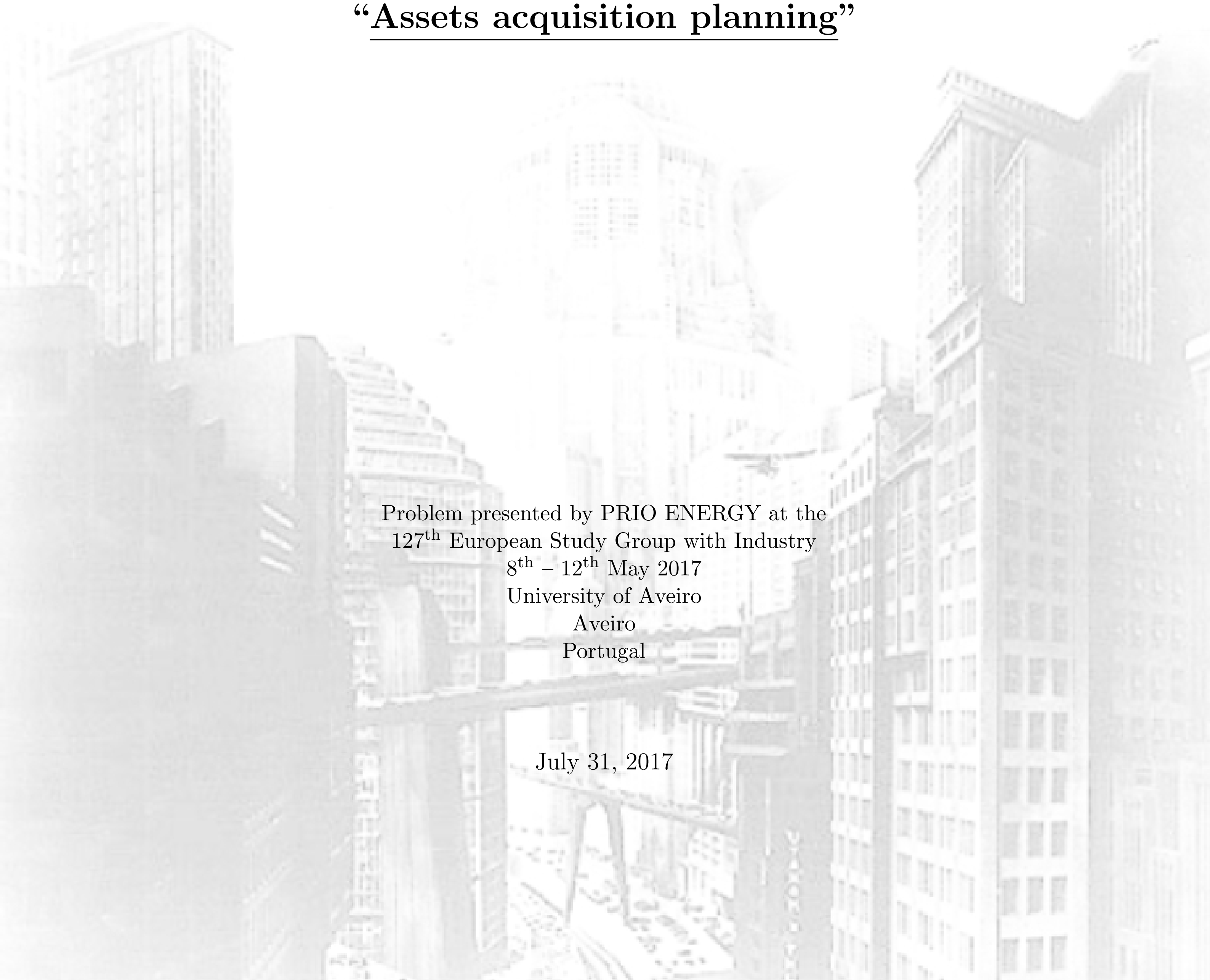
Portuguese Study Groups' Reports

Report on
"Assets acquisition planning"



Problem presented by PRIO ENERGY at the
127th European Study Group with Industry
8th – 12th May 2017
University of Aveiro
Aveiro
Portugal

July 31, 2017

Problem presented by: PRIO ENERGY

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Executive summary

At the 127th European Study Group with Industry an energy sector company proposed an industrial challenge that consisted on the asset acquisition planning for its liquefied petroleum gas (LPG) cylinder business, one of the most recent business areas in the company. This business area is still in a growing phase and to satisfy the market needs and assure a sustainable growth a very tight control of the main assets, the LPG cylinders, is of paramount importance. Therefore, a detailed planning of all the assets acquisition is required, taking into consideration several variables: sales growth rate, seasonality, cylinder rotation and corresponding return rate to the filling plant.

The challenge was to develop a model for the assets acquisition planning. In order to tackle this challenge, it was necessary to forecast the demand. For that purpose, time series techniques were used, in particular, moving averages and exponential smoothing. The results show that the seasonality does not explain all the variation of the demand, therefore it is necessary to use a model that would consider other possible explanatory variables.

According to several authors, gas consumption may be influenced by several aspects, such as, atmospheric temperatures, heliophany (a measure of the day luminosity), wind, relative humidity, rains, minimum and maximum temperatures, demand in previous periods, and prices.

The forecast of bottled propane gas sales and return rate was also addressed through multivariate linear regression. Regression models for the monthly number of bottles of types A and B were obtained, having presented good percentages of explained variability with the variables under study.

The main goal of the challenge, the acquisition plan, was addressed using inventory models with reverse logistics. Several deterministic approaches have been considered to enable different aspects in the framework. A new inventory model has been developed to contemplate the three possible destinations of returned bottles: cleaning, requalification, or disposal. The models were implemented in Excel and can be tested, using PRIO estimates of holding costs and fixed setup costs, and the forecasts of sales and return rate computed previously.

Keywords Industrial Mathematics, LPG bottles demand, time series, exponential smoothing forecast, moving averages, multivariate linear regression, inventory model, EOQ model, reverse logistics.

Contents

1 Introduction

PRIO ENERGY is a Portuguese company which started its activity in 2006 focusing in the production and distribution of biofuel. Since then, the company has been continuously growing, extending its business areas to other fuels, now operating at a national level.

The liquefied petroleum gas (LPG) cylinder business started its activity in 2012, and since then it has experienced a continuous growth. In this business, the LPG cylinder is the main asset and a correct planning of its needs is critical. The company currently commercializes propane gas and has two types of cylinders, henceforth named A and B, with different capacity.

PRIO ENERGY wants to find a model to forecast the demand of each type of LPG cylinders. These forecasts are crucial for the company to define an assets acquisition plan, i.e., to determine the amount of LPG cylinders to acquire, and when to acquire them.

The approach that the ESGI workgroup used to find the solution of this problem can be divided in three phases. First, it is necessary to forecast demand, sales and the return of LPG bottles. This data can be used in an Economic Order Quantity (EOQ) model for inventory management. At last, because it is necessary to consider the return rate of LPG bottles, reverse logistic models and closed loop supply chain models were explored.

This report is organized in the following way. We devote the next section to review the approaches used in literature for these type of problems. In Section 4 we determine the sales forecast for bottle propane gas for the Portuguese market and for PRIO ENERGY. Then, in Section 5, the forecast of the return rate of the LPG cylinder is addressed. Section 6 is devoted to explore several inventory models with reverse logistics for planning the assets acquisition and we end this report by concluding and presenting some recommendations in Section 7.

2 Review of the approaches used in literature

A recent review on forecasting natural gas consumption, mostly applicable in this challenge, is by Soldo [22] where the author divides papers in the literature according to forecasting area, forecasting horizon, used data, and forecasting tools. For the national/regional level, Soldo suggests for future research the use of classic forecasting tools combined with optimization.

The most common models to forecast demand and sales are time series approaches, either using exponential smoothing methods or autoregressive models.

A moving average is a technique that calculates the overall trend in data and is very useful for forecasting short-term trends. It is the average of a number of time periods and it is called moving because as a new demand number is obtained for a time period, the oldest number in the set falls off, keeping the time period locked. Exponential smoothing focuses more on most recent data, giving more weight to the most recent observations. Exponential smoothing has been extensively used in forecasting [10, 12, 14, 13, 24, 26, 27].

Simple exponential smoothing, however, does not work well when there is a trend in the data. In these cases, double exponential smoothing (Holts method [13] being the most common example) can be used to forecast. The method requires separate smoothing constants for the level smoothing factor α, and for the trend smoothing factor β.

While time series models try to forecast future values taking into account only the patterns in the historical data, they do not consider any factors that can influence the future. For a more complete introduction on time series see, for example, [4, 11]. On the opposite, Linear Regression techniques [9] consider several covariates to model the data, i.e., they allow deriving an equation which can be used to predict future values (for the dependent variable) given independent variables' data. In simple linear regression there is a single independent variable, while in multiple linear regression (MLR) there are several independent variables [8, 17, 16].

In order to apply the latter approach it is therefore important to try to determine which variables may influence gas consumption.

