<u>Shell - Annular Breakthrough</u>

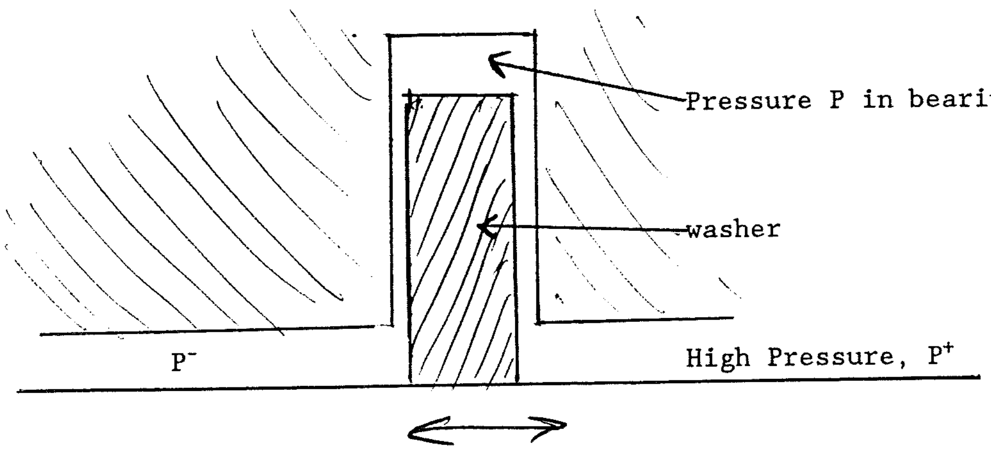
During the meeting several mechanisms for annular breakthrough were proposed, some of which incorporated a tilting motion of the 'washer' allowing gas escape. The model which received most support, however, treated the bearing as a 'canal lock' mechanism.

In this model the two competing effects

(i) large pressure gradients across the washer and

(ii) frictional forces (which move the washer against the pressure for part of the cycle)

are assumed to be of roughly the same order. Referring to figure 1 (a simplified model for one bearing, although our model could be extended to any number) there is a high pressure, P^+, to the right of the washer;



<u>Figure 1</u>

this pressure is a known function of time. To the left is an unknown and lower pressure P^-. Within the bearing cavity is an again unknown pressure P. The length scales in the diagram have been exaggerated: the gaps between the washer and walls are much less than other length scales . As the shaft moves to the right the washer is held against the right hand wall and gas can only travel through the left hand gap. This situation is reversed when the washer is held against the left hand wall. The competing effect of a high pressure gradient forcing the washer to the left means that the washer spends more time against the left hand wall than against the right - indeed if this gradient is sufficiently large the washer will never reach the right hand wall.

The bearing thus acts as a 'lock' since gas is typically stored in the cavity for part of the cycle before moving to the left.

The mass flux through the narrow gap will be assumed to obey a 'lubrication law'; that is, mass flux is proportional to pressure jump and the third power of the gap width. This law is a consequence of 'slow flow' and the Reynolds equation [Cameron, Basic Lubrication Theory].

We prescribe the high pressure, P^+, so that

$$P^+(t) = A + B \cos \omega t. \tag{1}$$

(since sufficient data was not available during the meeting neither A,B nor any of the constants to follow can be given numerical values).

The washer is assumed to move according to a 'wet friction' law

$$\frac{d}{dt}(x(t) - C \cos \omega t) = P^-(t) - P^+(t). \tag{2}$$

Here x(t) is the horizontal coordinate for the centre of the washer. We must have

$$|x| \le L \tag{3}$$

since the washer is constrained by the two walls. Equation (2) holds until $|x| = L$ when equation (2) is 'switched off' until it predicts dx/dt changing sign.

The mass flow into the bearing cavity is given by a flux from the right 'reservoir' minus a flux into the left 'reservoir' so that

$$\frac{dP}{dt} = D\{(P^+ - P)(L-x)^3 + (P^- - P)(L+x)^3\}. \qquad (4)$$

Note that we have implicitly assumed that the pressure increase, dP/dt, is proportional to the mass flux.

Finally, the equation for the pressure in the left reservoir is

$$\frac{dP^-}{dt} = - E(P^- - P)(L+x)^3 - \lambda P^- . \qquad (5)$$

Again this represents a lubrication flow. The last term in equation (5) is a 'leakage' term. In the absence of any movement of the shaft the pressure P^- decreases exponentially to zero (atmospheric). Equations (1) to (5) form our system. The system can be extended to any number of bearings, with P^+ for the second bearing being equal to P^- for the first etc.

