Detecting Geometric Faults from Measured Data

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Abstract

Manufactured artefacts such as major aircraft components (wings, fuselage, tailplane) are defined at the concept and design stages using a variety of methods, namely Computer Aided Design (CAD), NACA aerofoil definitions or purely analytical descriptions (polynomials, splines, etc.). At the end of the design and development the final manufactured artefact can only be verified if it is measured. The measured data is always a set of discrete points commonly described as a point cloud (x, y, z coordinates). Our goal here is to detect the faults from point cloud and reconstruct the measured object with as few points as possible. We can then insert this minimal reconstruction into CAD, and use analytical methods, to verify if the design intent was achieved: that is if the faults interfere with flight.

1 Introduction

To work from a simple set up, we assume that if the wing was manufactured correctly it would be the plane $y = 0$. This can easily be achieved if there exists an analytic description of what the wing should be, this is called the *design intent*. We say we *unwrap* the data by applying the inverse of the design intent to the real measured data of the manufactured wing. The real measured data is called the *point cloud* as it is a set of points with no regularity or even spacing. In this unwrapped configuration points that do not have $y = 0$ are due to manufacturing errors. We further simplify by working only in 2D, where if the wing was manufactured correctly it would be the line $y = 0$. This 2D data is often called a *slice* of the point cloud. The method we will describe below was applied to the data after the unwrapping process. The basic idea is to build a set of vectors, $\mathcal{B}$, or *fault vectors*, that represent the faults we wish to find. we take the unwrapped data and project it onto these fault vectors in $\mathcal{B}$ and then choose the fault vectors with the largest contribution to the projection. Finally we re-project the original unwrapped data onto these important vectors from $\mathcal{B}$. Hopefully at this point we will have a good representation of the wing along with the information of what and where are the faults. It is possible that to apply an analogous method to the full 2D surface wing could be infeasible, however, if we describe the faults on an arbitrary slice it should be possible to stitch together these slices to describe the total wing Faults.

2 A Simple Example

To design these fault vectors we need to first recognize what the faults will look like after the unwrapping. If the wing was produced with no flaws whatsoever the unwrapped data slice, to be referred to as data from now on, would be the line $y = 0$. We being with a simple example, we imagine that two plates were joined together with a slight error, but both were angled correctly, then the unwrapping would produce a step, see Figure 1 below. Note that errors are introduced in the measurement, which at times can be on the same scale as the fault we wish to find. To locate this step we construct a discrete basis for all possible steps and then project the data onto these steps. An example of a few of these basis can be seen in Figure 2. Mathematical speaking, each discrete step will be represented by $\mathbf{b}_0^n$ a vector in $\mathbb{R}^N$, where N are the number of points in the data slice, such that

$$b_0^n(m) := \begin{cases} c_0 & \text{if } m \geq n, \\ 0 & \text{if } m \langle n, \end{cases} \tag{1}$$

where c_0 is a constant that can de adjusted. Let f represent the data, then to find a best fit with these step basis' we need to find α_0^n's such that