In [25], Vitullo et. al. look at the financial implication of forecasting natural gas, the nature of natural gas forecasting, and the factors that impact natural gas consumption. The authors suggest the most important factors influencing gas consumption are:

- Temperature
- Prices
- Wind
- Demand on the previous month
- Humidity
- Precipitation
- Luminosity

The authors in [25] also present a survey of the mathematical techniques and practices used to model natural gas demand. These authors argue that the most common mathematical modelling techniques used to forecast daily demand are MLR and artificial neural networks (ANN) (for further details the reader is refered to [24]).

Concerning cylinder returns, Carrasco-Gallego and Ponce-Cueto [5] have addressed returns forecasting techniques in the LPG sector of a country with similar characteristics, Spain. The authors conclude that when a direct replacement policy is in place (as is the case in PRIO ENERGY's challenge), the monthly forecast of returns is similar to the monthly forecast of sales, not adding significant value the use of dedicated forecasting models for returns. Note, however, that in the case addressed by these authors the market share of the company analysed was stable at nearly 80%. Therefore, as with gas demand forecast, time series and regressions models were explored in this challenge for cylinder returns.

Once the forecast of demand, sales and return of LPG cylinders is determined, an economic order quantity (EOQ) model can be used for inventory management [3, 7, 19].

The EOQ model is an attempt to estimate the best order quantity by balancing the conflicting costs of holding stock and of placing replenishment orders. The effect of order quantity on stock-holding costs is that, the larger the order quantity for a given item, the longer will be the average time in stock and the greater will be the storage costs. On the other hand, the placing of a large number of small-quantity orders produces a low average stock, but a much higher cost in terms of the number of orders that need to be placed and the associated administrative and delivery costs [19]. A drawback of this approach is that it does not take into account reverse logistics, which in this challenge (i.e. the return of cylinders) plays an important role.

In [18], Richter extended the EOQ model to allow the incorporation of used products, which were repaired and incorporated in production. It assumes a stationary demand in a model with two shops, where the first shop is producing new products and repairing products used by the second shop. Also considering deterministic demand and reverse logistics is the model proposed by Teunter [23], differing in allowing to consider varying disposal rates and disaggregating holding costs. The inner workings of these EOQ extensions are provided in greater detail in Section 6.

Other developments on the EOQ model are by Alivoni et al. [1]. They propose a stochastic model where production or purchase of new items integrates product reuse, in order to identify the need of placing a production/purchasing order to avoid stock-out situations.

3 Dataset and exploratory analysis

The dataset provided by the company included information about sales, return number of cylinders, operational stock of assets, total number of assets in the market, of the two types of propane cylinders (A – small bottles and B – large bottles), between January Year 0 (Jan/Y0) and December Year 1 (Dec/Y1). The forecast for sales during Year 2 was also provided by the company. Note that, for confidentiality reasons, the data used in this work was masked (with a conversion factor). In this report, the Year 0 corresponds to 2015 and Year 2 corresponds to 2017.

Additional data about consumption of propane in Portugal were also collected from the Portuguese Association of Petroleum Companies – APETRO[1]. Meteorological data such as temperature, humidity, precipitation, and wind were collected from OGIMET[2] and from the Portuguese Institute for Sea and Atmosphere – IPMA[3], respectively.

Figure 1 and Figure 2 show this data.

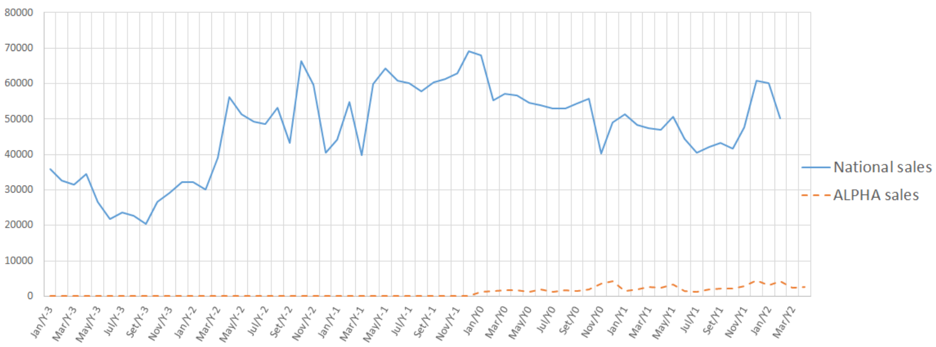


Figure 1: Comparison between national sales and PRIO ENERGY sales.

Figure 1 shows a general increase in the sales of PRIO ENERGY, however, it still represents a small percentage of the national market. Furthermore, the national and PRIO ENERGY sales of propane have different behaviours, thus in principle this is not a good indicator to forecast the company's sales when based on the national ones.

Figure 2 depicts the values of propane national sales and, simultaneously, the average temperature[4] in Portugal in the same period is shown. Since propane gas is used mostly for cooking and water heating, it is expected that whenever temperature decreases there is an increase in gas consumption. However, from Figure 2, this is not always true. In fact, for January Year -1 there is a decrease of the temperature and also a decrease of the consumption of gas.

In order to study the existence of seasonality in sales of type A and B cylinders, Figure 3 presents the sales for the three years. In this figure an increasing linear trend of PRIO ENERGY's sales, due to the company's market expansion, is observed. There is also some indicators of seasonality, because over the years the variations appear to be similar. In order to

[1] `http://www.apetro.pt`
[2] `http://www.ogimet.com`
[3] `https://www.ipma.pt`
[4] The air temperature in ($^\circ C$) was multiplied by a constant factor for a better comparison with the sales.

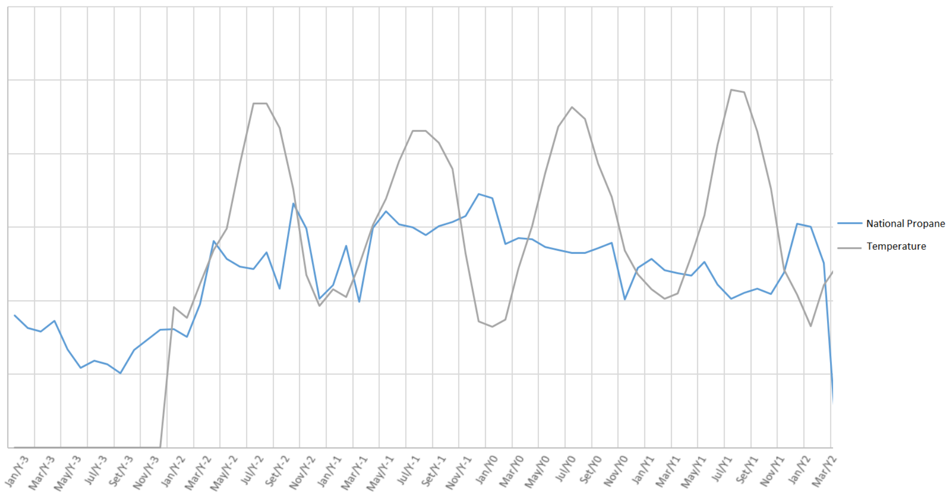


Figure 2: Average temperature and national sales.

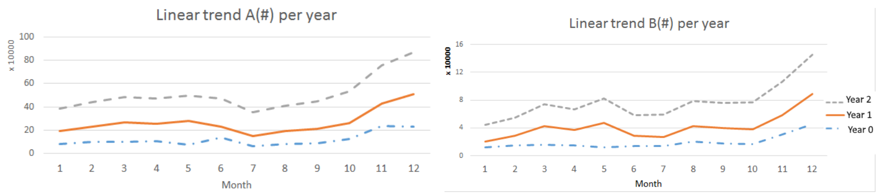


Figure 3: PRIO ENERGY sales of type A and B cylinders.

analyse the seasonality, moving averages seasonality coefficients (e.g. [26, 13]) and exponential smoothing forecasts (for review see e.g. [10, 12, 14]), are calculated in the next Section.

4 Bottled propane gas sales forecast

In order to make a good acquisition plan, forecasting demand and sales is of the foremost importance. This Section is devoted to understand the variation of LPG sales and explore several methods for forecasting the future sales of bottled propane gas.

4.1 Time series models

4.1.1 Estimating seasonality coefficients

To study the seasonality of the data, seasonality coefficients for total propane sales of PRIO ENERGY (in tons); sales of type A and B PRIO ENERGY assets; and sales of butane, propane and total in Portugal were calculated.