Note that

$A > B$ (P^+ is always greater than atmospheric pressure)

and $D \gg E$ (the bearing cavity is much smaller than the left hand reservoir).

Equations (1) to (5) are easily solved numerically.

With $A = 0.5$, $B = 0.1$, $C = 1$, $D = 1$, $E = 0.2$, $\omega = 1$, $L = 1$, $\lambda = 1$ and x, P and P^- set at 1 initially the pressures $P^+(t)$ and $P(t)$ are shown in fig.2. In fig.3 is shown the corresponding $x(t)$.

The one bearing system as it stands does not exhibit a breakthrough instability but rapidly settles down to a periodic motion. This work could form the basis for a more sophisticated, many bearing analysis.

JND
JRK
AAL
DAS
PW.

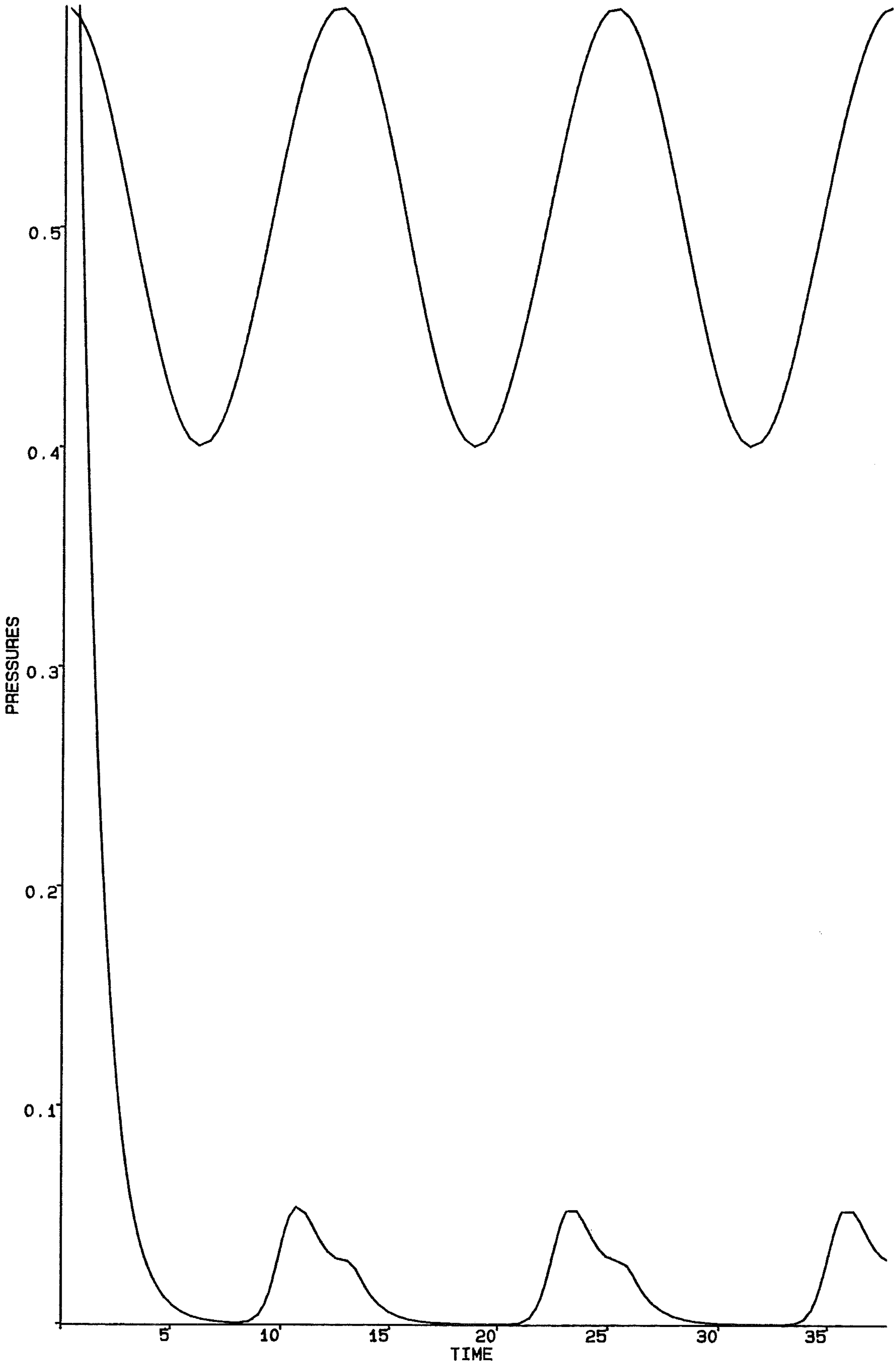


Fig 2

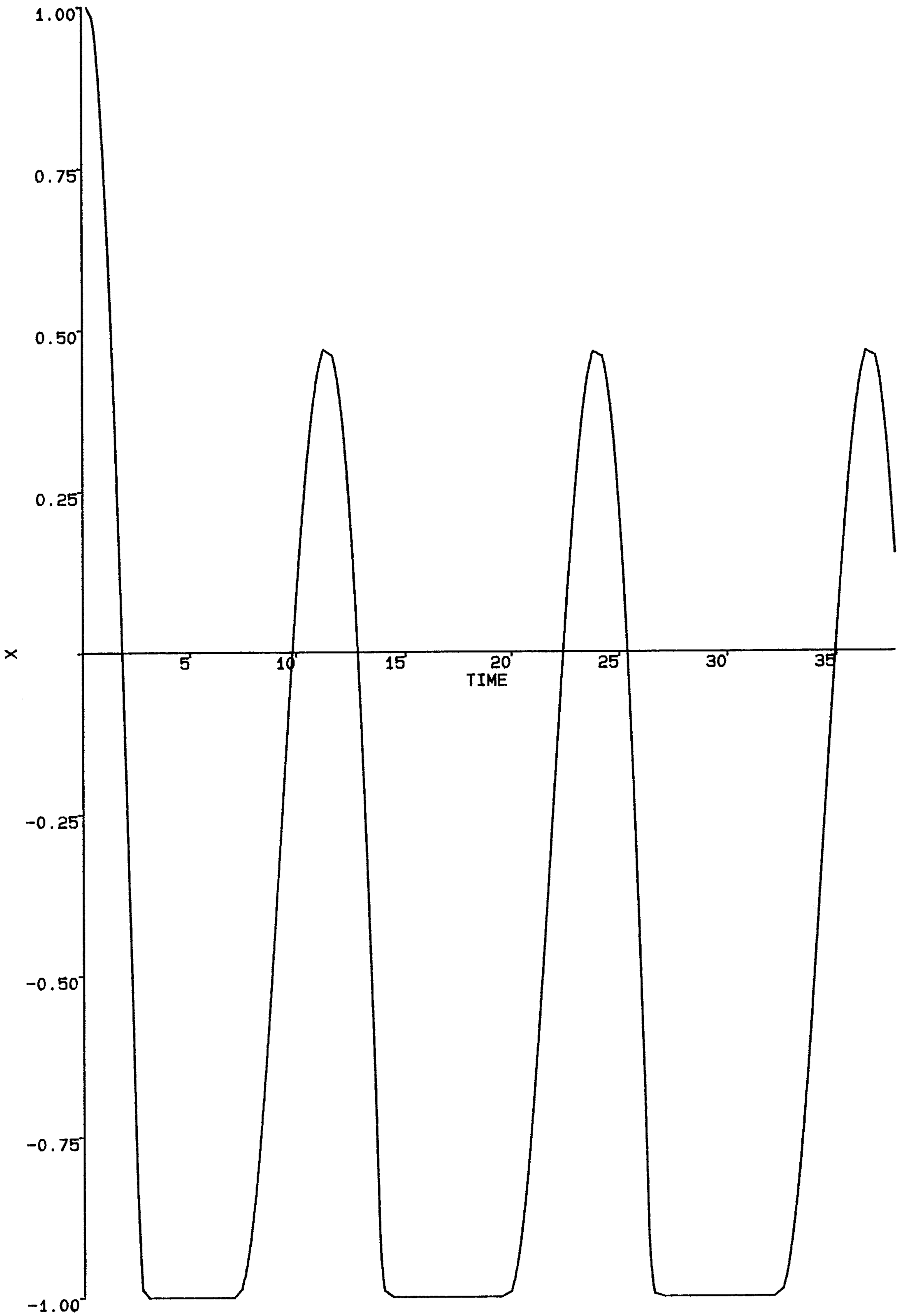


Fig 3