$$\min_{\alpha_0^n} F(\alpha_0^1, \alpha_0^2, \dots, \alpha_0^N), \tag{2}$$

where

$$F = \|f - \sum_{k=0}^{N} \alpha_0^n \mathbf{b}_0^n\|, \tag{3}$$

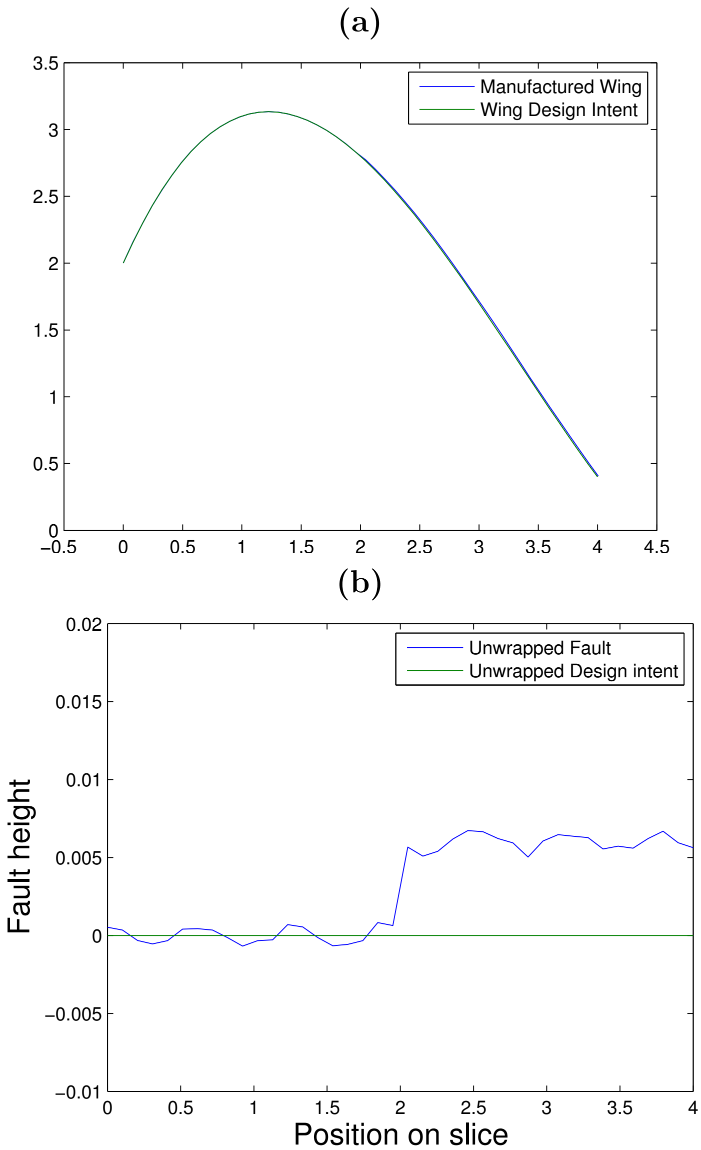


Figure 1: **(a)** shows both the wing design intent with the measurement of the manufactured wing, they are almost indistinguishable from one another, **(b)** shows the unwrapped design intent and the measured wing.

and $N =$ number of data points. We call the α_0^n's the coefficients of the step basis. This is accomplished by finding the critical point, i.e. find α_0^n such that

$$\frac{\partial F}{\partial \alpha_0^n} = 0, \text{ for every } n \text{ from } 1 \text{ to } N \,. \tag{4}$$

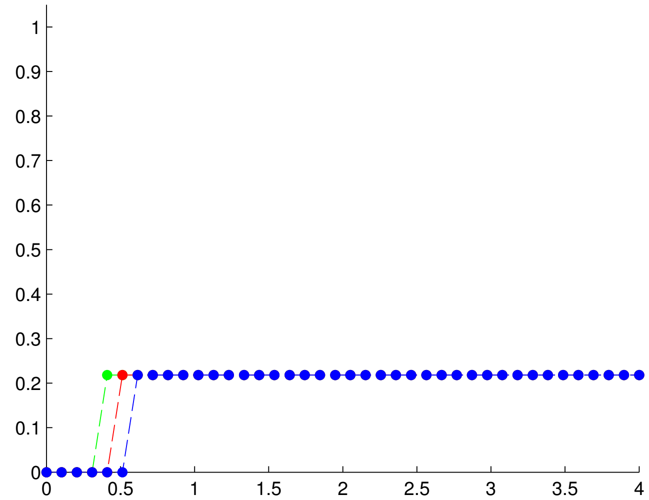


Figure 2: the balls represent the actual values of the step vectors.

Then we say that $\sum_n \alpha_0^n \mathbf{b}_0^n$ is the projection of f onto the step vectors. See Figure 3 for an example of the α_0^n's distribution for the fault in Figure 1. Clearly in this case the most relevant coefficient is $\alpha_0^{N/2} \approx 0.027$, the step that starts at $x = 2$, thus we choose only this coefficient and plot $\alpha_0^{N/2} \mathbf{b}_0^{N/2}$ with the original fault in Figure 4.

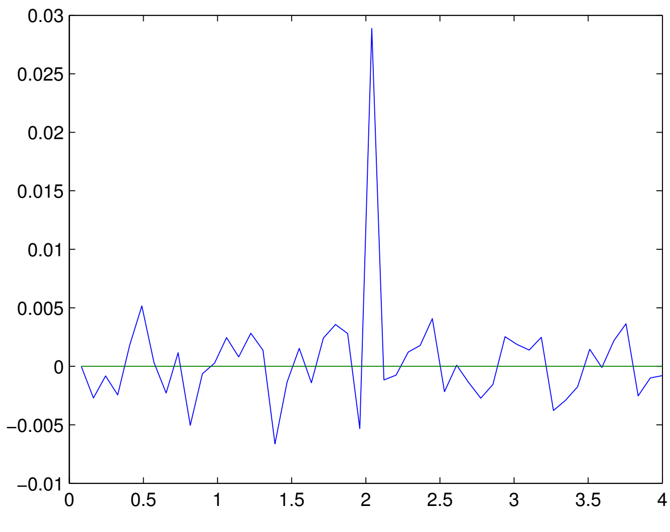


Figure 3: the value of the vector $\alpha_0^n \times x$.