Let us consider, without loss of generality, PRIO ENERGY sales of propane between January Year 0 and December Year 2. The number of observations is given by $N = k \times s$, where k is the number of years and s the number of periods in the year, i.e. months. In this case $N = 3 \times 12 = 36$. Using the centered moving averages method, it is possible to calculate 12 seasonal indexes (one for each month) that express the amount of sales in each month that are superior (or inferior) to the global mean sales. Using the multiplicative method, the seasonal indexes work as a percentage. The non-normalized estimates of the seasonal component at time i of each year are:

$$\bar{S}_i = \frac{1}{k-1} \sum_{j=1}^{k} S^*_{i+s(j-1)} \ , \qquad i = 1, 2, \ldots, s,$$

with

$$S^*_t = \frac{X_t}{M_t},$$

where M_t are the centered moving averages of the sales series X_t:

$$M_t = \frac{1}{s}\left(\frac{1}{2}X_{t-\frac{s}{2}} + X_{t-\frac{s}{2}+1} + ... + X_{t+\frac{s}{2}-1} + \frac{1}{2}X_{t+\frac{s}{2}}\right), \quad t = \frac{s}{2}+1, \ldots, N-\frac{s}{2}.$$

Finally, the standardized estimates of the seasonal components are:

$$\hat{S}_i = \bar{S}_i \cdot \frac{1}{\sum\limits_{j=1}^{s} \bar{S}_j} \ , \qquad 1, 2, \ldots, s.$$

Using these formulas it is possible to estimate the seasonal indexes of the demand and sales (see Figure 4).

Similarly, the seasonal coefficients presented in Table 1 were obtained. The computations of the seasonal coefficients were implemented in an Excel file, depicted in Figure 15 in the Appendix.

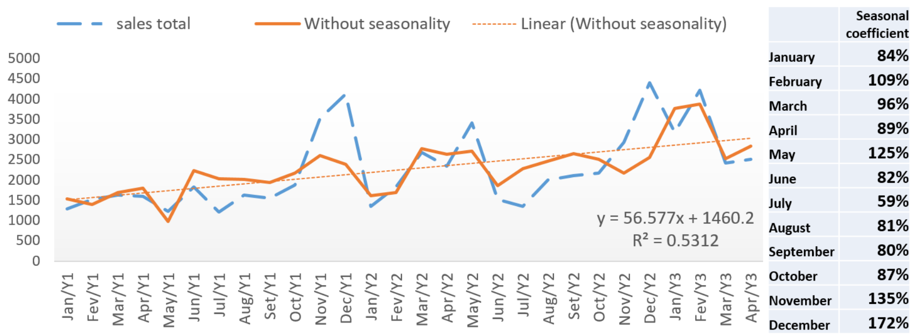


Figure 4: PRIO ENERGY sales (in tons).

The national coefficients (PT butane, PT propane, PT total) do not have significant variations from month to month. In fact, for the national sales, the PT butane seasonal coefficients present a minimum of 84% in October and a maximum of 111% in May, while for PT propane and PT total the minimum is attained in September with 92% and 91%, respectively, and the maximum occurs in April with 111% and 110%.

On the other hand, PRIO ENERGY sales show more significant variations. The seasonal coefficients of the total sales vary from 59% in July to 172% in December, while for type A cylinder from 55% in July until 163% in December, and finally, type B cylinder sales vary from 65% in July to 183% in December (see Table 1). The months with high coefficients (above 100%) are February, May, November and December, and the ones with smallest coefficients (below 100%) are June, July, August and September whose correspond to summer season.

Table 1: Seasonal coefficients of LPG sales

	PT			PRIO ENERGY		
Month	butane	propane	total	total	type A	type B
January	100%	99%	99%	84%	96%	69%
February	102%	95%	97%	109%	110%	108%
March	103%	93%	96%	96%	95%	98%
April	107%	111%	110%	89%	92%	84%
May	111%	110%	110%	125%	121%	132%
June	96%	98%	98%	82%	91%	70%
July	109%	98%	101%	59%	55%	65%
August	101%	98%	99%	81%	69%	97%
September	90%	92%	91%	80%	74%	87%
October	84%	109%	102%	87%	91%	81%
November	88%	99%	96%	135%	144%	124%
December	108%	98%	101%	172%	163%	183%

Therefore, forecast for sales can be done using these seasonal coefficients.

4.1.2 Exponential smoothing forecast

Exponential smoothing forecast is a widely used method for time series forecast, including sales forecasting [14, 15, 6].

Using the seasonal coefficients obtained in the previous subsection to desseasonalize the data, then Holt's method was used to forecast type A bottles sales. The model obtained using Holt's method [27] was such that: AIC = 671.9332, BIC = 678.5942, RMSE = 25697.11 for the training set, level coefficient $\alpha = 0.0253$ and trend coefficient $\beta = 0.0253$. Further details of this model are depicted in Table 2). In Figure 5, the observed values of the type A bottles sales (from Jan/Y0 to Apr/Y2) are presented alongside with the estimated values (from Jan/Y0 to Apr/Y2), and the forecast sales for May/Y2 until Dec/Y3.

Table 2: Details of the model for type A bottles sales obtained using Holt's method.

sigma:	25697.11						
	AIC	AICc	BIC				
	671.9332	674.6605	678.5942				
Error measures:							
	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training set	-4739.01	25697.11	18961.96	-6.6708	14.9323	0.71456	0.05174

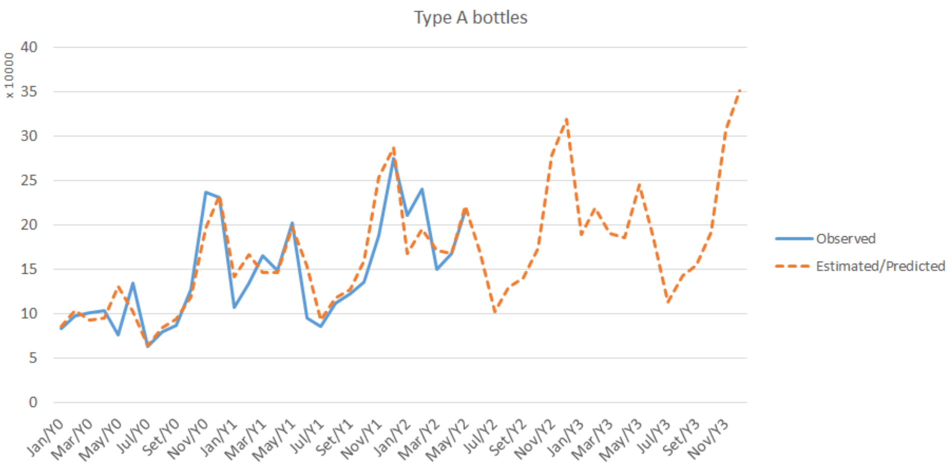


Figure 5: Forecast of type A bottle sales using Holt's method.

4.2 Regression Models

Data provided by PRIO Energy concerning sales from January 2015 to April 2017 was used to estimate the regression models for:
(i) total amount of propane gas sold by PRIO;
(ii) the number of type A propane gas bottles national sales;
(iii) the number of type A propane gas bottles Aveiro sales;
(iv) the number of type A propane gas bottles Lisbon sales;
(v) the number of type B propane gas bottles.

Subsequently these models were used to forecast the company sales for the period from May 2017 to December 2018.

The predictor variables that were considered in some of the previous mentioned models are

- `Temperature`: Temperature, in °C;
- `Prio_A_t_1`: Number of type A bottles sold on the previous month;
- `Prio_B_t_1`: Number of type B bottles sold on the previous month;
- `PRIOPromotionalCampaign` – PRIO's Promotional Campaign– 0 if no promotional campaign or 1 if there is a promotional campaign;
- `PRIO_SalesObjective` – PRIO's Sales Objective – 0 for no or 1 for yes;
- `ExpectationOfPriceIncrease` – PRIO's Expectation Of Price Increase – 0 for no or 1 for yes;
- `t` – Time, in months, since January 2015;
- `Wind` – the monthy mean Wind velocity (km/h);
- `Month` – Month with Month= 1 for January, ..., Month= 12 for December.

Table 3: Model Summary

Model	R	R^2	R^2_a	Std. Error of the Estimate
1	0.590 a.	0.349	0.324	785339.989
2	0.732 b.	0.537	0.499	675572.838
3	0.804 c.	0.647	0.603	601889.927
4	0.850 d.	0.723	0.675	544305.423

a. Predictors: (Constant), PRIO_SalesObjective
b. Predictors: a. + ExpectationOfPriceIncrease
c. Predictors: b. + PRIO_PromotionalCampaign
d. Predictors: c. + Temperature

Model for total sales of propane gas

The best adjusted model, whose details are shown in Table 4.2, has $R_a^2 = 0.675$, therefore 67.5% of the variability of the total sales of propane gas, in tones, is explained by `PRIO_SalesObjective`, `ExpectationOfPriceIncrease`, `PRIO_PromotionalCampaign`, `Temperature` and `Month`.

Table 4: Estimated coefficients of the regression model for total sales of propane gas, in tones.

	Unstandardized Coefficients		Standardized Coefficients		
Model	B	Std. Error	Beta	t	Sig.
(Constant)	2510736.027	268427.634		9.353	0.000
PRIO_SalesObjective	1414770.415	469878.578	0.389	3.011	0.006
ExpectationOfPriceIncrease	2347781.668	509504.229	0.465	4.608	0.000
PRIO_PromotionalCampaign	971928.126	306322.579	0.321	3.173	0.004
Temperature	-74832.797	19798.401	-0.465	-3.780	0.001

From Table 4, an increase in the mean temperature there is a decrease on the propane gas sales. While the existence of PRIO Sales Objective, Expectation of Price Increase or PRIO Promotional Campaign, yields an increase on the propane gas sales. Also for an increase in time (months) there is an increase on the propane gas sales.

