is proportional to the jet velocity, except at very low velocities when viscosity is important. At velocities above V_{crit} the nature of the instability changes from varicose to sinuous (figure 3) and the jet no-longer breaks into drops of uniform size, hence this mode of instability should be avoided. At very high velocities aerodynamic effects become important and the jet breaks up into a fine spray.

The critical velocity V_{crit} depends not only on the Reynolds and Weber numbers, but also on the nozzle design. Indeed it is possible to observe all three kinds of break-up (varicose, sinuous and fine spray) at the same Reynolds and Weber numbers with different nozzles. Short nozzles produce flows which have larger values of V_{crit} and are more stable to disturbances than longer nozzles. The velocity at the outlet of a short nozzle has a plug profile, whereas that at the outlet of a long nozzle has a fully developed Poiseuille profile. The Poiseuille profile subsequently relaxes to a plug profile through a radial transfer of momentum which destabilises the jet. The values of Re and We for the Elkem jet appear to be above V_{crit} for certain nozzle designs and so short nozzles are necessary to ensure varicose break-up.

Linear Stability Theory

The linear stability of a liquid jet to axisymmetric disturbances was first analysed by Rayleigh (1879). Subsequently Weber (1931) and Stirling & Sleicher (1975) have modified this theory to include respectively the effects of fluid viscosity and aerodynamic drag. The relative importance of viscosity to surface tension is proportional to $\sqrt{We}/Re$, which is small, and hence viscosity can be neglected. Aerodynamic effects are also negligible because the density of the liquid metal is much greater than that of air.

Rayleigh considers the growth rate, s, of an axial disturbance of wavelength λ. In cylindrical polar coordinates (r, z) in a frame moving at the unperturbed velocity of the jet, the jet radius r_j is taken to be

$$r_j = \frac{d}{2} + \hat{\eta} \exp\left(\frac{2\pi i z}{\lambda} + st\right).$$

The fluid velocity $\mathbf{u}$ and pressure p are then of the form

$$\mathbf{u} = \hat{\mathbf{u}}(r) \exp\left(\frac{2\pi i z}{\lambda} + st\right),$$

$$p = \frac{2\sigma}{d} + \hat{p}(r) \exp\left(\frac{2\pi i z}{\lambda} + st\right).$$

The linear growth rate, s, is found to be given by

$$s^2 = \frac{8\sigma}{\rho d^3} \frac{I_1(\xi)}{I_0(\xi)} \xi(1 - \xi^2), \qquad \text{where} \qquad \xi = \frac{\pi d}{\lambda}$$

and $I_n(x)$ is the modified Bessel function of order n. s has real solutions for $0 < \xi \leq 1$ and so all wavelengths above πd are unstable (shorter wavelength disturbances are stable as they increase the surface area). A plot of the dimensionless growth rate $(s\sqrt{\rho d^3/8\sigma})$ is shown by the solid curve in figure 4. The maximum growth rate occurs for $\lambda \simeq 4.5d$, and the break-up length is proportional to $d\sqrt{We}$ $(\propto V d^{3/2})$ and so increases linearly with V, as observed in experiment.

The Effect of Rotation

In order to consider the effect of rotation on the stability of the jet, we have modified Rayleigh's theory to include solid body rotation. In a frame which is translating and rotating with the undeformed jet, the linearised equations of motion are

$$\rho \frac{\partial \mathbf{u}}{\partial t} + 2\boldsymbol{\Omega} \wedge \mathbf{u} + \boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \mathbf{r}) = -\nabla p$$

$$\nabla . \mathbf{u} = 0$$

and the boundary conditions at the jet surface $(r = r_j)$ are

$$\mathbf{u}.\mathbf{n} = \frac{\partial r_j}{\partial t}$$

$$p = \sigma \nabla . \mathbf{n},$$

where $\mathbf{n}$ is the unit normal. We now seek a solution to the above equations of the form

$$r_j = \frac{d}{2} + \hat{\eta}\exp(ikz + st)$$
$$\mathbf{u} = (\hat{u}(r), \hat{v}(r), \hat{w}(r))\exp(ikz + st)$$
$$p = \frac{2\sigma}{d} + \frac{\Omega^2}{2}\left(r^2 - \frac{d^2}{4}\right) + \hat{p}(r)\exp(ikz + st),$$

where $\quad k = \frac{2\pi}{\lambda}.$

The equations of motion for the linear perturbation then become

$$\rho s\hat{u} - 2\rho\Omega\hat{v} = -\frac{\partial\hat{p}}{\partial r}$$
$$s\hat{v} + 2\Omega\hat{u} = 0$$
$$\rho s\hat{w} = -ik\hat{p}$$
$$\frac{1}{r}\frac{\partial}{\partial r}(r\hat{u}) + ik\hat{w} = 0.$$