Note that for this example, if we were to remove the errors of the measurement and compare the real fault with the projection of the measured

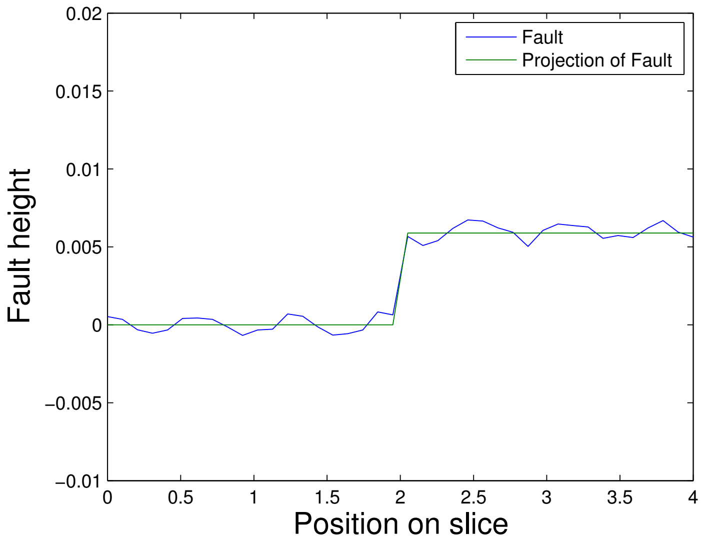


Figure 4: the projection of the measured fault compared with the measured fault itself.

fault: they would be almost identical, that is, taking only the most relevant coefficients of the projection we somewhat ignore the error measurements.

3 The Minimal Projection Method

The faults we wish to detect include plates being joined incorrectly, bolts and an unwated waveiness. To capture this range of faults we shall introduce polynomial fault vectors to be used in the projection method. Let the slice data be composed of the points $\{(x_0, f(x_0)), (x_1, f(x_1)), \ldots, (x_N, f(x_N))\} = (\mathbf{x}, f(\mathbf{x}))$ and N is the number of points. Then the order-k fault vector starting on the x_n is given by

$$\mathbf{b}_k^n(m) := \begin{cases} c_k(x_m - x_n)^k & \text{if } m \geq n, \\ 0 & \text{if } m \langle n, \end{cases} \tag{5}$$

where c_k is such that $\|\mathbf{b}_k^{N/2}\| = 1$. Later on we shall see that c_k must not vary with k so as to accurately locate the position of the faults, it will be a result of all the fault vectors of order-k having the same growth from their "starting" point, for example the step vectors will have the same height. For the record, c_k will be the typical L_2 norm of the vector

$$\mathbf{c}_k(m) := \begin{cases} (x_m - x_{N/2})^k & \text{if } m \geq N/2, \\ 0 & \text{if } m \langle N/2. \end{cases} \tag{6}$$

Given these vectors, one may deduce from the introduction the following method: project the data onto step vectors $\mathbf{b}_0^n$'s, then choose only the most relevant coefficients β_0^n's, store them and let $p_0 \leftarrow \sum_n \beta_0^n \mathbf{b}_0^n$, then remove this projection from the data $f \leftarrow f - p_0$. Now again project this remainder data f onto the fault vectors of order-1, the line basis, choose only the most relevant coefficients β_1^n's, store them and so on and so forth. At the end of this process all you need do is consult the coefficients β_k^n's to find what faults are present and their location. The major flaw in this method lies in attempting to carefully define "the most relevant coefficient", for the $\mathbf{b}_k^n$'s are not linearly dependent: typically we will need basis vectors up to $k = 4$, which would give us $4N$ fault vectors to represent a N-dimensional space. So the same fault will be clearly detected by different order fault vectors, it would then be unclear as to what criteria we could use to determine if a coefficient α_k^n is relevant enough. Another crucial feature is that we want the least number of coefficients to characterize the faults. To summarize, we want to represent the data in terms of all these basis vectors while choosing only the most relevant coefficients, mathematically speaking, we want to find α_k^n's such that,

$$\min_{\alpha_k^n} \; F(\alpha_0^1, \alpha_0^2, \ldots, \alpha_K^N), \tag{7}$$

where

$$F(\alpha_0^1, \alpha_0^2, \ldots, \alpha_K^N) = \Big\| \sum_{n=1}^{N} \sum_{k=0}^{K} \alpha_k^n \mathbf{b}_k^n - f \Big\|^2 + \omega \Big\| \sum_{n=1}^{N} \sum_{k=0}^{K} \alpha_k^n \Big\|^2 \tag{8}$$