National type A bottles sales

In terms of the national sales of type A bottles different regression models were tested. The best model found was:

$$\begin{aligned} Prio\_A\_number &= 140068.9 - 4351.5\,Temperature \\ &+86935.7\,PRIO\_PromotionalCampaign \\ &+99561.8\,PRIO\_SalesObjective \\ &+59204.7\,ExpectationOfPriceIncrease \\ &+3439.3\,t + \varepsilon \qquad (1) \end{aligned}$$

where `Prio_A_number` represents the number of assets of type A. The ajusted coefficient of determination was $R_a^2 = 0.902$, that is 90.2% of the variability of the number of bottles of type A is explained by the variables that are included in the model. The application of regression analysis is based on the normality assumption of errors (ϵ), therefore in order to validate this assumption some normality statistical tests were applied. From Shapiro-Wilk normality test, the null hypothesis of normal distribution of the residuals is not rejected (W = 0.98115, p-value = 0.877). Also from Figure 6, the model from Eq. 1 can be considered adequate.

Having into account the linearity of model in Eq. 1, one can interpret each one of its parameters, assuming the others fixed. Therefore:

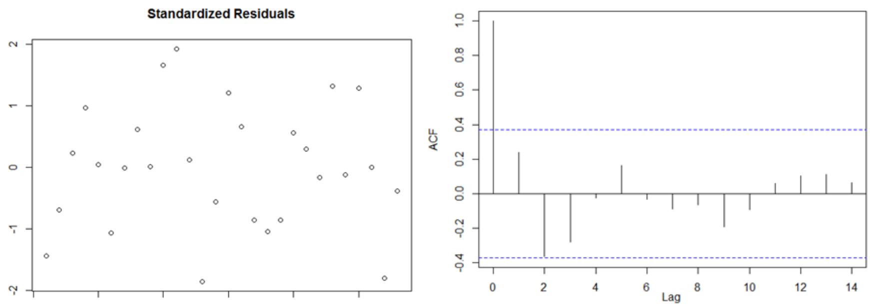


Figure 6: Analysis of the residuals of the model for national type A bottles sales (Eq 1).

- an increase of 1° of mean temperature will result in decrease of the sale of type A bottle by approximately 4352 units;
- when PRIO has a promotion the units of type A bottle increase of sales of approximately 86936
- when sales objectives are set there is an increase of sales of approximately 99561.8;
- whenever there is expectation of prices increase there is an increase of sales of approximately 59204.7 units;
- for every month that passes after January 2015 there is an increase on the sales of approximately 3439.3 units.

Figure 7 presents, in black, the real observed number of type A propane gas bottles from January 2015 to May 2017 and the company's estimates from June to December 2017, while in red are presented the estimates from January 2015 to May 2017 obtained with model in Eq. 1. The existence of promotional campaigns presents a relatively high impact on the sales. For this reasons, two different scenarios are consider. In scenario 1 the existence of the company's promotional campaigns in June and November, is considered. While in scenario 2 it is assumed that no promotional campaigns occur.

The estimated values of the type A bottles sales closely replicate the behavior of the past observed sales. For both scenarios the forecast obtained using the regression model in Eq. 1 presents lower values than the ones presented by the company, therefore the company expectation of growth is larger than the one predicted by the proposed regression model (Figure 7 and Table 5). Since we do not have any information concerning the company growth strategy. Perhaps the company is aiming at further promotional campaigns or other actions that are expected to increase the volume of sales.

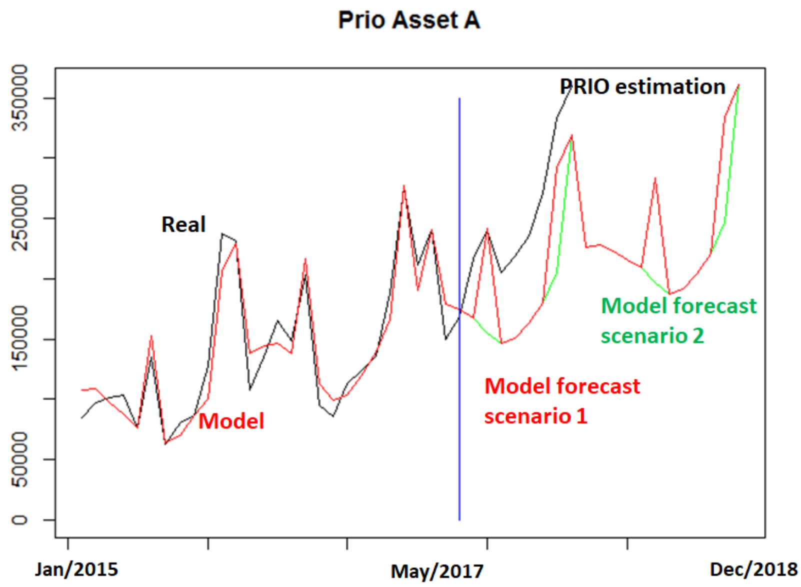


Figure 7: Observed and estimated sales, and forecast of the national type A bottles sales using the model from Eq. 1, for scenarios 1 and 2. See text for further details.

PRIO's Regional sales

The previously presented models concern the national sales without any regional characteristics consideration. Nevertheless, the working group considers that the model should considered regional aspects. In fact, for example, in terms of the air temperature there are large variations within the Portuguese territory. Also, variables such as number of inhabitants per region, the fact that region has a high number of seasonal visitant such as tourists, the existence of local promotional campaigns, should be consider.

In addition to the previous possible predictors consider previously, for the estimation of the regional regression models the mean, minimum and maximum temperatures in Lisbon and Aveiro were consider.

Next, as a preliminary study, regression models for the region of Lisbon and Aveiro are consider. Figure 8 presents a summary of the regional models. It is also possible to compare the two region models with the national model. For the regional and national models the existence of promotional campaigns and the number of months since January 2015 present a positive impact on the sales of type A bottles. For the national and Lisbon models, the existence of PRIO_SalesObjetive yields an increase on the number of type A bottles,

Table 5: Predicted values obtained by the regression models (for scenario 1 and 2) and the company's sales expectation.

Year	Month	Model forecast Scenario 1	Scenario 2	Company's Forecast
2017	May	167693	167693	216545
	June	**241891**	**154956**	**239624**
	July	146102	146102	204792
	August	150640	150640	218204
	September	162935	162935	235039
	October	179526	179526	270725
	November	**292453**	**205517**	**333270**
	December	319451	319451	360066
2018	January	225896	225896	
	February	228135	228135	
	March	223184	223184	
	April	215925	215925	
	May	208965	208965	
	June	283163	196227	
	July	187373	187373	
	August	191912	191912	
	September	204206	204206	
	October	220798	220798	
	November	333724	246789	
	December	360723	360723	

while for the Aveiro region it is not significant. Expectation of prices increase has a positive impact on the sales, only in national terms, however this is not significant for the Lisbon and Aveiro region. At national terms the mean temperature presents a significant and positive impact on the sales. For the Lisbon models it is the maximum temperature that presents a positive impact, while for Aveiro is the minimum temperature.

National type B bottles sales

In terms of the national sales of type B bottles different regression models were tested. The best model found was:

$$\begin{aligned} Prio\_B\_number \;=\;& 11821.9 + 9527.0\, PRIO\_PromotionalCampaign \\ & +23969.1.8\, PRIO\_SalesObjective \\ & +21849.3\, ExpectationOfPriceIncrease \\ & +467.1\, t + \varepsilon \end{aligned} \qquad (2)$$

where `Prio_B_number` represents the number of assets of type B. The ajusted coefficient of determination was $R_a^2 = 0.766$, that is 76.6% of the variability of the number of bottles of type B is explained by the variables included in model. of Eq. 2. In this model the temperature revealed not

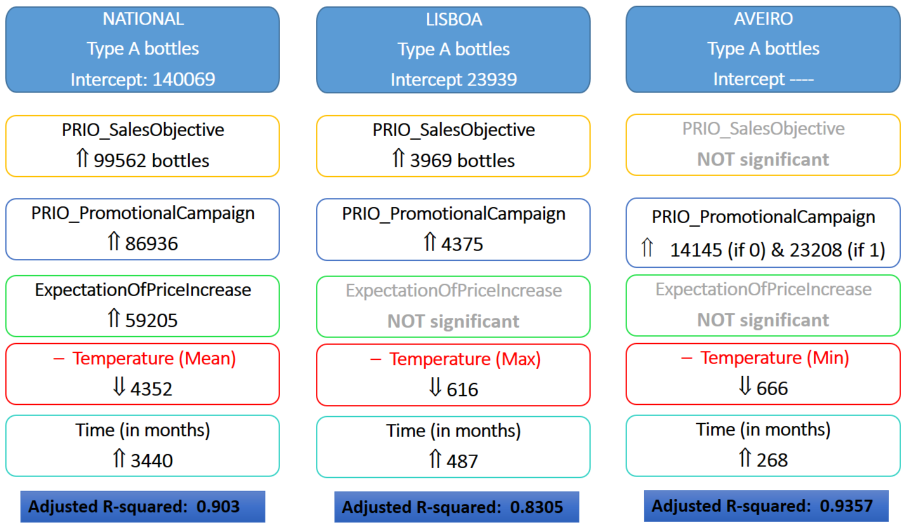


Figure 8: Comparison of the regression models for sales at national level (on the left), sales on the Lisbon area (on the midle) and on the region of Aveiro (on the right).