Eliminating $\hat{v}$ from the top two equations we find that

$$\hat{u} = -\frac{1}{\beta^2\rho s}\frac{\partial\hat{p}}{\partial r}, \qquad \text{where} \qquad \beta = \sqrt{\left(1 + \frac{4\Omega^2}{s^2}\right)}$$

and substituting for $\hat{u}$ and $\hat{w}$ in the final equation gives

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\hat{p}}{\partial r}\right) - \beta^2k^2\hat{p} = 0.$$

$\hat{p}$ must remain bounded at $r = 0$, and so

$$\hat{p} = p_0 I_0(\beta k r)$$

The linearised boundary conditions are

$$\hat{u}(d/2) = s\hat{\eta}$$
$$\hat{p}(d/2) = -\frac{4\sigma}{d^2}\left(1 - \frac{k^2d^2}{4}\right)\hat{\eta},$$

and substituting for $\hat{u}$ and $\hat{p}$ we obtain

$$s^2 = \frac{8\sigma}{\beta\rho d^3}\frac{I_1(\beta\xi)}{I_0(\beta\xi)}\xi(1 - \xi^2), \qquad \text{where} \qquad \xi = \frac{kd}{2} = \frac{\pi d}{\lambda}$$

Note that since β depends on s that this is now an implicit expression for s. In the limit of no rotation, when $\beta \rightarrow 1$ we recover Rayleigh's formula.

In figure 4, the dimensionless growth rate for $\Omega = \sqrt{2\sigma/\rho d^3}$ is compared to that for $\Omega = 0$. As before, all wavelengths above πd are unstable, but rotation reduces the growth rate and increases the wavelength of the most unstable mode. Thus we would expect to find an increase in the break-up length with rotation.

That rotation stabilises axisymmetric disturbances may at first appear to be counter-intuitive, however the following physical argument shows it to be correct. In the unperturbed state, there is an axial pressure gradient which balances the centrifugal force. Now, if a section of the jet expands radially it must rotate slower in order to conserve angular momentum. This in turn reduces the centrifugal force, which no longer balances the pressure gradient, and leads to a net inward force on the jet.

In order to test this theory, we conducted some (qualitative) experiments to determine the effect of rotation on the break-up length. We observed that jet break-up for a rotating fluid is via a helical instability rather than an axisymmetric mode, and that the break-up length is shorter than for non-rotating fluid. Thus the above tnheory may be of little practical interest.

Satellite Drops and Forcing

Careful experimental observations (Bogy 1979) find that in varicose jet break-up small satellite drops are formed in between the main drops. These satellite drops are particularly undesirable in ink-jet printing and various one- and three-dimensional non-linear theories have been developed in order to understand their formation (these are reviewed by Bogy 1979). It is found experimentally that if the jet is forced to oscillate at a wavelength in the range 5 to 8 jet diameters, no satellite drops are formed provided that the amplitude of the forcing is sufficiently large. Forcing also has the desirable effect of reducing the break-up length and, by varying the disturbance wavelength, provides a means of controlling drop size.

In order to produce a disturbance of wavelength $6d$ forcing at approximately 20Hz is required. Various methods of forcing the jet were proposed:

Loudspeaker This is the method used in ink-jet printing, but because of the larger scale and harsher enviroment of liquid metal production this method may not be practical.

Air-jets This method involves the use of radially directed air-jets to create the disturbance. One possible drawback with this method is that it is likely to produce a fine spray. This method would also be difficult to implement if an array of nozzles were used. (In order to maintain current production levels approximately 50 nozzles would be required).

Mechanical Vibration This method has already been tried by Elkem in preliminary experiments. Our own qualitative experiments showed that this method was quite effective at reducing break-up length.