where ω is a weight that can be chosen. To minimize we equate

$$\frac{\partial F}{\partial \alpha_i^j} = 0$$

for every i and j, resulting in

$$\begin{aligned}\frac{\partial F}{\partial \alpha_i^j} =& \langle \mathbf{b}_i^j, \sum_{n=1}^{N} \sum_{k=0}^{K} \alpha_k^n \mathbf{b}_k^n - f \rangle + \omega \alpha_i^j = 0 \implies \\ & \sum_{n=1}^{N} \sum_{k=0}^{K} \left(\omega \delta_{ik} \delta^{jn} + \langle \mathbf{b}_i^j, \mathbf{b}_k^n \rangle \right) \alpha_k^n = \langle \mathbf{b}_i^j, f \rangle,\end{aligned} \tag{9}$$

where δ_{ik} and δ^{ik} equal 1 if $i = k$ and zero otherwise. If we denote

$$\boldsymbol{\alpha} = (\alpha_0^1, \alpha_0^2, \ldots, \alpha_0^N, \alpha_1^1, \alpha_1^2, \ldots, \alpha_1^N, \alpha_2^1, \ldots, \alpha_K^N)^T,$$

then equation (9) can be rewritten as a matrix equation,

$$\left(\mathbf{B}^T \mathbf{B} + \omega \mathbf{I} \right) \boldsymbol{\alpha} = \mathbf{B}^T f, \tag{10}$$

where,

$$\mathbf{B} = \begin{pmatrix} \mathbf{b}_0^1 & \mathbf{b}_0^2 & \ldots & \mathbf{b}_0^N & \mathbf{b}_1^1 & \mathbf{b}_1^2 & \ldots & \mathbf{b}_1^N & \mathbf{b}_2^1 & \ldots & \mathbf{b}_K^N \end{pmatrix}, \tag{11}$$

and $\mathbf{I}$ is a KN-dimensional identity matrix. Note that there is always a ω large enough such that $\mathbf{M}$ is invertible. For the fault vectors we have defined in equations (5) choosing $\omega = 1$ will guarantee that $\mathbf{M}$ is invertible. Hence,

$$\boldsymbol{\alpha} = \left(\mathbf{B}^T \mathbf{B} + \omega \mathbf{I} \right)^{-1} \mathbf{B}^T f. \tag{12}$$

If $\mathbf{B}$'s colomns was composed only of fault vectors of order-k and $\omega = 0$, then the above formula would give the coefficients for the projection of the data onto the order-k fault vectors.

Let us examine the distribution of the α_i^j's for the data given in Figure 5. Using equation (12) we can find $\boldsymbol{\alpha}$, shown in Figure 6. Note that different fault vectors perceive the same events. We choose the most representative coefficients as being the largest local maximum of vector $|\boldsymbol{\alpha}|$, the absolute

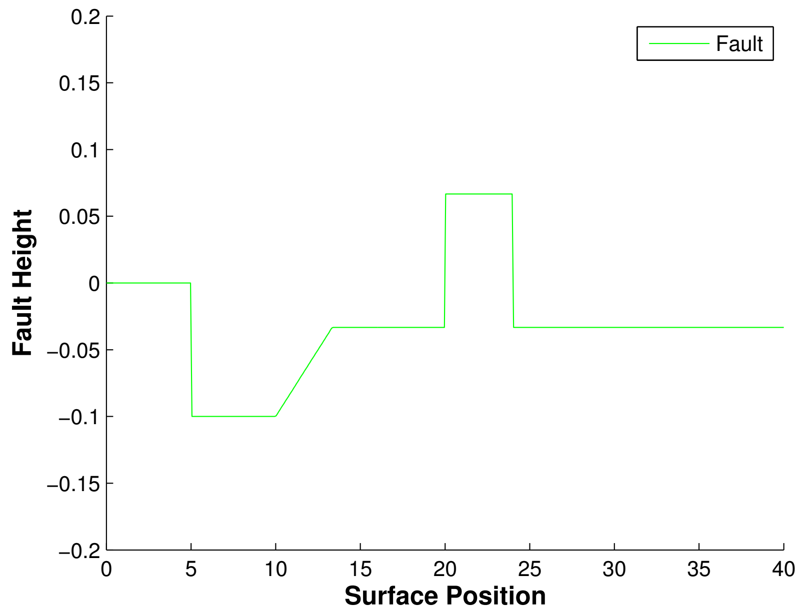


Figure 5: A Fault example.