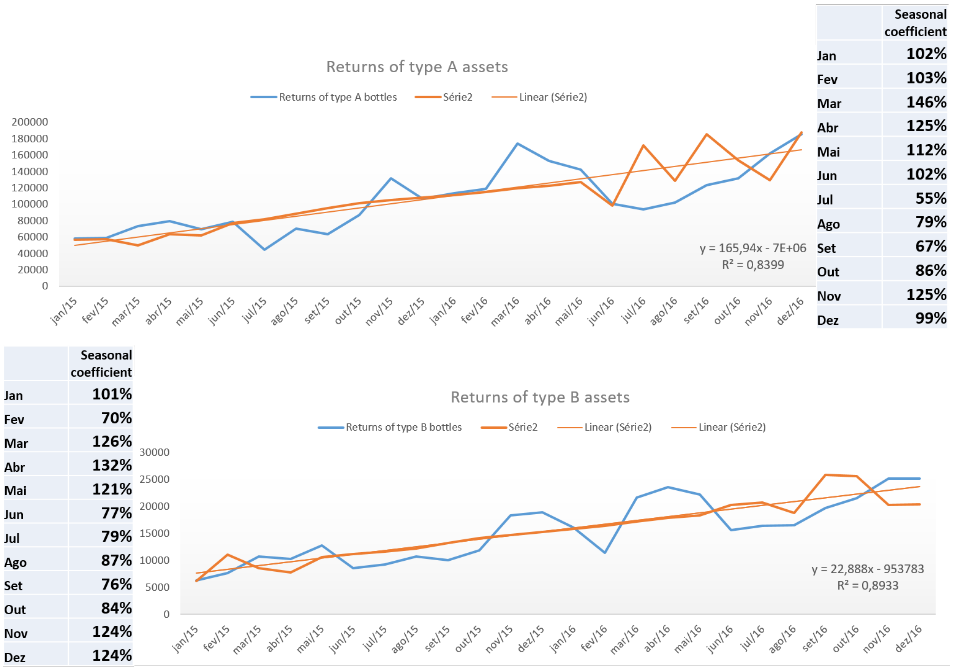
to be significant, which can be due to the fact that these kind of bottles are used mostly in industries and not so dependent on the temperature of the month. The analysis of residuals were also made and their normality distribution was not rejected through the application of the Shapiro-Wilk test ($W = 0.958$, p-value=0.324). In terms of forecasting sales for assets B, the model in Eq. 2 can be used to construct several scenarios that have into account the existence or not of sales promotional campaigns, sales objective and also the expectation of price increase.

5 Return rate forecast

To correctly plan the acquisition of new bottles from the supplier, we not only need to know the demand, but also the reverse logistic flows. The empty bottles being returned to PRIO can be reinserted in the system, filled again and sold to the clients. As the acquisition of new bottles is expensive, reusing is the key. This Section is devoted to understand the fluctuation of returns and to forecast the return rate.

5.1 Time series models

Similar to Section 4, we can calculate the seasonal coefficients for returns of type A and B bottles. Figure 9 contains these values.



	Seasonal coefficient
Jan	102%
Fev	103%
Mar	146%
Abr	125%
Mai	112%
Jun	102%
Jul	55%
Ago	79%
Set	67%
Out	86%
Nov	125%
Dez	99%

	Seasonal coefficient
Jan	101%
Fev	70%
Mar	126%
Abr	132%
Mai	121%
Jun	77%
Jul	79%
Ago	87%
Set	76%
Out	84%
Nov	124%
Dez	124%

Figure 9: Forecast of type A and B bottles returns using only the seasonal coefficients

We implemented these models in an Excel file, which is depicted in Figure 16 in the Appendix.

The obtained seasonal coefficients are in Table 6. In the summer months the returned bottles are usually well below average, while in the beginning of Winter and in Easter the returned bottles have its peak.

Table 6: Seasonal coefficients for the returned bottles.

Month	typeA returns	typeB returns
January	102%	101%
February	103%	70%
March	146%	126%
April	125%	132%
May	112%	121%
June	102%	77%
July	55%	79%
August	79%	87%
September	67%	76%
October	86%	84%
November	125%	124%
December	99%	124%

5.2 Regression Models

The best regression model obtained for Return of type A bottles, at national level, is the following:

$$ReturnA = 20473 + 0.3584PRIO\_A\_number + 2944t + \varepsilon \qquad (3)$$

Approximately 79.9% of the variability of the Return of type A bottles is explained by PRIO_Sales and time in months from January 2015.

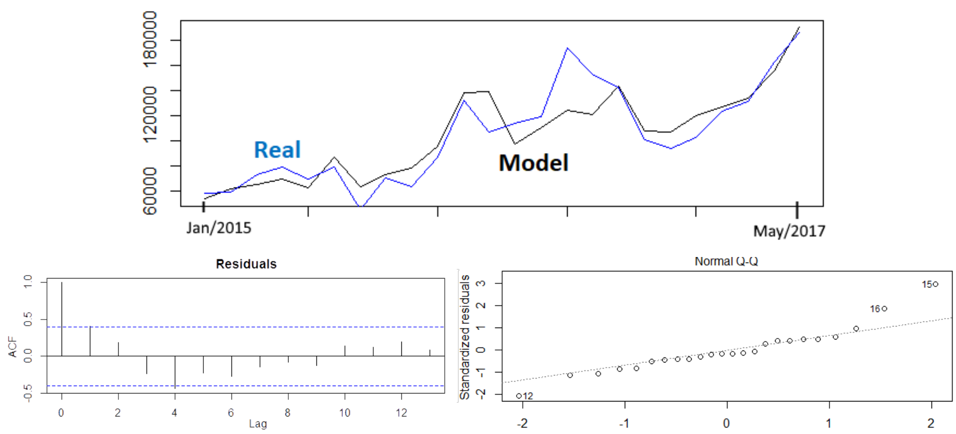


Figure 10: Return rate of type A bottles.

In terms of Return of type B bottles, at national level, the best regression model obtained was:

$$ReturnB = 5956.25 + 0.2971PRIO\_B\_number + 582.5117t + \varepsilon. \qquad (4)$$

Although Prio_SalesObjetive variable do not present significant parameter at a 5% level, they have a negative effect (-5095.82) in terms of the returned bottles of B if we assume a 10% level. Approximately 81.56% of the variability of the Return of type B bottles is explained by PRIO_Sales and time in months from January 2015.

6 Inventory models with reverse logistics

The assets acquisition plan should determine when to order to the external supplier new LPG bottles (Order Point) and how many should be bought (batch size). Classical inventory models such as the Wilson model determine the Economic Order Quantity (EOQ) as the batch size that minimizes the total cost of stock management. This plan should take in account the empty bottles that are returned to PRIO, which can be either reused or disposed of. Therefore we started by applying to the PRIO data two inventory models with reverse flows found in literature. The forecasts computed in the previous sections are here used as the input values of demand and return rate.

6.1 Inventory models from Richter and Teunter

The inventory model developed by Richter [18] is based on the classical EOQ formula, but accommodates the reverse logistic flow of items that can be recovered. Figure 11 illustrates the dynamics of the Richter inventory model, using two shops with three production setups and one repair setup.

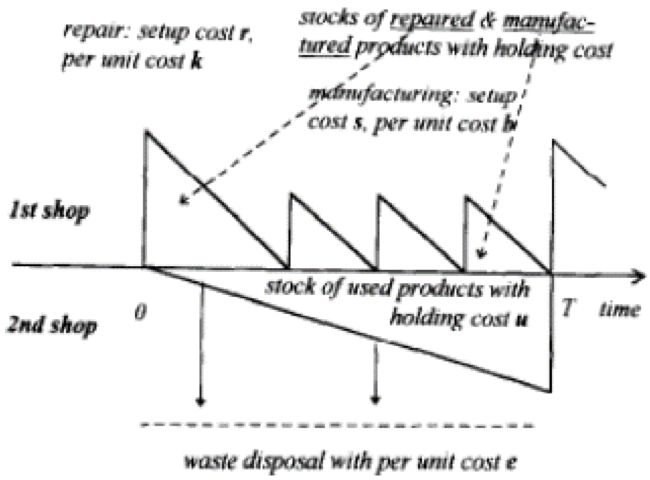


Figure 11: Inventory stock model for two shops with $m = 1$ and $n = 3$, according to Richter model [18]

This model considers deterministic demand and return rates, and also a constant disposal rate. Different holding costs at first and second shop are considered in the formula. In this model, the returned items may be either reused or disposed off.