Magnetic Pinching This uses an alternating axial magnetic field produced by a coil around the nozzle outlet to create the disturbance. The magnetic field induces azimuthal surface currents producing an inward Lorentz force. As the magnitude of the magnetic field has two maxima per cycle the frequency of the magnetic field should be half that of the instability. This method has the advantage that it requires no moving parts, and for multiple nozzles the coils could be connected in series.

References

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Postscript

In a letter received after the Study Group, Patrick Parkes suggested a modification to Elkem's original procedure for producing drops (in which a jet of liquid metal falls onto a convex 'rock'; a sheet of liquid is formed which subsequently breaks up into droplets). He suggests using a concave stone (Fig.5) to improve the drop formation. Some work has been carried out on this configuration by G.N. Lance and published in the 1950's.

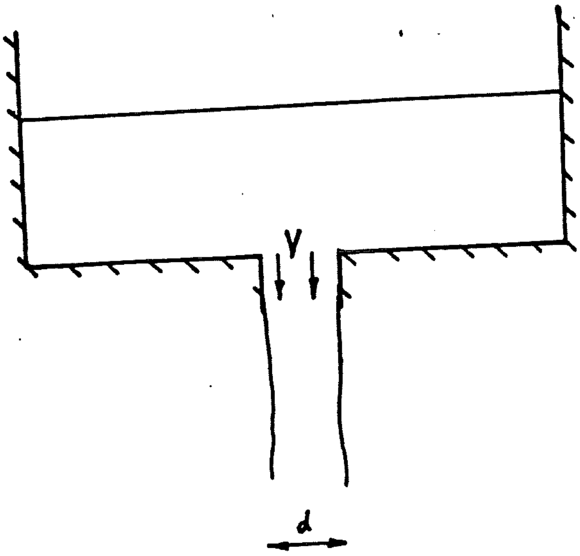

Fig. 1 Flow Geometry.

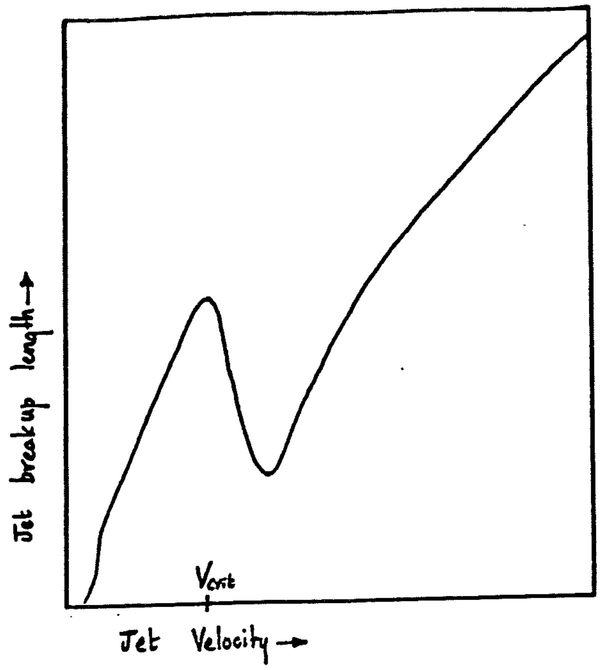

Fig. 2 Jet stability curve (schematic)