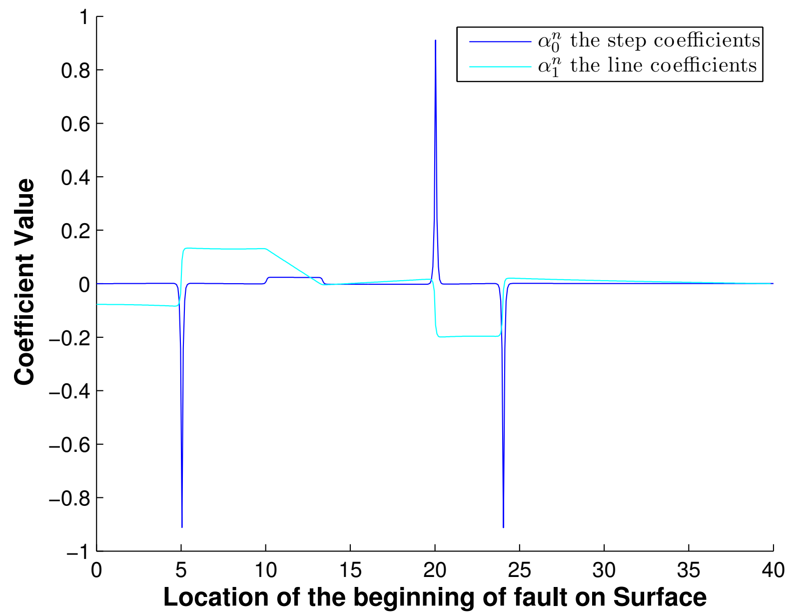


Figure 6: step and line fault Coefficients, where the line coefficients have been magnified 20 so as to easily see them.

value of the components of the coefficient vector $\boldsymbol{\alpha}$, this is done separately

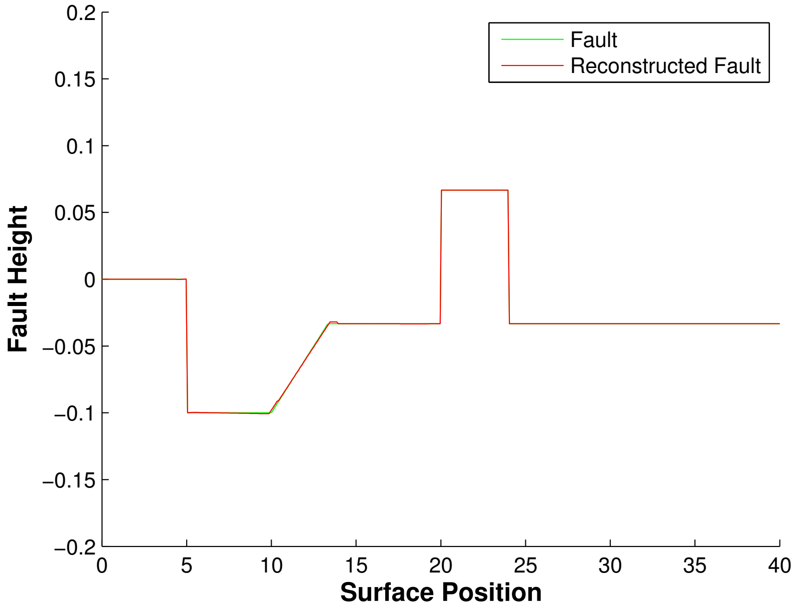


Figure 7: fault and reconstructed fault.

for the coefficients of each fault vector order, then we project the data onto the fault vectors of these coefficients. Note, if c_k's in equations (5) were not constants then the critical points in Figure 6 would be dislocated and would not represent the position of the fault. The result, which we call the reconstructed fault is shown in Figure 7 below.

When measurement errors are present it is necessary to use higher order fault vectors, defined in equations (5), to obtain better results. Figure 8 shows an example of a recontructed fault where the data include measurement errors. When dealing with measurement errors possible the best procedure would be to smooth out the data, though we can apply the method directly to the data.