The formula for computing the total cost per unit of time is:

$$K = (mr + ns)\frac{d}{x} + \frac{x}{2}\left[h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta + \frac{u\beta^2(m-1)}{m}\right] \tag{5}$$

And for computing the optimal batch size:

$$x(\alpha) = \begin{cases} \sqrt{\frac{2ds}{\alpha^2 h + \beta u + \beta^2 u}}, & \alpha \in I \\ \sqrt{\frac{2d(r+s)}{(\alpha^2+\beta^2)h + u\beta}}, & \alpha \in J \\ \sqrt{\frac{2dr}{\beta(\beta h + u)}}, & \alpha \in L \end{cases} \tag{6}$$

where

- d – constant demand rate (units/unit of time)
- r – repair fixed cost
- s – production fixed cost
- b – manufacturing unit cost
- k – repairing unit cost
- e – disposal unit cost
- h – holding cost per unit per unit time at shop 1 (where the production occurs)
- u – holding cost per unit per unit time at shop 2 (where the repair occurs)
- x – total lot size
- T – collection interval (cycle time), $T = \frac{x}{d}$
- α – disposal rate, $\alpha = 1 - \beta$
- β – repair rate (equivalent to the recovery rate of Schrady[21])
- n – number of production setups
- m – number of repair setups
- K_2 – total cost of EOQ-related cost factors for time interval
- K – total cost per time unit for the producer, $K = \frac{K_2}{T}$
- R – linear production, waste disposal, and repair costs per unit time (non-EOQ related cost factors)

- G – overall cost per time unit, $G = K + R$
- I – lower waste disposal rates
- J – median waste disposal rates
- L – higher waste disposal rates

This model was implemented in an Excel file, depicted in Figure 17 in the Appendices.

The inventory model developed by Teunter [23] is also based on the EOQ formula, and considers the return flow of items that can be recovered. It also uses deterministic demand and return rates. The difference is that this model considers a varying disposal rate instead of a constant rate. Different holding costs for manufactured, recoverable and recovered items can be considered separately. Returned items may also be either reused or disposed of. In this model, M manufacturing batches and R recovery batches succeed each other, but there can be only two cases: either $M = 1$ or $R = 1$.

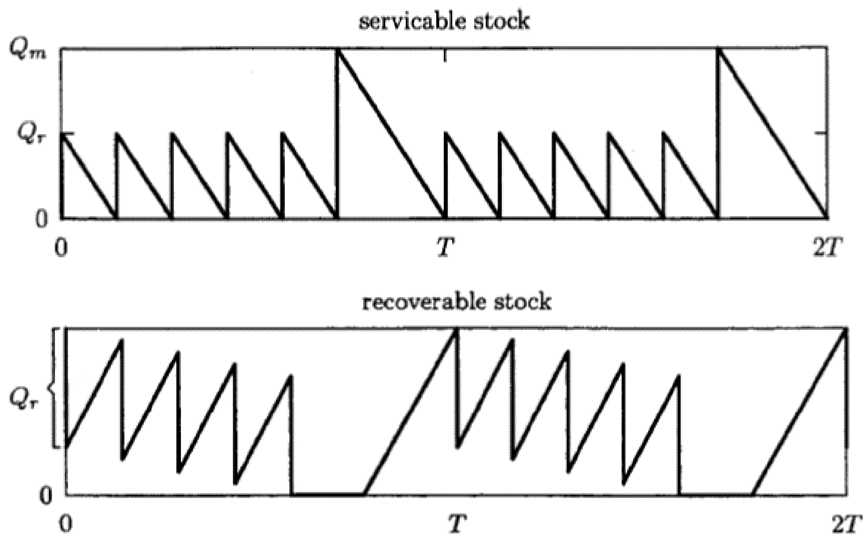


Figure 12: Inventory stock model according to Teunter model [23]

The formula for computing the total cost per unit of time (case M=1) is:

$$TC_{M=1} = \frac{K_m\lambda(1-\beta)}{Q_m} + \frac{K_r\lambda\beta}{Q_r} + h_r\frac{1}{2}\beta Q_r + \\ + h_m\frac{1}{2}(1-\beta)Q_m + h_n\frac{1}{2}\left(\beta Q_r + \left(\beta - \frac{g-\beta}{1-\beta}.\frac{\beta}{g}\right)Q_m\right) + \\ + \lambda\left((1-\beta)c_m + \beta c_r + (g-\beta)c_d\right) \tag{7}$$

The formulas for computing the optimal batch size for manufacturing Q_m and for recovery Q_r are, respectively:

$$Q_{m_{M=1}} = \sqrt{\frac{2K_m\lambda(1-\beta)}{h_n(1-\beta) + h_n(\beta - \frac{g-\beta}{1-\beta}\frac{\beta}{g})}} \tag{8}$$

$$Q_{r_{M=1}} = \sqrt{\frac{2K_r\lambda}{h_r + h_n}} \tag{9}$$

The number of recovery batches is:

$$R = \frac{\beta}{1-\beta}\frac{Q_{m_{M=1}}}{Q_{r_{M=1}}} \tag{10}$$

where:
λ – demand(continuous and deterministic)
g – return percentage ($0 < g < 1$)
β – reuse/recovery percentage($0 < \beta < 1$)
λg – items returned
$\lambda\beta$ – items reused, where the units disposed are $\lambda(g-\beta)$
t – continuous time variable
c_m – cost of manufacturing an item
c_r – cost of recovering an item
c_d – cost for disposal of an item
K_m – setup cost for manufacturing

This model was implemented in an Excel file, depicted in Figure 18 in the Appendices.

6.2 Inventory model developed for PRIO based on Wilson model

The models presented before do not contemplate all the specifications required in this case study. In the PRIO case, the returned items can have three different destinations. Most of the returned LPG bottles (98%) only need cleaning, and some of them (about 2%) need requalification. At the moment, because this business is relatively new for PRIO, there is no LPG bottles that need to be disposed of, but in the future this situation can also

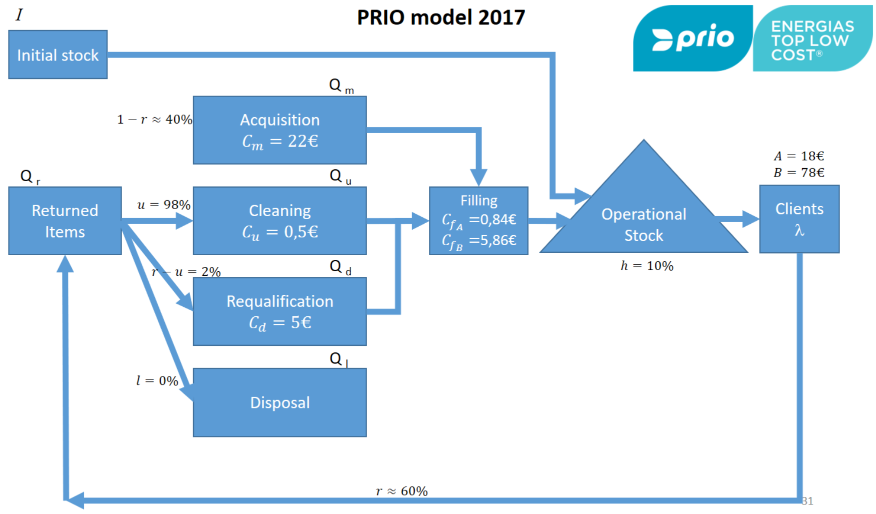


Figure 13: Inventory stock model developed

occur. The costs and time for each of these processes are different. The process is depicted in Figure 13.

As in the classical EOQ formula, in this model the total costs considered are the sum of the acquisition costs C_A, setup costs C_S and holding Costs C_H.

The acquisition costs in Equation 11 consider the cases where new bottles are acquired from the supplier with a cost C_m, the bottles are reused with just a cleaning cost C_u, or the case where the returned bottles have to be requalified with a cost C_d. In this three cases, a constant filling cost is also included. In the future, a disposal cost C_l could also be considered.

In this case study we consider that the rate of bottles returned and disposed of (l) is zero.

$$C_A = C_m(1-r)(\lambda - I) + C_u u(\lambda - I) + C_d(r-u)(\lambda - I) \tag{11}$$

where

- λ is the constant demand rate (units/units of time)
- I is the initial stock
- r is the return rate

- u is the rate of bottles returned and cleaned

- $d = r - u$ is the rate of bottles returned and requalified.