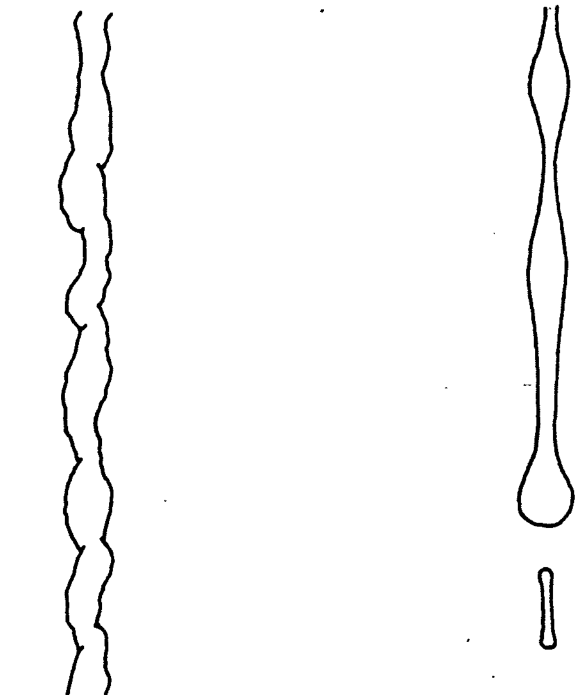

Fig 3. Modes of jet breakup

a) sinuous mode

b) varicose mode

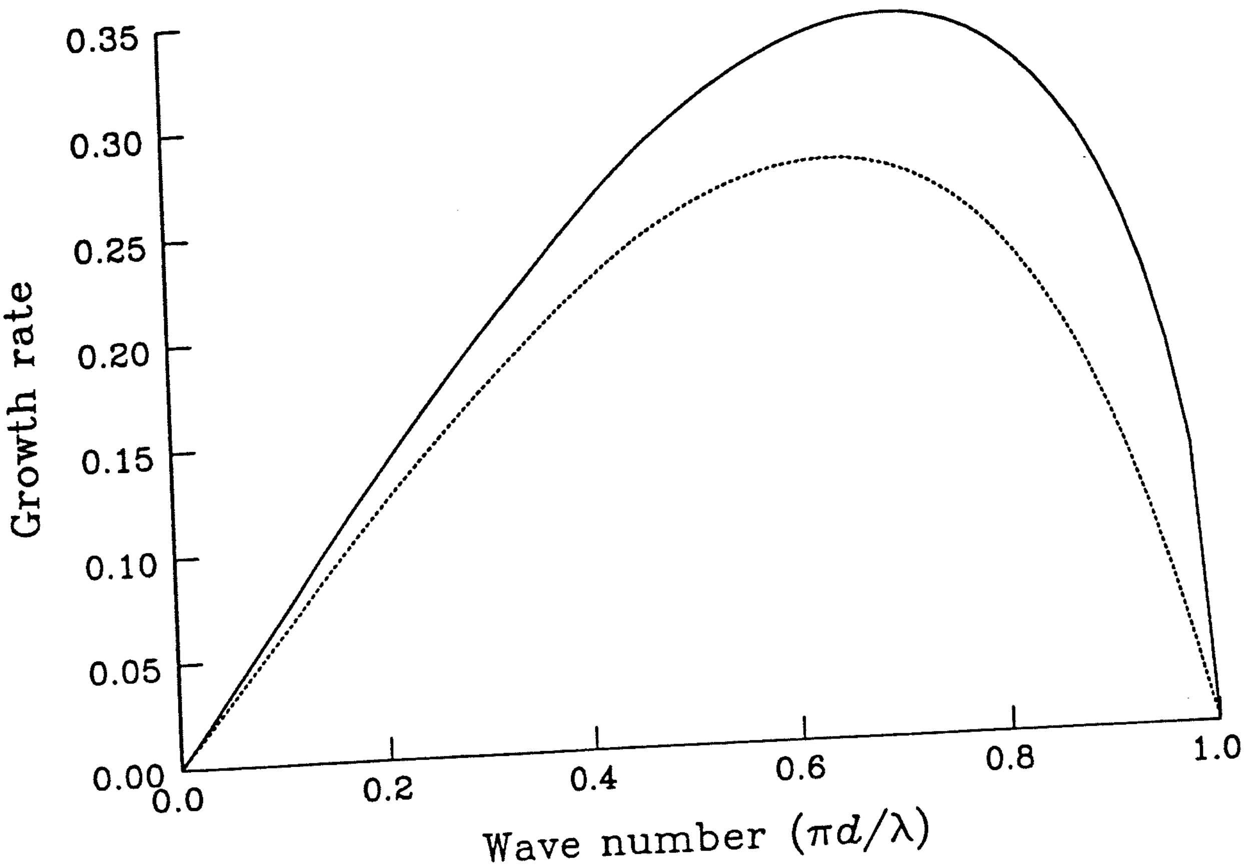

Figure 4: Plot of dimensionless growth rate ($s\sqrt{\rho d^3/8\sigma}$) against wavenumber: —— $\Omega = 0$, $\cdots\cdots$ $\Omega = \sqrt{2\sigma/\rho d^3}$.

Fig 5