For the minimal projection method presented below let,

K is the largest order fault vector defined by equations (5),
M_k the number of representative coefficients for the fault vectors of order-k,
$|\boldsymbol{\alpha}_k| = (|\alpha_k^1|, |\alpha_k^2|, \ldots, |\alpha_k^N|)$,

then the method is illustrated by the algorithm below,

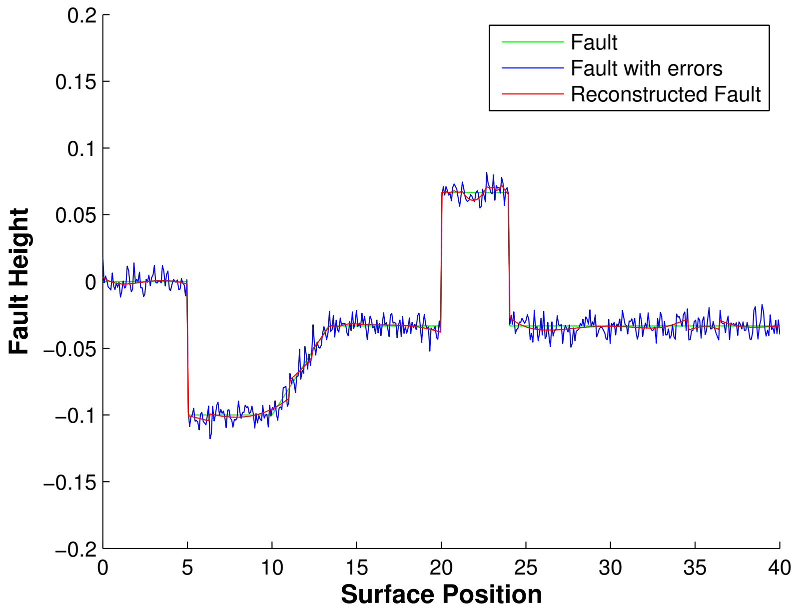


Figure 8: fault with measurement errors and reconstructed fault.

	Minimal Projection Method
1.	Choose K.
2.	Choose each M_k for k from 1 to K.
3.	For data f use equation (12) to obtain $\boldsymbol{\alpha}$.
4.	Let $\boldsymbol{\beta}_k$ be the M_k largest local maximum of $\lvert\boldsymbol{\alpha}_k\rvert$.
5.	Let Pf be the Projection of f onto the fault vectors of all the $\boldsymbol{\beta}_k$'s.
6.	Check that Pf accurately approximates f, if not increase the M_k's that need more representatives and go back to item 4.

4 An Example in Proportion

With the help of Dr. Richard Burguete we designed some synthetic data that is closer to the scales and possible events that occur on a slice of measured data from a wing. The chosen data imitates a bolt head, a undesired waviness and a misaligned plate see Figure 9. If we consider the extension of this slice to be 40cm then: the bolt head protrudes 0.04mm above the plate, the wave's amplitude is 0.04mm and the plates are misaligned by 0.027mm. We have used only 1500 points on the slice so that there are 10 measured points on the bolt's head.

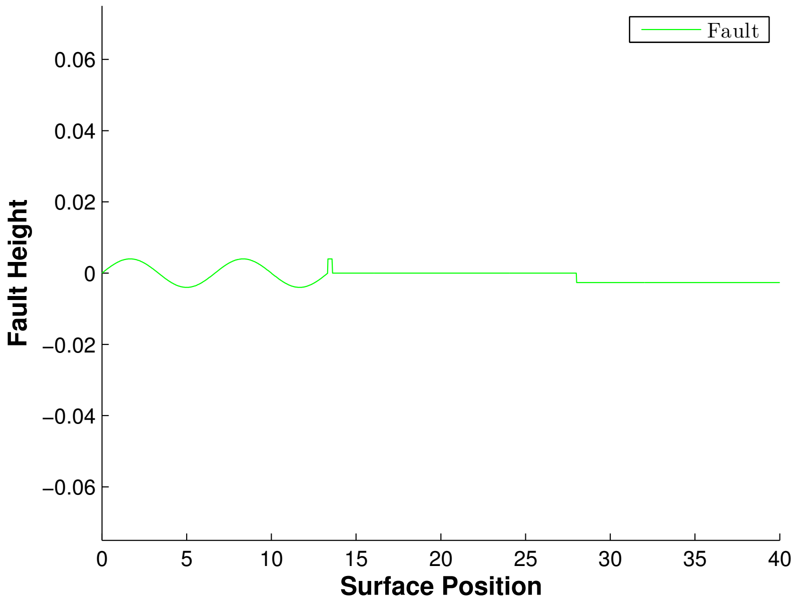


Figure 9: synthetic data with a bolt head, a wave and a misalignment of two plates.