The setup costs are:

$$C_S = \frac{K_m(\lambda - I)(1-r)}{Q_m} + \frac{K_u(\lambda - I)u}{Q_u} + \frac{K_d(\lambda - I)(r-u)}{Q_d} \tag{12}$$

where

- K_m is the production fixed setup costs
- K_u is the reuse fixed setup costs
- K_d is the requalification fixed setup costs
- Q_m is the batch size for buying new bottles
- Q_u is the batch size for reusing bottles
- Q_d is the batch size for requalifying bottles

The holding costs:

$$C_H = h_m\frac{(1-r)Q_m}{2} + h_u\frac{uQ_u}{2} + h_d\frac{(r-u)Q_d}{2} + h_i\frac{I}{2} \tag{13}$$

where

- h_m is the holding cost per new item bought per year
- h_u is the holding cost per reused item per year
- h_d is the holding cost per requalified item per year
- h_i is the holding cost per existent item in stock per year

By minimizing the total costs, it is possible to obtain the expression for the optimal quantities Q_m^*, Q_u^* and Q_d^*. The optimal batch size for buying new bottles is:

$$Q_m^* = \sqrt{\frac{2K_m(\lambda - I)}{h_m}} \tag{14}$$

The optimal batch size for reuse is:

$$Q_u^* * = \sqrt{\frac{2K_u(\lambda - I)}{h_u}} \tag{15}$$

For requalification, the optimal batch size is:

$$Q_d^* * = \sqrt{\frac{2K_d(\lambda - I)}{h_d}} \tag{16}$$

This model was implemented in an Excel file, depicted in Figure 19 in the Appendices.

6.3 Inventory model developed for PRIO based on continuous replenishment

The previous model considered that the all the acquired, and returned bottles arrive at discrete moments periodically in time, but actually that only happens with the acquired bottles. The returned bottles arrive continuously to PRIO wharehouse, and are continuously cleaned and requalified and filled. Therefore, a continuous replenishment model could be adapted to this case study.

As depicted in Figure 14, this kind of models considers that there is a period T_1 where there is simultaneously continuous replenishment of bottles (with rate u) and demand being satisfied (with rate λ), and a period T_2 where replenishment is interrupted and there is only demand being satisfied.

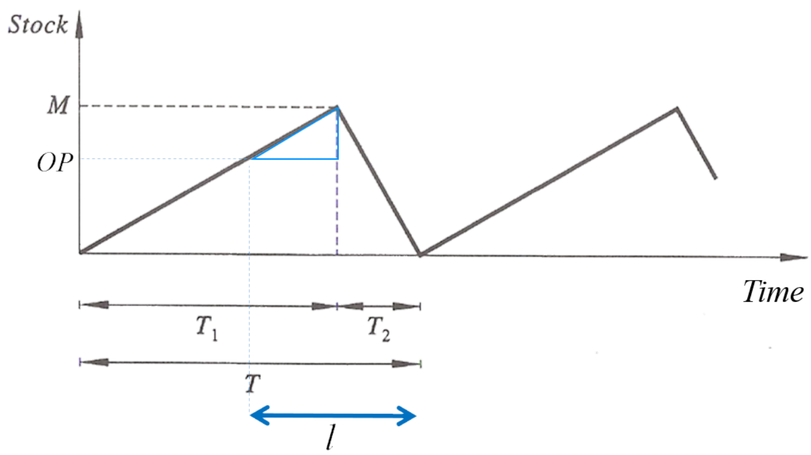


Figure 14: Inventory stock model for continuous replenishment

Therefore, from the slopes of the main triangles, we have Equations 17:

$$T_1 = \frac{M}{u - \lambda} \qquad T_2 = \frac{M}{\lambda} \tag{17}$$

where M is the maximum stock level:

$$M = Q - \lambda \cdot T_1 = Q\left(1 - \frac{\lambda}{u}\right) \tag{18}$$

and the batch size corresponds to the total production during period T_1, i.e., $Q = uT_1$.

The total costs are given by:

$$CT(Q) = C_a D + C_s \frac{D}{Q} + C_h \frac{Q}{2}\left(1 - \frac{\lambda}{u}\right) \tag{19}$$

being D the demand for the planning horizon (year) and λ the daily demand and p the daily production.

By minimizing the total costs, the optimal quantity is given by:

$$Q^* = \sqrt{\frac{2C_s D}{C_h}}\sqrt{\frac{u}{u-\lambda}} \tag{20}$$

If the lead time l is longer than the period of demand ($l > T_2$) then from the slope in the blue triangle in Figure 14 we can derive the formula 21.

$$\frac{M - OP}{l - T_2} = u - \lambda \Leftrightarrow OP = M - (u - \lambda)(l - T_2) \tag{21}$$

Then, simplifying Equation (21) and replacing M using the Equation (17) and T_2 using Equation (18), we can obtain the order point OP as a function that depends only on the quantity of bottles Q, the demand rate and reutilization rate, and lead times:

$$OP = Q\left(1 + \frac{u}{\lambda}\right) + l\,(\lambda - u) \tag{22}$$

7 Conclusions and recommendations

The goal of this work was to develop a model for the LPG assets acquisition planning. For that purpose, it is necessary to forecast the sales of propane gas cylinders, and use it to plan the assets acquisition necessity.

First, time series techniques, namely exponential smoothing and moving averages, were used to compute the seasonal coefficients and to forecast the demand and the number of returned bottles.

This approach allowed to see that the national seasonal coefficients are quite distinct from the ones observed for the sales of the company. PRIO ENERGY's sales present a larger variability in the seasonal coefficients than the total national sales. For the company the higher coefficients were observed in May, November and December, while the smaller were in July.

Since national and PRIO ENERGY seasonality coefficients are different, some other possible explanatory variables should be considered in order to forecast the demand with better accuracy. For that reason, several data has been collected, such as atmospheric temperatures, demand in previous periods, objectives of sales, expectation of price increase, and among others variables. Taking into account this data, for the period from January 2015 to May 2017, multiple regression models were estimated for the total sales of propane, for the number of type A and B bottles at a national level and only for Lisbon and Aveiro regions. These models were used to forecast the values for the period from June 2017 to December 2018. Similar procedure was followed to forecast the number of returned bottles.

Two tailored-made inventory models, based on the discrete EOQ formulas with reverse logistic from Richter [18] and Teunter [23] were developed for the PRIO case and implemented in an Excel file. A continuous replenishment inventory model was also proposed for this case study to deal with the continuous return flow of bottles.

We did not use Neural Networks or Geostatistical models, but it is a possible approach for forecasting demand and sales. We also did not use SVARIMA model due to the limited number of observations.

A possible broader approach would be to adapt the closed loop supply chain model with reverse flows from [2, 20] that can simultaneously model the demand and return and acquisition plan.

All the inventory models presented were deterministic, using a constant demand rate. If the company considers that the demand or return rate has a stochastic nature, then non deterministic inventory models should be adapted instead. The working group considers that developing a model for a mixed scenario, regarding stochastic demand and return rate, with continuous returns and periodic discrete acquisition of new bottles from the supplier, would be an approach that would better fit the particularities of the challenge proposed by PRIO.

Acknowledgements

This work was partially supported from COST Action TD1409, Mathematics for Industry Network (MI-NET), supported by COST (European Cooperation in Science and Technology).

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A Appendix

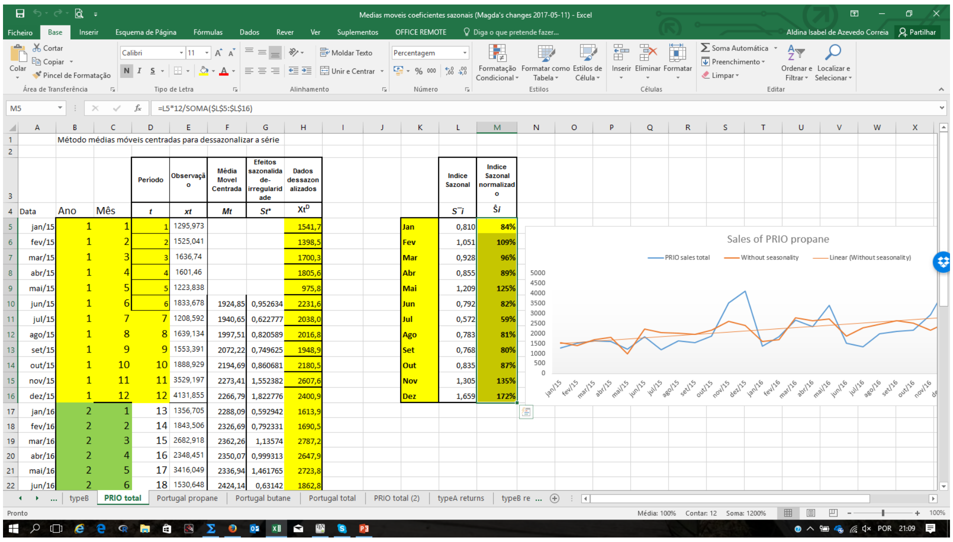

Figure 15: Implementation of the time series model developed for PRIO sales

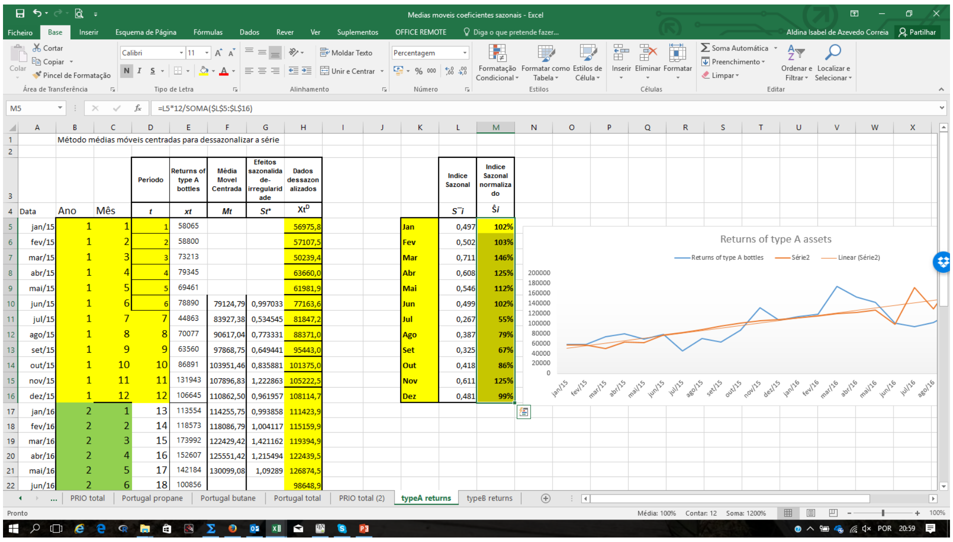

Figure 16: Implementation of the time series model developed for PRIO returned bottles

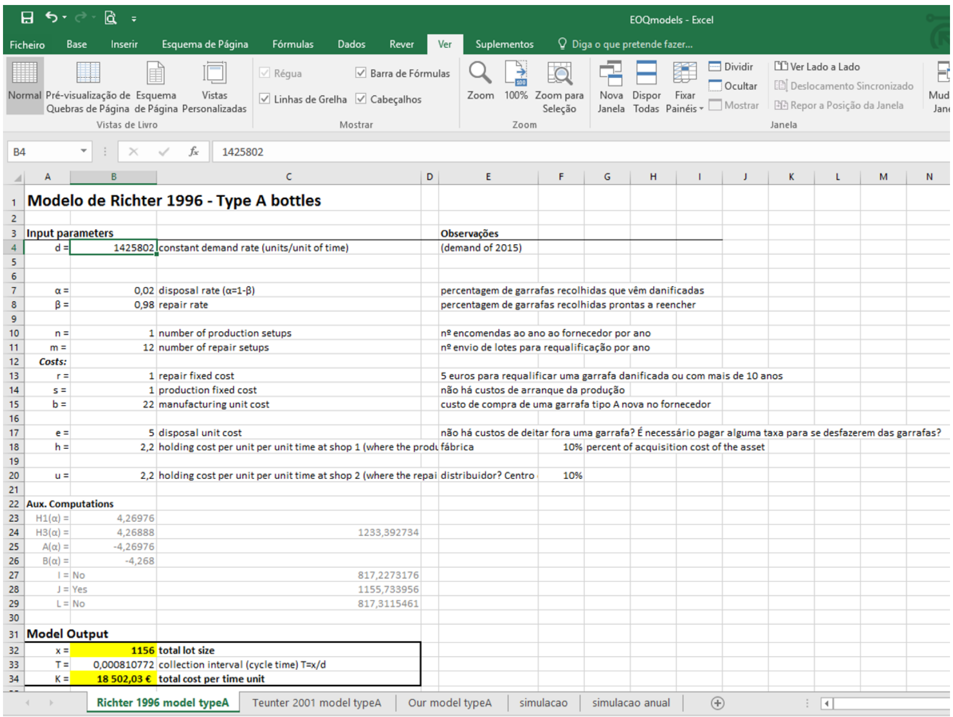

Figure 17: Implementation of the Richter inventory model

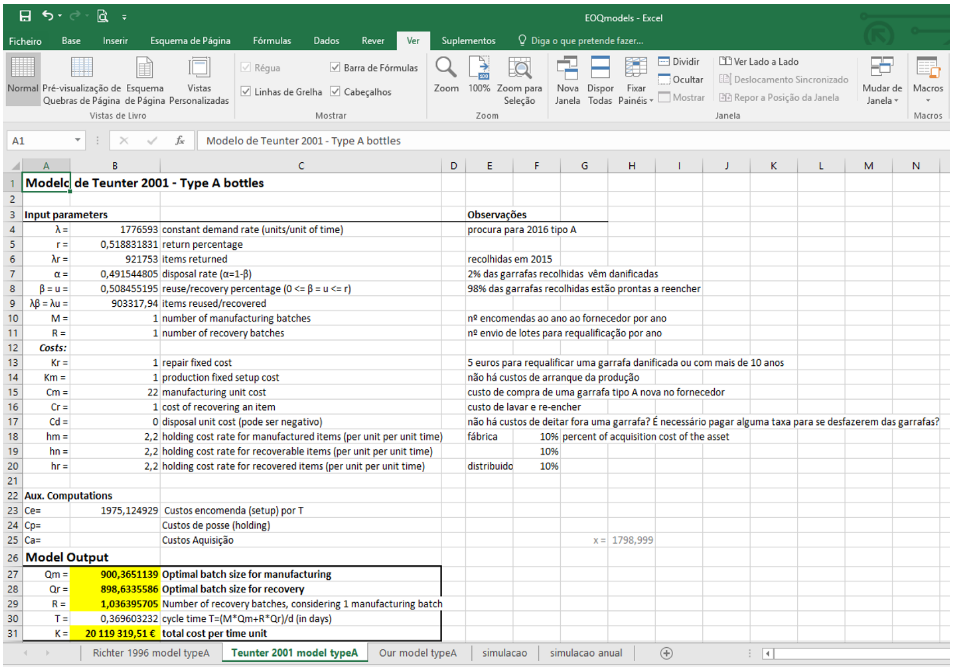

Figure 18: Implementation of the Teunter inventory model

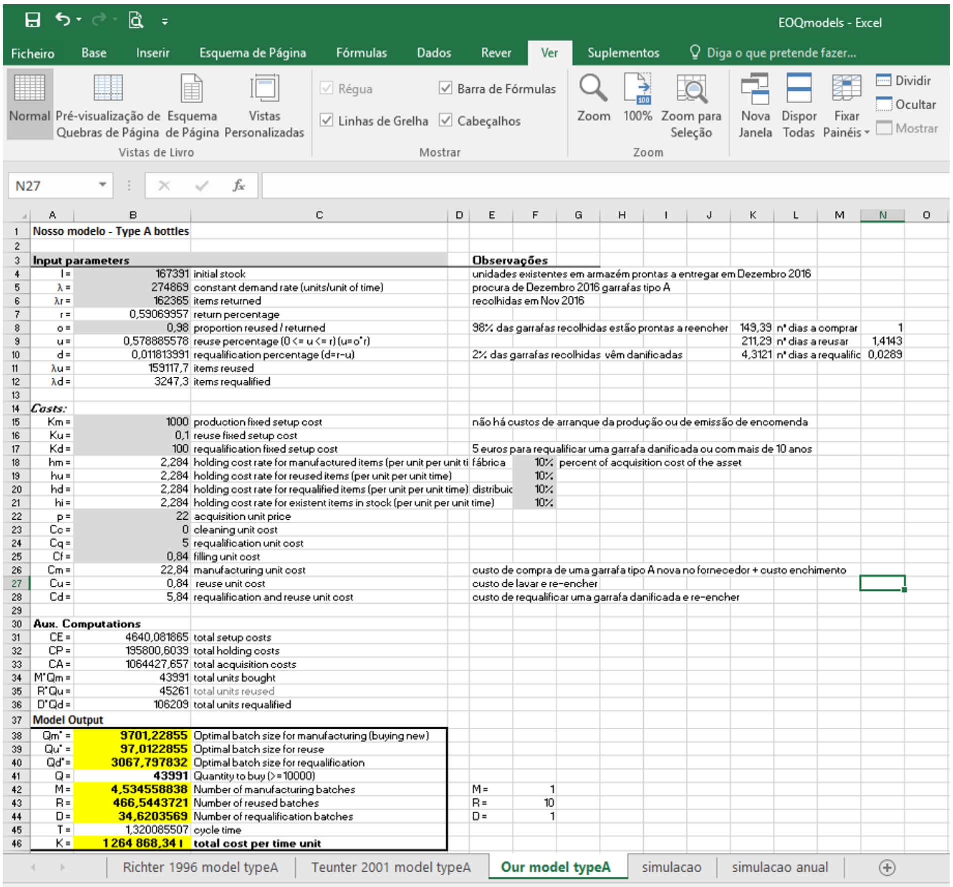

Figure 19: Implementation of the inventory model developed for PRIO