We have modelled the measurement errors by Gaussian white noise with an average amplitude equal to a third of the bolt head height. Now we apply the minimal projection method to this data, that contains the measurement errors, using fault vectors up to order-5, defined by equations 5, and choose the 5 most representative coefficients for each order of fault vector, i.e. $M_0 = M_1 = M_2 = M_3 = M_4 = M_5 = 5$. We compare the output of the method (reconstructed data), the data and the original fault in Figure 10 and onwards.

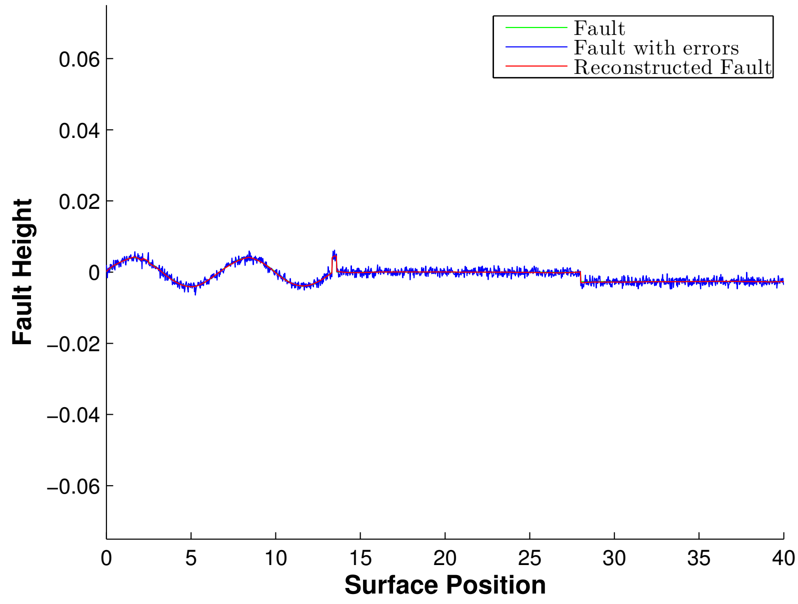
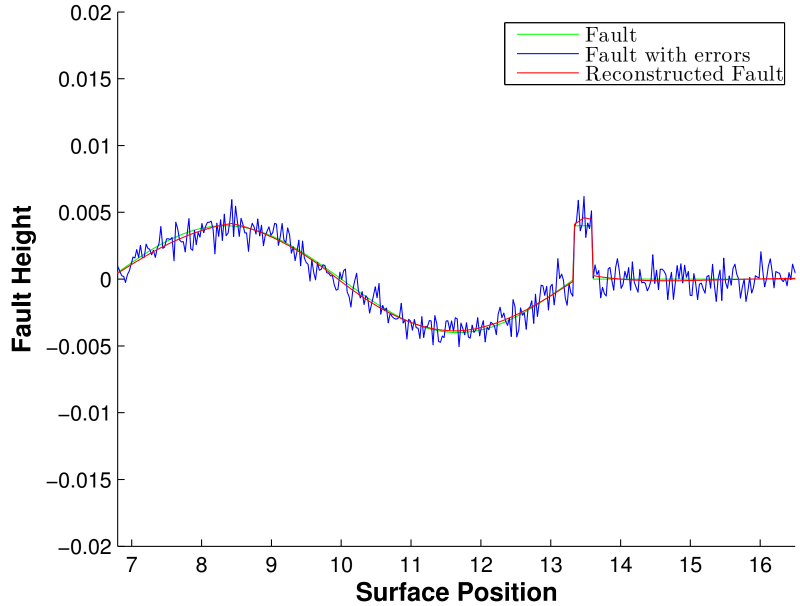
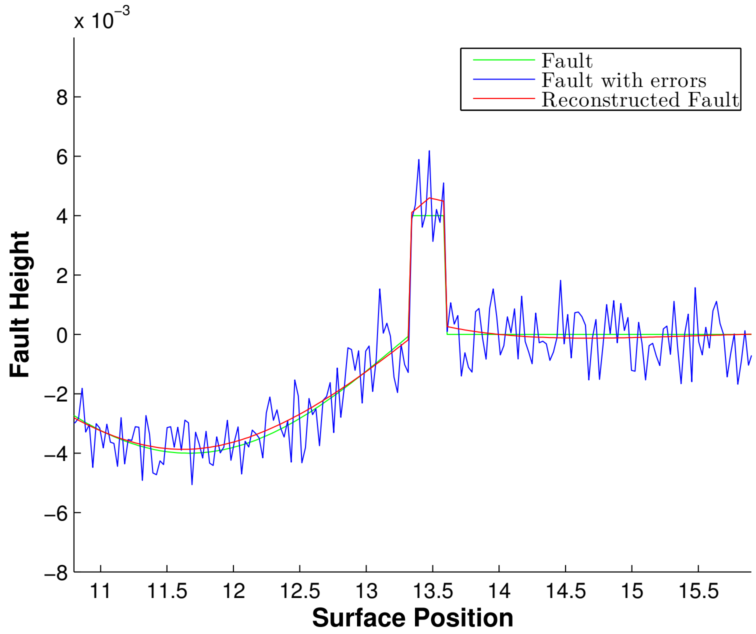
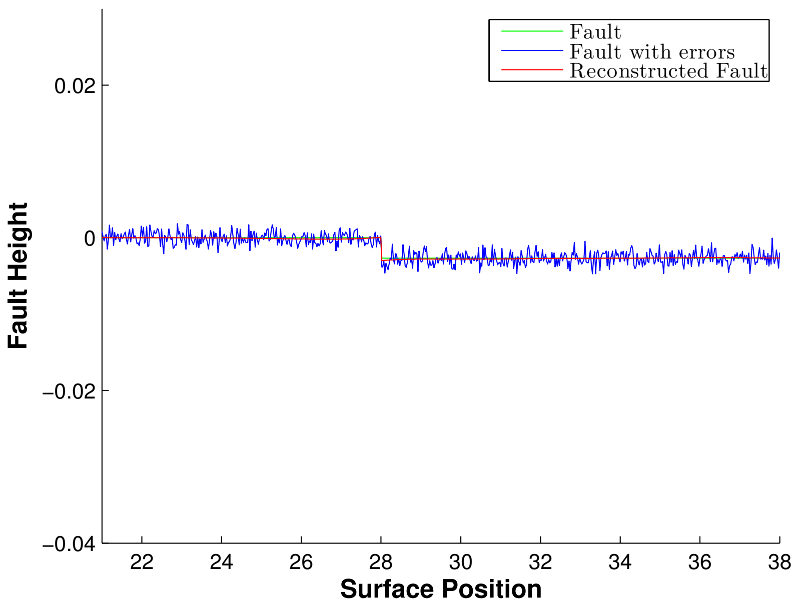
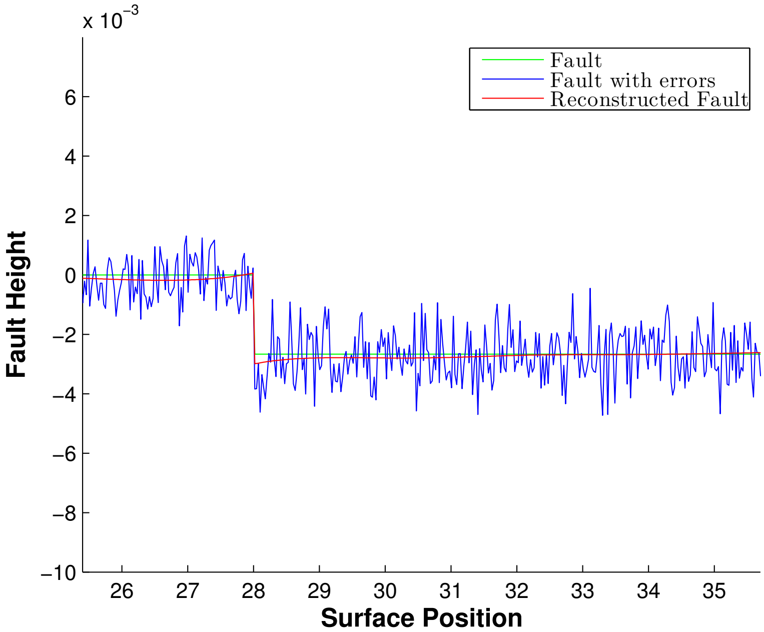


Figure 10: the green line is the fault, the blue is the measured data and red the output of the method.




x 10^−3
Fault
Fault with errors
Reconstructed Fault
Fault Height
Surface Position
8
6
4
2
0
−2
−4
−6
−8
11
11.5
12
12.5
13
13.5
14
14.5
15
15.5


Fault
Fault with errors
Reconstructed Fault
Fault Height
Surface Position
0.02
0
−0.02
−0.04
22
24
26
28
30
32
34
36
38



5 Conclusion

We feel that this minimal projection method works very well and we predict it would have better results if the measured data was to be treated to smooth out these error measurements. Another possible method is to project the data onto the fault vector of different orders, one-by-one, record all the local minimums and maximums of the coefficients α_n^k's called $\boldsymbol{\beta}$, then finally project the data onto the fault vectors of all coefficients in $\boldsymbol{\beta}$.